

GRADE

12

CAPS

3 in 1

Mathematical Literacy

Susan Nicol, *et al.*

Maths Literacy 3 in 1 includes the following

- Notes and Worked Examples
- Questions per Topic
- Detailed Answers with Explanations

Published by
THE ANSWER
50 Imam Haron Road
Claremont, 7708
T: (021) 671 0837
F: (021) 671 2546

info@theanswerseries.co.za

www.theanswer.co.za

Printed and bound by CTP Printers

2015 publication
ISBN: 978-1-920568-70-2

IT IS ILLEGAL TO PHOTOCOPY ANY PAGES FROM THIS BOOK
WITHOUT THE WRITTEN PERMISSION OF THE PUBLISHER.



the
ANSWER
series...your key to exam success

PUBLICATIONS & ACKNOWLEDGEMENTS

ABOUT THIS PUBLICATION

The Answer proudly offers this new Answer Series publication in *Mathematical Literacy*. Susan Nicol wrote this study guide according to the National Curriculum and Assessment Policy Statement for Mathematical Literacy. Her extensive experience and expertise contributed to this successful learning programme.

WHO IS THE ANSWER?

The Answer is an educational publishing company based in Cape Town that specialises in producing study guides for high school learners throughout South Africa. Established in 1975, we are constantly expanding our range of guides and updating the content to stay abreast of curriculum changes.

The guides are designed to enrich the understanding of learners and provide essential exam technique and practice. Learners can work independently, or the guides can be used in the classroom. Written by subject specialists, they provide invaluable support for all learners wanting to make a success of their matric exams.

ACKNOWLEDGEMENTS

On a personal note, our sincere thanks go to:

- Kirsten Whiting for her valuable contribution to this book
- The DTP team who has worked creatively and tirelessly on this publication
- Olivia Rumble for her many cheerful creations
- The production team for their dedication and willingness to meet deadlines

YOUR PARTICIPATION!

Visit our website and submit your suggestions, comments and criticism. We rely on these as we aim to provide the best material possible.

COPYRIGHT



THE WHOLE OR ANY PART OF THIS PUBLICATION MAY NOT BE REPRODUCED OR TRANSMITTED BY ANY MEANS WITHOUT WRITTEN PERMISSION FROM THE PUBLISHER. THIS INCLUDES ELECTRONIC OR MECHANICAL, INCLUDING PHOTOCOPYING, RECORDING, OR ANY INFORMATION STORAGE AND RETRIEVAL SYSTEM.

OUR CAPS RANGE FOR GRADE 8 & 9 LEARNERS

- Mathematics
- Economic and Management Sciences

OUR CAPS RANGE FOR GRADE 10 LEARNERS

- Mathematics
- Mathematical Literacy
- Physical Sciences
- Life Sciences
- Accounting
- Business Studies
- History
- Economics
- Consumer Studies
- English Home Language
- English 1st Additional Language

OUR CAPS RANGE FOR GRADE 11 LEARNERS

- Mathematics
- Mathematical Literacy
- Physical Sciences
- Life Sciences
- Accounting
- Business Studies
- English 1st Additional Language

OUR CAPS RANGE FOR GRADE 12 LEARNERS

- Mathematics
- Mathematical Literacy
- Physical Sciences
- Life Sciences
- Accounting
- Business Studies
- History
- Economics
- Consumer Studies
- Geography
- English Home Language
- English 1st Additional Language



OUR IEB RANGE INCLUDES

- Gr 10 Life Sciences
- Gr 11 Life Sciences
- Gr 12 Life Sciences

The Answer

THE PUBLISHER

CONTENTS

Exam Assessment.....	i
----------------------	---

NOTES with QUESTIONS and ANSWERS:

MODULE 1: Numbers and Calculations with Numbers 1 - 21

1: Number formats and conventions	1
2: Operations on numbers and calculator skills	2
3: Rounding	5
4: Ratios	8
5: Proportion	11
6: Rates	16
7: Percentages	17

ANSWERS	A1 - A4
---------------	---------

MODULE 2: Patterns, Relationships and Representations..... 22 - 52

1: Making sense of graphs that tell a story	22
2: Patterns and relationships	24

ANSWERS	A5 - A9
---------------	---------

MODULE 3: Finance..... 53 - 108

1: Financial documents	53
2: Tariff systems	61
3: Income, expenditure, profit/loss, Income-and-Expenditure statements and budgets	68
4: Cost price and selling price	73
5: Break-even analysis	75
6: Interest	78
7: Banking, loans and investments	82
8: Inflation	95
9: Taxation	98
10: Exchange rates	105

ANSWERS	A10 - A19
---------------	-----------

MODULE 4: Measurement 109 - 146

1: Measurement systems	109
2: Measuring length and distance	109
3: Measuring mass (weight)	113
4: Measuring volume	119
5: Measuring temperature	123
6: Measuring time	124
7: Calculating perimeter, area, total surface area and volume	133

ANSWERS	A19 - A31
---------------	-----------

MODULE 5: Maps, Plans and Representations..... 147 - 173

1: Scale	147
2: Maps	155
3: Floor, elevation and design plans	165
4: Instructions and assembly diagrams	168
5: Models	171

ANSWERS	A32 - A37
---------------	-----------

MODULE 6: Data Handling 174 - 198

1: Developing questions	174
2: Collecting data	175
3: Classifying and organising data	176
4: Summarising data	179
5: Representing data	187
6: Interpreting and analysing data	196

ANSWERS	A38 - A42
---------------	-----------

MODULE 7: Probability 199 - 207

1: Expressions of probability	199
2: Prediction	201
3: Representations for determining possible outcomes	203
4: Evaluating expressions involving probability.....	207

ANSWERS	A42 - A44
---------------	-----------

EXAM ASSESSMENT

TIME AND MARK ALLOCATION

	TIME	MARKS
PAPER 1	3 hours	150
PAPER 2	3 hours	150

WEIGHTING OF TOPICS

	TOPIC	WEIGHTING (%)
BASIC SKILLS TOPICS	Interpreting and Communicating Answers and Calculations	• No weighting provided
	Numbers and Calculations with Numbers	• Assessed in an integrated way throughout the Application Topics
	Patterns, Relationships and Representations	
APPLICATION TOPICS	Finance	35% ($\pm 5\%$)
	Measurement	20% ($\pm 5\%$)
	Maps, Plans and Representations	15% ($\pm 5\%$)
	Data Handling	25% ($\pm 5\%$)
	Probability	Minimum 5%

FORMAT OF THE QUESTION PAPERS

	PAPER 1 'SKILLS' familiar contexts	PAPER 2 'APPLICATIONS' familiar and unfamiliar contexts
INTENTION	• 'Basic skills' paper • Assesses proficiency of core content and/or skills • Limited to CAPS	• 'Applications' paper • Assesses ability to use both mathematical/non-mathematical techniques to explore familiar and unfamiliar contexts
STRUCTURE and SCOPE of CONTENT and/or SKILLS	5 Questions Questions 1 - 4 deal with contexts relating to first 4 topics: • Finance • Measurement • Maps, Plans and Representations • Data Handling • Probability - assessed in one/more questions • Basic Skills Topics - integrated in all other questions Question 5: integrates all topics	4 or 5 Questions All questions deal with contexts drawing integrated content from two/more topics: • Finance • Measurement • Maps, Plans and Representations • Data Handling • Probability - assessed in one/more questions • Basic Skills Topics - integrated in all other questions
DISTRIBUTION OF MARKS TAXONOMY LEVELS	Level 1 knowing	60%
	Level 2 routine procedures	35%
	Level 3 multi-step procedures	5%
	Level 4 reasoning & reflecting	40%
Contexts	'familiar' limited to CAPS context	'familiar' and 'unfamiliar' not limited to CAPS context

1 NUMBERS AND CALCULATIONS WITH NUMBERS

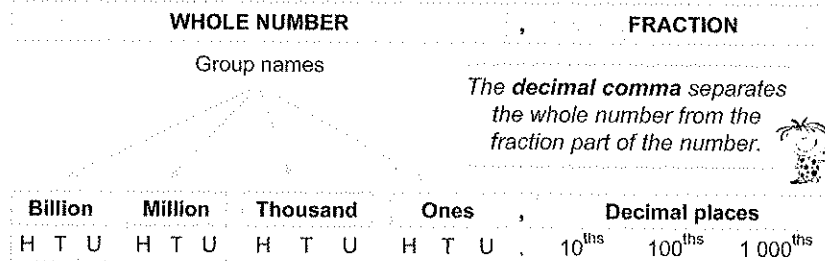
UNIT

1

NUMBER FORMATS AND CONVENTIONS

Working with numbers as digits and words

Numbers are grouped in threes from right to left starting at the comma. Each group of three digits has a name, i.e. **Ones**, **Thousand**, **Million**, **Billion**, and is made up of **Hundreds**, **Tens** and **Units**. (HTU respectively)



Worked Examples

1. Write the following number in words: 721 482,5

Thousand	Ones	Decimal places
H T U	H T U	10 ^{ths}
7 2 1	4 8 2	5

∴ Seven hundred and twenty-one thousand, four hundred and eighty-two, comma five.

2. Write the following number using digits: one million, forty-eight thousand and three.

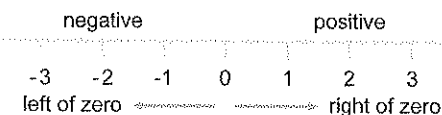
Million	Thousand	Ones
H T U	H T U	H T U
1	0 4 8	0 0 3

∴ 1 048 003



Numbers have direction

We can say that numbers have direction.



Positive and negative numbers can be used as directional indicators.

A positive number has no sign in front of it (15 means: +15) whereas a negative number has a negative sign in front of it (-15).

- 3°C indicates 3 degrees below zero °C.
- R1 000 indicates credit (money owned) while -R1 000,00 indicates debit (money owed).



Worked Examples

- Determine Emily's bank balance if she had R2 500 in her account; then went shopping and spent R400; R1 200; R850 and then R380. Thereafter, on the way to her car, she picked up R200 and then paid R25 for parking.

$$R2\ 500 - R400 - R1\ 200 - R850 - R380 + R200 - R25 = -R155$$
- If the temperature was 36°C in Upington at midday; and it then dropped by 42°C by midnight; what was the temperature recorded at midnight?

$$36^{\circ}\text{C} - 42^{\circ}\text{C} = -6^{\circ}\text{C}$$

Numbers can be used in fixed ways

Numbers can also be used in places where they will never be added, subtracted, multiplied or divided.

Examples

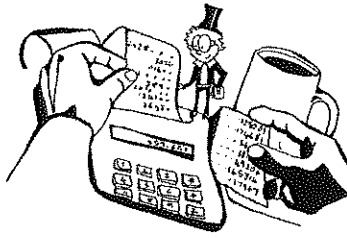
- Flat number 821 means flat 21 on the 8th floor.
- The set of digits 5-2-0-4-6-1 indicates the number of runs scored off each ball in one over (6 balls) of cricket.



Test Your Understanding



1. The provincial budget for education is R4 827 030. Write the number in words.
2. A stock broker made nine hundred and fifty thousand rand profit on a deal. Write this number using digits.
3. How would you write the following using digits:
 - 3.1 A bank balance that is in credit of two hundred rand.
 - 3.2 A debit bank balance of one hundred and fifty rand.
4. How would you indicate the following, using digits:
 - 4.1 Five degrees below zero.
 - 4.2 The tenth flat on the fifth floor of an apartment block.
 - 4.3 Diving to a depth of eighty metres below sea level.



UNIT

2

OPERATIONS ON NUMBERS AND CALCULATOR SKILLS

9/10/11 12

Estimation



Estimation is a valuable tool that gives an idea of what the final answer should be. It is useful to estimate before calculating, so that if the answer differs greatly, it could be an indication of a potential error.

How to estimate:

In order to estimate, first convert the numbers to values that are easier to work with, e.g. units, 10's, 50's, 100's, 500's, etc., and then do the calculations on these converted values. See *Rounding - Unit 3*, for guidelines on how to convert.

Where estimation is useful:

A situation where estimation is useful is for totalling a basket or trolley of goods when shopping within a budget, for example:

Milk R18,95; Bread R13,99; Yoghurt R22,99; Apples R7,99 and Eggs R33,95

Estimation converting to 5 rand:

$$\approx 20 + 15 + 25 + 10 + 35$$
$$= R105$$

Estimation converting to 10 rand:

$$\approx 20 + 10 + 20 + 10 + 30$$
$$= R90$$

Calculated answer:

$$18,95 + 13,99 + 22,99 + 7,99 + 33,95$$
$$= R97,87$$

NOTE! If you've mostly had to round down, then the estimated amount will be less than the calculated amount (and vice versa).



Worked Examples

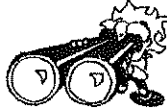


Estimate the answer to each of the following and compare it to the calculated answer:

1. 692×28

$$\approx 700 \times 30$$
$$= 21\,000$$

(calculated answer: 19 376)



2. $891 - 244 - 487$

$\approx 900 - 250 - 500$
 $= 150$

(calculated answer: 160)



3. $328 \div 10,4$

$\approx 300 \div 10$
 $= 30$

(calculated answer: 31,538)

Calculator operations



Addition and multiplication:

- The order of operations involving addition and multiplication does not matter.

$8 + 3 = 11$ and $3 + 8 = 11$

$7 \times 5 = 35$ and $5 \times 7 = 35$



- Some useful skills to remember when adding numbers:

$R82 + R53 = (R80 + R2) + (R50 + R3)$
 $= R80 + R50 + R2 + R3$
 $= R130 + R5$
 $= R135$

Break numbers down
 into easier groups,
 such as tens and units.



- Some useful skills to remember when multiplying numbers:

$6(8 + 9 - 3) = 6(14)$
 $= 84$

Always simplify inside brackets first,
 before multiplying out the brackets.



Subtraction and division:

- The order of operations involving subtraction and division must *not* be changed.

$9 - 3 = 6$ and $3 - 9 = -6$ $\therefore 9 - 3 \neq 3 - 9$

positive 6 has a different meaning to negative 6
 e.g. you have R6 (+6) vs you owe R6 (-6)

$12 \div 3 = 4$ and $3 \div 12 = \frac{1}{4}$ $\therefore 12 \div 3 \neq 3 \div 12$

the whole number 4 has a different meaning to the fraction number $\frac{1}{4}$

e.g. you get 4 slices of pizza vs you only get $\frac{1}{4}$ of a slice of pizza.

BODMAS

9 10 11 12



BODMAS is used to determine the correct order of operations, when calculating a sum.

Scientific calculators do BODMAS automatically, but it is still important to know how to use BODMAS, as you will still need to do some calculations manually.

BODMAS stands for the following:

B: Brackets (always work inside-out)

O: Of (multiply)

D: Divide

M: Multiply

A: Add

S: Subtract



Worked Examples



1. $(8 + 6 \div 2) + \frac{1}{2} \times 10 - 5$

Do the brackets first, but remember to do the division in the brackets before you add.

$= (8 + 3) + \frac{1}{2} \times 10 - 5$

$= 11 + \frac{1}{2} \times 10 - 5$

Do the multiplication before you add and subtract.

$= 11 + 5 - 5$

$= 11$



2. $80 \div 2 \times 5 + \frac{1}{4}$ of 60

$80 \div 2 \times 5 + \frac{1}{4} \times 60$

$= 80 \div 2 \times 5 + 15$

$= 40 \times 5 + 15$

$= 200 + 15$

$= 215$



Powers and Roots

Powers

- Powers are a way of writing repeated multiplication of the same number by itself.

$$8^2 = 8 \times 8 \\ = 64$$

When the exponent is a '2',
we say the number is squared.

$$5^3 = 5 \times 5 \times 5 \\ = 125$$

When the exponent is a '3',
we say the number is cubed.



Roots

- Roots are the opposite of powers.
- To find the square root of a number you need to find which number multiplied by itself results in the number under the square root sign.

$\sqrt{\quad}$ or $\sqrt[2]{\quad}$ means square root.

$$\sqrt{64} = \sqrt[2]{64} \\ = 8$$

What times itself will give you 64? $8 \times 8 = 64$



Worked Examples

$$1. (3 + 2)^2 - 5 \times 3 + 2^3$$

$$= 5^2 - 5 \times 3 + 2^3 \\ = 25 - 5 \times 3 + 2^3 \\ = 25 - 15 + 2^3 \\ = 25 - 15 + 8 \\ = 10 + 8 \\ = 18$$



$$2. \sqrt{16} + (\sqrt{25})^2 - 3^2 \times 2 \div 6$$

$$= 4 + 5^2 - 9 \times 2 \div 6 \\ = 4 + 25 - 18 \div 6 \\ = 4 + 25 - 3 \\ = 26$$



Squaring and square-rooting
are opposite operations

$$(\sqrt{3})^2 = \sqrt{3^2} = \sqrt{9} = 3$$



Using the calculator

The Basic Calculator

- Rewrite the common fraction as a division sum

$$\text{e.g. } \frac{1}{4} = 1 \div 4 = 0.25$$

- Use repeated multiplication for powers

$$\text{e.g. } 8^2 = 8 \times 8 = 64$$

$$2^3 = 2 \times 2 \times 2 = 8$$

- Use multiplication for 'OF'

$$\text{e.g. } \frac{2}{5} \text{ of } 150 = \frac{2}{5} \times 150 = 60$$

- Calculate brackets from the inside out; one step at a time

e.g. $27 - [35 - 2 \times (34 - 27)] \rightarrow$ first type $34 - 27$ and then press the = key (Answer: 7)

then type 2×7 and then press the = key (Answer: 14)

then type $35 - 14$ and then press the = key (Answer: 21)

and finally type $27 - 21$ and then press the = key (Final answer: 6)

- Use the root sign ($\sqrt{\quad}$) to calculate the square root of a number

e.g. $\sqrt{144} \rightarrow$ type 144 and then press $\sqrt{\quad}$ key (Answer: 12)

Scientific calculator

$\sqrt{144} \rightarrow$ press $\sqrt{\quad}$ key
then type 144 then press =

- Don't forget BODMAS!

(This often requires a sum to be done in multiple steps.)

$$\text{e.g. } 50 + 4 \times \frac{1}{8} \text{ of } 240$$

$$= 50 + 4 \times \frac{1}{8} \times 240$$

$$= 50 + 4 \times 30$$

$$= 50 + 120$$

$$= 170$$

'of' is equivalent to 'x'

Of comes before Multiplication

Multiplication comes before Addition



The Scientific Calculator

- Type the expression in as it appears in the question.
- The calculator will do BODMAS for you!



Worked Examples

1. 26% of R585,99

$$= 26 \div 100 \times 585,99$$

$$\approx R152,36$$

% means
 $\div 100$



3. $356,24 + 2,3^2 \times \sqrt{121}$

$$= 356,24 + 2,3 \times 2,3 \times (\sqrt{121})$$

$$= 356,24 + 5,29 \times 11$$

$$= 356,24 + 5,29 \times 11$$

$$= 356,14 + 58,19$$

$$= 414,43$$



REMEMBER!

Calculators don't make mistakes, BUT people do, so always enter the information carefully.
Estimating the answer is a good idea so that there is a certain expectation of what the answer should be.

Test Your Understanding

1. Estimate the total value of the following shopping basket:

- Eggs : R12,99
- Bread : R14,50
- Cheese : R27,85
- Tomato : R3,22
- Bacon : R19,89



REMEMBER!

Round off to the nearest number that is easy to work with, i.e. nearest R1 or R5.



2. Estimate how many people attended the fête, if the hall was 27% full of its total capacity of 150 people.

3. Calculate the following:

3.1 $\frac{46 + 6 \times 4}{55 - 5 \times 7}$

3.2 14% of R63,15

3.3 $5 + \frac{2}{3}$ of 150

3.4 $48,6^2 \div 10 + \sqrt{121}$ (to 2 decimal places)

3.5 $17\pi \times \frac{11}{100}$ (to 2 decimal places)

NOTE! π needs to be approximated to 3 decimal places
 $\therefore \pi \approx 3,142$



UNIT

3

ROUNDING

9 10 11 12

How to round off to the nearest whole number



- First look at the digit in the **1st decimal place** (i.e. the digit to the right of the decimal comma).
 - if the digit in the first decimal place is a **1, 2, 3 or 4**, the digit in the **unit position remains unchanged** when rounding (i.e. these numbers will **round down** to their nearest whole number). **OR**
 - if the digit in the first decimal place is a **5, 6, 7, 8 or 9**, the digit in the **unit position increases by 1** when rounding (i.e. these numbers will **round up** to the next whole number, their nearest whole number).

Worked Examples

Round the following off to the nearest whole number:

1. 6,2 $\approx$ 6
2. 4,8 $\approx$ 5
3. 8,314 $\approx$ 8
4. 2,78 $\approx$ 3
5. 649,5 $\approx$ 650

The digit in the unit position is a 9, which means the 649 has to round up to 650 because 50 is 1 more than 49.



How to round off to a specified number of decimal places



- First look at the digit to the right of the specified decimal place (the **neighbouring decimal place**).
 - if the digit in the neighbouring decimal place is a **1, 2, 3 or 4**, the digit in the **specified decimal place remains unchanged** when rounding (i.e. these numbers will **round down**). **OR**
 - if the digit in the neighbouring decimal place is a **5, 6, 7, 8 or 9**, the digit in the **specified decimal place increases by 1** when rounding (i.e. these numbers will **round up**).

Worked Examples



Consider the following number:

4,81350

1. Round off to 1 decimal place.

4,8(1)350

want to round off to here ...

... so we consider this number.

If the 2nd decimal place is

- 0 - 4, the 1st decimal is not influenced (i.e. stays the same).
- 5 - 9, the 1st decimal is influenced (i.e. will become one more).

(1) is between 0 and 4,

∴ the 1st decimal is not influenced

∴ rounding off to 1 decimal place results in **4,8**.



2. Round off to 2 decimal places.

∴ we consider the 3rd decimal place.

4,81(3)50

want to round off to here ...

... so we consider this number.

Money is always rounded off to 2 decimal places.



(3) is less than 5.

∴ the 2nd decimal is not influenced.

∴ rounding off to 2 decimal places results in **4,81**.

3. Round off to 3 decimal places.

∴ we consider the 4th decimal place.

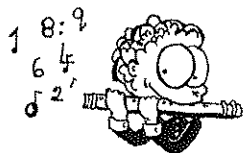
4,813(5)0

want to round off to here ...

... so we consider this number.

Since the 4th decimal is (5) the 3rd decimal is influenced.

∴ rounding off to 3 decimal places results in **4,814**.



How to round off to the nearest 5



This concept is useful when working with money.

In order to round off to the nearest 5 cents

- First look at the digit in the 2nd decimal position (i.e. the last digit when working with money):
 - if this digit is a 1 or a 2, it must **become a zero (0)** when rounding (i.e. these numbers will **round down**) **OR**
 - if this digit is a 3, 4, 6 or 7, it must **become a 5** when rounding (i.e. the numbers with a 3 or 4 in the last digit position will **round up** and the numbers with a 6 or 7 in the last digit position will **round down**) **OR**
 - if the digit in the neighbouring position is an 8 or a 9, it must **become a zero (0) with the digit to the left of it increasing by 1** when rounding (i.e. these numbers will **round up**)

In order to round off to the nearest R5

- First look at the digit in the **units position** (i.e. the last digit before the decimal comma) and follow the same instructions as above.



Worked Examples

1. Round the following off to the nearest 5 cents:

$$1.1 \text{ R}43,21 \approx \text{R}43,20$$

$$1.2 \text{ R}899,68 \approx \text{R}899,70$$

$$1.3 \text{ R}6\,321,94 \approx \text{R}6\,321,95$$

2. Round the following off to the nearest R5:

$$2.1 \text{ R}755,92 \approx \text{R}755$$

$$2.2 \text{ R}8,15 \approx \text{R}10$$

$$2.3 \text{ R}1\,062,99 \approx \text{R}1\,060$$

How to round off to the nearest 10, 100, 1 000



This concept is useful when estimating.

- First assess the context of the question.
 - if rounding to the nearest **10**: look at the digit in the **units position** **OR**
 - if rounding to the nearest **100**: look at the digit in the **tens position** **OR**
 - if rounding to the nearest **1 000**: look at the digit in the **hundreds position**.
- Apply the same instructions as for rounding off to the nearest whole number. If the neighbouring digit is a 1, 2, 3 or 4, round the number **down** and if the neighbouring digit is a 5, 6, 7, 8 or 9, round the number **up**.

Worked Examples

Round the following off to the nearest 10:

1. $494 \approx 490$
2. $777 \approx 780$

Round the following off to the nearest 100:

1. $1\ 395 \approx 1\ 400$
2. $860 \approx 900$

Round the following off to the nearest 1 000:

1. $64\ 720 \approx 65\ 000$
2. $8\ 199 \approx 8\ 000$

Rounding up or down

The context in which the rounding occurs determines the way in which a value has to be rounded:

- Money is always rounded off to two decimal places, e.g. R3,50
- When buying quantities of paint, tiles etc. you will always need to round the quantity up because these items are not sold in parts of the whole, and if you round down you won't have enough paint or tiles.
- Calculations involving people are rounded either up or down to the nearest whole number depending on the context.

Worked Examples

1. How many 2 l tins of paint does Paula need if she is painting three walls and each wall requires 1,5 l of paint?

$$\text{Number of litres of paint} = 1,5 \times 3 = 4,5 \text{ l}$$

$\therefore$ Paula needs 3 tins of paint.

(i.e. $2 \text{ l} \times 3 = 6 \text{ l}$ of paint but $2 \text{ l} \times 2 = 4 \text{ l}$ would not be enough!)

2. Mary runs a lift club and decides that each member should pay R180,00 each month. How many people does she need in the lift club if she needs R1 200 per month?

She needs $R1\ 200,00 \div R180,00$, which is 6,66666... people.

She would therefore have to have 7 people in the lift club to cover the costs.

(She will collect a little more than the R1 200,00 she needed.)

If she rounded down to 6, she would only collect
 $6 \times R180 = R1\ 080$; which isn't enough!

Test Your Understanding

1. Round off to one decimal place:

1.1 $235,02$

1.2 $12,9477$

1.3 $38,98$

2. Round off to two decimal places:

2.1 $14,111$

2.2 $134,6983$

2.3 $7\ 482,5104$

3. Round off to three decimal places:

3.1 $7,9999$

3.2 $15,9983$

3.3 $6,7053$

4. Round off to the nearest 5 cents:

4.1 $R335,73$

4.2 $R109,98$

4.3 $R34,56$

5. Round off to the nearest rand:

5.1 $R199,45$

5.2 $R39,98$

5.3 $R2\ 719,35$

6. Round off to the nearest R5:

6.1 $R4\ 532$

6.2 $R2\ 462,15$

6.3 $R3\ 617,95$

7. Round off to the nearest 10:

7.1 894

7.2 788

7.3 $2\ 174$

8. Round off to the nearest 100:

8.1 $38\ 192$

8.2 $7\ 135$

8.3 448

How about
rounding off
this section
with ice cream!

9. Round off to the nearest 1 000:

9.1 $1\ 241$

9.2 $94\ 355$

9.3 $6\ 551$

10. How many dried fruit bars can you buy with R65 if each bar costs R9,95?

11. Roy loves reading. He decides to build a bookshelf that has a length of 1,3 m (130 cm). His average book has a width of 3 cm. How many books will Roy be able to fit on his bookshelf?

What is a ratio?

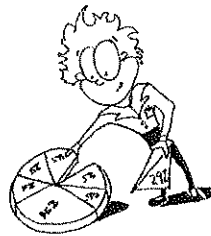
- A **ratio** compares two or more quantities that are expressed in the same unit, $\therefore$ it is not necessary to include the units.
- A ratio may be written as $a : b$ or $\frac{a}{b}$ or referred to by the phrase 'a to b'.
e.g. $1 : 4$ OR $\frac{1}{4}$ OR '1 to 4'

- The **order** in which the values in the ratio are written is **VERY** important.

e.g. $1 : 4 = \frac{1}{4}$ (i.e. a quarter of a pizza)

vs

$4 : 1 = \frac{4}{1}$ (i.e. 4 whole pizzas)



Equivalent ratios

- Equivalent ratios** are ratios that represent the same comparison of quantities using different values.
- Equivalent ratios can be found by **multiplying or dividing both** values in the ratio by the **same number**.

e.g. $\begin{array}{ccc} \times 5 & 7 : 6 & \times 5 \\ & 35 : 30 & \end{array}$ $\frac{7}{6} = \frac{7 \times 5}{6 \times 5} = \frac{35}{30}$
 $\therefore 7 : 6 = 35 : 30$

e.g. $\begin{array}{ccc} \div 3 & 33 : 12 & \div 3 \\ & 11 : 4 & \end{array}$ $\frac{33}{12} = \frac{33 \div 3}{12 \div 3} = \frac{11}{4}$
 $\therefore 33 : 12 = 11 : 4$



Simplifying ratios

- Ratios must always be written in their **simplest form**.

This is done by **dividing both** values in the ratio by the **HCF**.

e.g. Marandi earns R12 000 a month, while Patrick earns R15 000 a month. The ratio of their income can be written as:

Marandi : Patrick

12 000 : 15 000

$\div 1\,000$

12 : 15

$\div 3$

4 : 5

$\therefore$ HCF = 3 000!

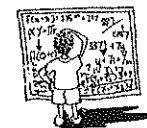
this is a common factor, but **not the highest** common factor

$\therefore$ you must keep dividing to get the ratio in its simplest form

- Ratios, given as **decimals**, must always be written as **whole numbers** in their **simplest form**.

This is done by **multiplying** both sides by whichever **factor of 10** will get rid of the decimal. **Thereafter**, simplify the ratio by **dividing** by the HCF.

e.g. Express the ratio 15,4 : 96,6 in its simplest form.



15,4 : 96,6

$\times 10$

154 : 966

$\div 14$

11 : 69

multiply both sides by 10, to get rid of the decimal

divide by HCF to simplify the ratio

- Ratios, given as **fractions**, must always be written as **whole numbers** in their **simplest form**.

Convert each fraction to an **equivalent fraction**, using a **denominator** that is **common to both**.

Thereafter, simplify the **numerators** of the ratio only by **dividing** by the HCF.

e.g. Express $\frac{5}{6} : \frac{3}{4}$ in its simplest form.

divide by the HCF to write ratio in simplest form

$\frac{5}{6} : \frac{3}{4}$

$\frac{20}{24} : \frac{18}{24}$

20 : 18

10 : 9

find equivalent fractions on both sides using a common denominator

i.e. $\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

drop the denominators as they are the same and simplify the numerators



Highest
Common
Factor
or
HCF

- Ratios, given in **different units**, must always be written in the **same units**. First converting the **bigger of the two units**, into the **equivalent smaller unit**, using your **conversion factors**.

Thereafter, simplify the ratio by **dividing** by the **HCF**.

Be sure to revise conversions in Module 4.



Simplify 15 minutes : 1 hour
15 minutes : 60 minutes

$$\begin{array}{l} 15 : 60 \\ \div 15 \\ \hline \therefore 1 : 4 \end{array}$$

divide by the HCF to get the ratio in the simplest form

hour is the bigger unit, so convert hours into minutes

$$1 \text{ hour} = 60 \text{ minutes}$$

since the units are the same (minutes), we drop them and only write the numbers

On some calculators, if you type $\frac{15}{60} =$, it will simplify it for you (giving $\frac{1}{4}$).

Remember to write your answer as a **ratio** (1 : 4) again!

Simplify 1,2 m : 35 cm

120 cm : 35 cm

$$\begin{array}{l} 120 : 35 \\ \div 5 \\ \hline \therefore 24 : 7 \end{array}$$

divide by the HCF to get the ratio in the simplest form

metres is the bigger unit, so convert metres into centimetres 1 m = 100 cm

since the units are the same (cm), we drop them and only write the numbers

Missing value ratio problems



- In this type of problem, we are given the **ratio of two quantities** and the **value of one of these quantities**. We therefore need to find the missing value of the other quantity. There are two methods to solve this type of problem:

METHOD 1 Multiplication / Division by 'what'?

e.g. A candy manufacturer needs to fill a box of sweets with green and yellow sweets in the ratio of 3 : 5. If the box contains 12 green sweets, how many yellow sweets should be in the box?

$$\begin{array}{l} 3 \times \text{'what?'} = 12 \\ \therefore 3 \times 4 = 12 \end{array} \quad \begin{array}{l} \text{green : yellow} \\ 3 : 5 \\ \times 4 \quad 12 : ? \quad \times 4 \end{array}$$



$$\therefore \text{Number of yellow sweets} = 5 \times 4 = 20$$

METHOD 2 Determining the value of 1 part

e.g. Ben and Jim decide to buy a pool business. They invest in the business in the ratio of 4 : 7. If Jim invests R182 000, how much does Ben invest in the business?

Ben : Jim

4 : 7

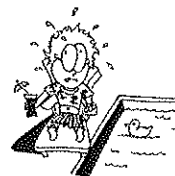
? : R182 000

$\therefore$ Jim's investment of R182 000 represents 7 parts

$\therefore$ 1 part = $R182\,000 \div 7 = R26\,000$

$\therefore$ Ben invests 4 parts in the business

$\therefore$ Ben's investment = $4 \times R26\,000 = R104\,000$



Sharing and dividing in a given ratio



- In this type of problem, we are given the **ratio of two quantities** and the **sum total of both quantities**. We therefore need to share the sum total into the given ratios. There are two methods to solve this type of problem:

METHOD 1 Finding the fraction of the whole

e.g. Sarah, Lilly and Paula make R1 200 profit from selling cakes. Determine how much profit each girl receives, if they invested in the ratio of 1 : 2 : 3.

Total no. of parts = $1 + 2 + 3 = 6$ parts

Sarah : Lilly : Paula

1 : 2 : 3

$\frac{1}{6} : \frac{2}{6} : \frac{3}{6}$

their fraction of their investment

$\therefore$ Sarah's profit = $\frac{1}{6}$ of R1 200

$$= \frac{1}{6} \times R1\,200$$

$$= R200$$

REMEMBER!
'of' means multiplication

$\therefore$ Lilly's profit = $\frac{2}{6}$ of R1 200

$$= \frac{2}{6} \times R1\,200 = R400$$

$\therefore$ Paula's profit = $\frac{3}{6}$ of R1 200

$$= \frac{3}{6} \times R1\,200 = R600$$



METHOD 2 Determining the value of 1 part

e.g. A herbicide needs to be mixed with water in the ratio of 2 : 5.

If Bill needs to make 14 l of the herbicide mixture, how many litres of herbicide and water is needed?

Total no. of parts = $2 + 5 = 7$ parts

herbicide : water

2 : 5

∴ Since 14 l needs to be divided into 7 parts

∴ One part = $14 \text{ l} \div 7 = 2 \text{ l}$

∴ Volume of herbicide = $2 \text{ l} \times 2 \text{ parts} = 4 \text{ l}$

∴ Volume of water = $2 \text{ l} \times 5 \text{ parts} = 10 \text{ l}$



Worked Examples

1. A Grade 12 Maths Literacy class has 12 boys and 36 girls.
Give the ratio of boys to girls in this class.

boys : girls

12 : 36

1 : 3

÷ HCF = 12

2. Express the ratio 12,4 : 4,8 in its simplest form.

12,4 : 4,8

124 : 48

31 : 12

× 10

÷ 4

multiply by a power of 10 (10, 100, 1 000, etc.)
to get rid of the decimal

divide by the HCF to get ratio in its simplest form

3. Simplify $\frac{2}{21} : \frac{5}{63}$

$\frac{2}{21} : \frac{5}{63}$

$\frac{6}{63} : \frac{5}{63}$

6 : 5

find equivalent fraction, using a common denominator

$$\text{i.e. } \frac{2}{21} = \frac{2 \times 3}{21 \times 3} = \frac{6}{63}$$

drop the denominator
because they are the same

4. If every 1 cm on a map is the equivalent to 100 km in real-life, what is the ratio of map : real-life?

map : real-life

1 cm : 100 km

1 cm : 10 000 000 cm

1 : 10 000 000

Distance Conversion Factors:

100 cm = 1 m
10 000 m = 10 km



drop the units since they are the same

5. Cement, stone and sand need to be combined in a ratio of 4 : 3 : 1 for a particular construction. If 900 g of stone is required, then what mass is required for the cement and sand?

METHOD 1 Multiplication/Division by 'what?'

cement : stone : sand

4 : 3 : 1

? : 900 g : ?

× 300 g

3 × 'what?' = 900 g

∴ 3 × 300 g = 900 g



∴ Mass of cement = $4 \times 300 \text{ g} = 1\,200 \text{ g}$

∴ Mass of sand = $1 \times 300 \text{ g} = 300 \text{ g}$

METHOD 2 Determining the value of 1 part

cement : stone : sand

4 : 3 : 1

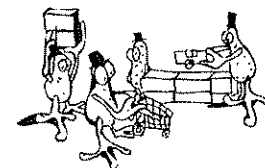
? : 900 g : ?

∴ The mass of 900 g of stone represents 3 parts

∴ 1 part = $900 \text{ g} \div 3 = 300 \text{ g}$

∴ Mass of cement = $4 \text{ parts} \times 300 \text{ g} = 1\,200 \text{ g}$

∴ Mass of sand = $1 \text{ part} \times 300 \text{ g} = 300 \text{ g}$



Test Your Understanding

1. Simplify the following ratios:

1.1 12 : 960

1.2 140 : 24

1.3 12,3 : 6,6

1.4 12 seconds : 1 hour

1.5 15 cm : 0,65 m

1.6 $\frac{3}{8} : \frac{2}{5}$

2. Daniel earns R15 000 per month more than his wife, Andrea, who earns R10 000 per month. Give the ratio of Daniel's income to Andrea's income.
3. Sally and David's heights are in the ratio 5 : 7. If David is 189 cm tall, how tall is Sally?

Revise
conversions
in Module 4.



4. A company has two shareholders who invested R300 000 and R500 000, respectively, in the company. The company's annual return of R1 200 000 must be shared according to the shareholders' investment ratio. How much does each shareholder receive?
5. The ideal ratio of the number of learners to the number of teachers is 25 : 1
- 5.1 If a school with 800 learners wants to employ enough teachers to satisfy this ratio, how many teachers are needed?
- 5.2 If the school with 860 learners can only afford to pay 20 teachers, what is the ratio of learners to teachers?
- 5.3 School A has 750 learners and 15 teachers and school B has 920 learners and 20 teachers. Which school has a smaller learner to teacher ratio?

UNIT 5

PROPORTION

What is Proportion?

- When two ratios are given as equal, e.g. $\frac{a}{b} = \frac{c}{d}$, we say that they are in **proportion**.

The following are examples of proportion:

a) $\frac{2}{5} = \frac{12}{30}$ because $\frac{2 \times 6}{5 \times 6} = \frac{12}{30}$

b) $\frac{3,5}{14} = \frac{7}{28} = \frac{1}{4}$ because $\frac{3,5 \times 2}{14 \times 2} = \frac{7}{28} = \frac{7 \div 7}{28 \div 7} = \frac{1}{4}$

- When ratios are in proportion, then their cross products are also equal.
 $\therefore$ when $\frac{a}{b} = \frac{c}{d}$ then $a \times d = b \times c$
- The order in which the numbers are written is very important.
- Proportion can be used to solve problems involving scale, exchange rates, best buys, fuel consumption, etc.

These are **equivalent fractions**.

$\therefore$ they are in the same proportion!

Direct Proportion

What is Direct Proportion?

Two quantities are in **direct proportion** (directly proportional) if the **ratio** between them remains **constant**, i.e. the one increases or decreases by the same factor as the other.

e.g. If one banana costs R2, what will 5 bananas cost?

The table below shows the cost of bananas:

Number of bananas	0	1	2	3	4	5
Cost (R)	0	2	4	6	8	10

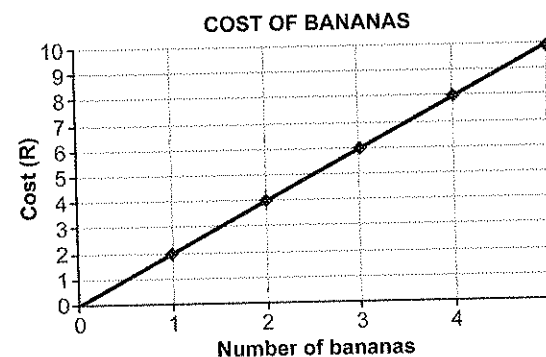
$\therefore$ The **constant ratio** between the number of bananas and the cost is

$$\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = 2$$

Direct Proportion Graphs

A direct proportion graph will always be an **increasing straight line** graph that **starts at the origin**.

e.g. The graph of the cost of the different number of bananas (previous example) is shown below:



Refer to Module 2 to refresh your memory of drawing graphs.

Why does the graph go through (0; 0)?

Because ... you don't have to pay anything if you don't buy any bananas!

Calculations with ratios in direct proportion

There are two methods to choose from when finding a missing value in purely mathematical proportion equations:

METHOD 1 Multiplication/Division by 'what?'

Rewrite the equation as a **ratio** and find the missing value (see p. 9).

e.g. $\frac{2}{25} = \frac{x}{175}$

$$\begin{array}{l} \times 7 \\ 2 : 25 \\ \times x : 175 \end{array} \times 7$$

$$\therefore x = 2 \times 7 = 14$$

$$25 \times \text{'what?'} = 175$$

$$\therefore 25 \times 7 = 175$$

$$\therefore 7 \text{ is the constant ratio}$$



$$\frac{4}{y} = \frac{2}{9}$$

$$\begin{array}{l} +2 \\ 4 : y \\ 2 : 9 \end{array} +2$$

$$\therefore y + 2 = 9$$

$$y = 9 - 2$$

$$y = 7$$

$$4 \div \text{'what?'} = 2$$

$$\therefore 4 \div 2 = 2$$

$$\therefore 2 \text{ is the constant ratio}$$



METHOD 2 Cross-Multiplication

Use the fact that the **cross products** are equal when ratios are in proportion. Equate the cross products and solve the equation.

e.g. $\frac{2}{25} = \frac{x}{175}$

$$\frac{2}{25} = \frac{x}{175}$$

$$25 \times x = 2 \times 175$$

$$25x = 350$$

$$x = 14$$

$$\frac{4}{y} = \frac{2}{9}$$

$$\frac{4}{y} = \frac{2}{9}$$

$$\therefore y \times 2 = 4 \times 9$$

$$2y = 36$$

$$y = 18$$

Direct Proportion in Context



- Write down the ratios of the two quantities. Check: as one quantity increases/decreases, the other will also increase/decrease by the same factor.
- Identify the constant ratio and use it to determine the missing value (Method 1) or cross multiply (Method 2) to solve it.
- Since direct proportion involves ratios, we can also use the method of 'Determining the value of 1 part' (see p. 9).

There are 3 different ways of solving the same sum!
Choose a method that suits you!



e.g. A 2 l can of paint can cover 15 m^2 . How many litres of paint are required to cover an area of $82,5 \text{ m}^2$?

METHOD 1 Multiplication/Division by 'what?'

$$\begin{array}{l} \text{litres : area} \\ 2 \text{ l} : 15 \text{ m}^2 \\ \times 5,5 \\ ? : 82,5 \text{ m}^2 \end{array} \times 5,5$$

$$\therefore \text{Number of litres} = 2 \text{ l} \times 5,5 = 11 \text{ l}$$

$$15 \times \text{'what?'} = 82,5$$

$$\therefore 15 \times 5,5 = 82,5$$

$$\therefore \text{Constant ratio} = 5,5$$



METHOD 2 Cross-Multiplication

$$\begin{array}{l} \text{litres : area} \\ 2 \text{ l} : 15 \text{ m}^2 \\ ? : 82,5 \text{ m}^2 \end{array}$$

$$\therefore \frac{2}{15} = \frac{x}{82,5}$$

$$15 \times x = 2 \times 82,5$$

$$15x = 165$$

$$x = 11 \text{ l}$$

Cross multiply
to solve



METHOD 3 Determining the value of 1 part

$$\begin{array}{l} \text{litres : area} \\ 2 \text{ l} : 15 \text{ m}^2 \\ ? : 82,5 \text{ m}^2 \end{array}$$

NOTE! How many **cans** of paint are needed?

Number of cans = $11 \text{ l} \div 2 \text{ l} = 5,5 \text{ cans}$

We can't buy half a can,
so we round **up** to 6 cans.

$\therefore 1 \text{ l}$ of paint could cover an area of $15 \text{ m}^2 \div 2 \text{ l} = 7,5 \text{ m}^2$

$\therefore$ Number of litres needed = $82,5 \text{ m}^2 \div 7,5 \text{ m}^2 = 11 \text{ l}$

Worked Examples



1. If you get 24 marbles in a 80 g bag; then how many marbles could you expect to get (assume that each marble has the same mass) in a 120 g bag?

METHOD 1

$$\begin{array}{l} \text{marbles : mass} \\ \div 80 \quad 24 : 80 \text{ g} \quad \div 80 \\ \quad \quad : 1 \text{ g} \\ \times 120 \quad ? : 120 \text{ g} \quad \times 120 \end{array}$$

$$\therefore \text{Number of marbles} = (24 \div 80) \times 120 = 36$$



METHOD 2

$$\begin{array}{l} \text{marbles : mass} \\ 24 : 80 \text{ g} \\ ? : 120 \text{ g} \end{array}$$

$$\therefore \frac{24}{80} = \frac{x}{120}$$

$$80 \times x = 24 \times 120$$

$$80x = 2\,880$$

$$x = 36$$

$$\therefore \text{Number of marbles} = 36$$

METHOD 3

$$\begin{array}{l} \text{marbles : mass} \\ 24 : 80 \text{ g} \\ ? : 120 \text{ g} \end{array}$$

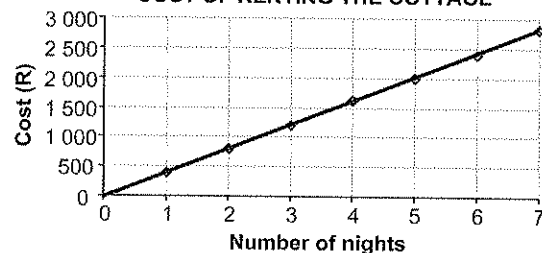
$$\therefore 1 \text{ marble weighs} = 80 \text{ g} \div 24 = 3,33 \text{ g}$$

$$\therefore \text{Number of marbles} = 120 \text{ g} \div 3,33 \text{ g} = 36$$

2. A bed and breakfast advertises R400 per night for a cottage.
- 2.1 Draw up a table to show the total cost of renting the cottage for 0 - 7 nights.
- 2.2 Using the table you completed in Question 2.1, draw the graph of the cost of renting the cottage.

2.1	Number of nights	0	1	2	3	4	5	6	7
	Cost (R)	0	400	800	1 200	1 600	2 000	2 400	2 800

2.2 COST OF RENTING THE COTTAGE



Indirect/Inverse Proportion



What is Indirect/Inverse Proportion?

Two quantities are in **indirect/inverse proportion** (inversely proportional) if their **product** remains **constant**, i.e. the one value increases by the same factor that the other decreases by and vice versa.

- e.g. It takes one learner 24 hours to make the decorations for the matric dance. How long would it take 12 learners?

Number of learners	1	2	3	4	6	8	12
Time taken (hours)	24	12	8	6	4	3	2
	$1 \times 24 = 24$	$2 \times 12 = 24$	$3 \times 8 = 24$	$4 \times 6 = 24$	$6 \times 4 = 24$	$8 \times 3 = 24$	$12 \times 2 = 24$

Two learners would take half the time, three learners, a third of the time, etc.

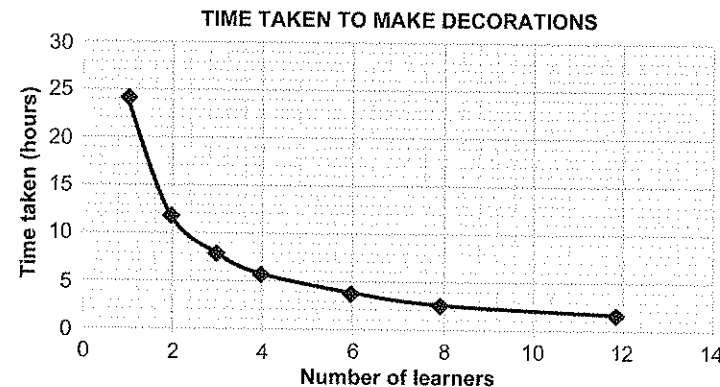


$\therefore$ The product remains constant at 24.

Indirect/Inverse Proportion Graphs

An indirect/inverse proportion graph will always be a **curved graph** that **never touches the x- or y-axis**.

- e.g. The graph below shows the time it takes learners to make decorations for the matric dance (see example above):



Refer to Module 2 to refresh your memory on drawing graphs.



Calculations with ratios in Indirect/Inverse Proportion

There are two methods to choose from when working with indirect/inverse proportion:

METHOD 1 Inverse Multiplication/Division Operations

Write down the ratios, then **multiply** one side, while **dividing** the quantities of the other side, in order to find the unknown.

e.g. 6 painters paint a house in 5 days. How many days will it take 12 painters to paint the same house?

$$\begin{array}{l} \text{painters : days} \\ \times 2 \quad 6 : 5 \quad \div 2 \\ \quad \quad 12 : ? \end{array}$$

$$\begin{aligned} \therefore \text{Number of days} &= 5 \div 2 \\ &= 2,5 \text{ days} \end{aligned}$$

NOTE! Opposite operations:
multiply on one side, **divide**
on the other side.



METHOD 2 Table with a constant product

Draw up a table and fill in the known values. Then multiply both quantities to find the constant product. Thereafter, use this constant product to solve for the unknown.

e.g. 6 painters paint a house in 5 days. How many days will it take 12 painters to paint the same house?

Number of painters	6	12
Number of days	5	?

$$6 \times 5 = 30$$

$$12 \times ? = 30$$

$$? = \frac{30}{12}$$



30 is the constant product

$$\therefore \text{Number of days} = 30 \div 12 = 2,5 \text{ days}$$



Worked Examples



1. Jim has volunteered to help renovate a crèche. If he had to do the renovations alone, it would take him 12 hours.

- 1.1 Complete the table below, showing the time it would take to renovate the crèche, depending on the number of people:

Number of people	1	2	3	4	6	12
Time taken (hours)	12					1

- 1.2 Using the table you completed in Question 1.1, draw the graph of the time it would take to complete the crèche renovations.

1.1

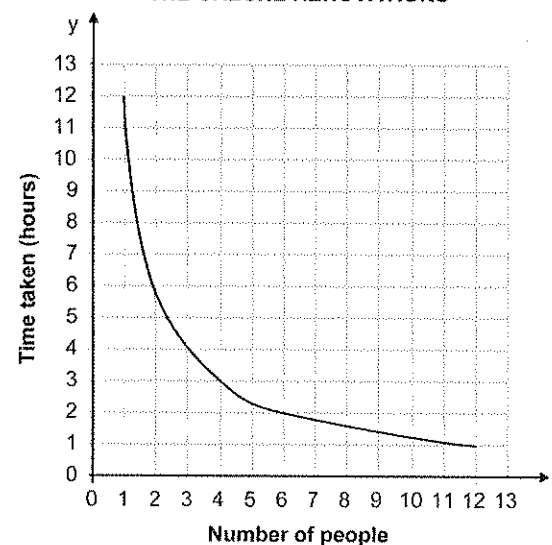
Number of people	1	2	3	4	6	12
Time taken (hours)	12	6	4	3	2	1

Constant product of 12 ... $1 \times 12 = 12$; $2 \times 6 = 12$ etc.



- 1.2

TIME NEEDED TO COMPLETE
THE CRÈCHE RENOVATIONS



2. If a swimming pool takes 8 hours to fill at a rate of 120 l per hour; how long will it take to fill if the rate of water flow is doubled?

METHOD 1

$$\begin{array}{l} \text{time : water flow} \\ 8 \text{ hrs : } 120 \text{ l/h} \\ + 2 \quad ? : 240 \text{ l/h} \end{array} \times 2$$

$$\therefore \text{Time taken} = 8 \text{ hrs} \div 2 = 4 \text{ hrs}$$

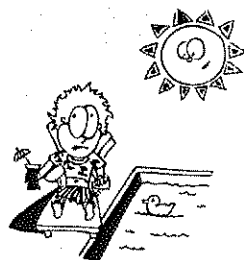
METHOD 2

Time (hrs)	8	?
Water flow (l/hr)	120	240

$$\begin{array}{l} 8 \times 120 = 960 \\ ? \times 240 = 960 \end{array}$$

Constant product!

$$\therefore \text{Time taken} = 960 \div 240 = 4 \text{ hrs}$$

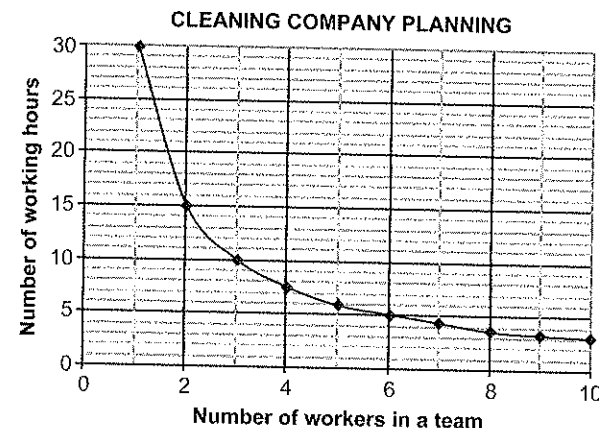


Test Your Understanding



- A bag of mielies that has a mass of 480 kg, costs R3 600. Calculate the price of a bag of mielies that has a mass of 200 kg.
- Twelve boy scouts decide to clean the camp. If it takes them 3 hours to clean up; how long would it take if there were only 8 boys?
- Examination rules specify that each learner is to be given a seating area in the examination venue of at least $1,6 \text{ m}^2$.
 - What is the minimum total area that is required for 52 learners sitting for an examination?
 - Calculate the maximum number of learners that can be accommodated in an examination venue with an area of 96 m^2 if the examination rules are adhered to.

4. Mrs White, the owner of a cleaning company, uses the graph below to determine how long it would take different teams of workers to clean a block of offices.



- How long will it take THREE workers to clean the same block of offices?
 - How many workers will she need to employ to complete cleaning the same block of offices in exactly SIX hours?
 - Estimate the number of hours it will take FOUR workers to clean the same block of offices.
5. Mr Khosa grows cabbages and carrots in his vegetable garden. He sells them in boxes. The table below shows the relationship between the number of cabbages and carrots in each box:

Relationship between the number of cabbages and carrots in each box

Number of boxes	1	2	5	B	15
Number of cabbages	2	4	10	24	30
Number of carrots	12	A	60	144	180

- The average mass of a cabbage is 2 kg and the average mass of a carrot is 0,12 kg. Calculate the total average mass of the cabbages and carrots in one box.
- Determine the following missing values:
 - A
 - B
- A customer placed an order for a number of boxes of vegetables which contained a total of 12 cabbages. How many carrots in total were included in these boxes?
- Using the table given above, draw the graph of the number of carrots included in every box.



What is a rate?



While a ratio is a comparison of two quantities that are measured in the same units, a rate is a **'special' ratio** that compares two quantities that are measured in **different units**.

Unlike ratios, when writing down rates, you must write down the different units of measurement that are being compared.

Example:

- the rate at which a car is travelling is measured in km **per** hour
- electricity is charged at R1,20 **per** kilowatt
- the exchange rate is R17 **per** Pound (£)

Working with rates



- It is often easy to first convert a rate into its **unit form**, by determining the **quantity per 1 unit**.
e.g. 12 potatoes cost R18
 $\therefore$ Cost of 1 potato = $R18 \div 12 = R1,50$
- To calculate variations based on a given rate, first determine the unit form and then **multiply both parts** of the ratio by the required number of units.
e.g. A Fiat Panda has an average petrol consumption of 5,7 l/100 km.
If the car has a 35 l tank, how far can it go on one tank of petrol?

$$\begin{array}{rcl} \text{litres : km} & & \\ \div 5,7 & 5,7 \text{ l : } 100 \text{ km} & \div 5,7 \\ \hline & 1 \text{ l : } 17,543... & \\ \times 35 & 35 \text{ l : ?} & \times 35 \end{array}$$

Never round off until your final answer!



$$\therefore \text{Number of km} = (100 \div 5,7) \times 35 = 614,04 \text{ km}$$

- When comparing rates (especially when shopping), determine the **unit form** of each item and then compare them.
e.g. Which is cheaper: 150 g box of candy for R12,50 or 200 g box of candy for R15,25?
 $\therefore$ 150 g box: $R12,50 \div 150 \text{ g} = R0,083.../g$
200 g box: $R15,25 \div 200 \text{ g} = R0,076.../g$
 $\therefore$ The 200 g box is cheaper per gram.

Worked Examples



- Jimmy's electrical bill stated that he used 1 438 kW of electricity for the month and the total due is R864,00. Determine the cost of electricity per kW.

R864,00 for 1 438 kW of electricity

$$\therefore \text{Cost per kW} = R864 \div 1\,438 \text{ kW} \\ = R0,60/\text{kW}$$

REMEMBER! State the units!

- Which cereal is cheaper?

Chocolate Puffs: 750 g box @ R18,99

Vanilla Twirls: 500 g box @ R14,95

$$\text{Chocolate Puffs: } R18,99 \div 750 \text{ g} = R0,025.../g$$

$$\text{Vanilla Twirls: } R14,95 \div 500 \text{ g} = R0,029.../g$$

$\therefore$ Chocolate Puffs cereal is cheaper.



Speed



- Speed is a common example of rate. It refers to how fast an object is travelling over time.
- We use kilometres per hour (km/h) to measure the speed of cars, trains and even planes, whereas the speed in the case of an athlete, for example, is usually measured in metres per second (m/s).
- The following triangle is useful in determining the relationship between distance, speed and time.



where D = distance
S = speed
T = time

Turn to Module 4 to refresh your memory on conversions.

This triangle is a summary of:

$$D = S \times T \quad (\text{distance} = \text{speed} \times \text{time})$$

$$S = \frac{D}{T} \quad (\text{speed} = \frac{\text{distance}}{\text{time}})$$

$$T = \frac{D}{S} \quad (\text{time} = \frac{\text{distance}}{\text{speed}})$$



Worked Examples

1. How far did Mpelo run if she ran for 15 minutes at 6 km/h?

$$15 \text{ minutes} = \frac{15}{60} = 0,25 \text{ hrs}$$

$$\begin{aligned} \therefore \text{Distance} &= \text{speed} \times \text{time} \\ &= 6 \text{ km/h} \times 0,25 \text{ h} \\ &= 1,5 \text{ km} \end{aligned}$$

Units must be the same!

2. Kirsten loves running. She completed a 10 km race in 48 minutes. Calculate her average speed in km/h.

$$48 \text{ minutes} = \frac{48}{60} \text{ hour} = 0,8 \text{ hour}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{10}{0,8} = 12,5 \text{ km/h}$$

Read the question carefully to make sure you answer in the **correct units**!

3. If Jeremiah cycled a distance of 48 000 m at a speed of 60 km/h; how long did Jeremiah cycle for?

$$48\,000 \text{ m} = 48\,000 \div 1\,000 = 48 \text{ km}$$

$$\begin{aligned} \therefore \text{Time} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{48 \text{ km}}{60 \text{ km/h}} = 0,8 \text{ h} \end{aligned}$$

NOTE! To convert hours to minutes, multiply by 60.

$$\begin{aligned} \therefore 0,8 \text{ h} &= 0,8 \times 60 \\ &= 48 \text{ minutes} \end{aligned}$$

Test Your Understanding

- Water is charged at R11,72/kℓ.
 - Calculate the cost of 342 ℓ of water.
 - How many kilolitres of water did one use if the water bill totalled R164,08?
- If a car can drive 500 km on a 35 litre tank of petrol, calculate the car's consumption per 100 km.

- Lusanda travels 25 km to work every day. If it takes her approximately 33 minutes to get to work, calculate her approximate speed in km/h.
- A turkey needs to be roasted for 30 minutes per 500 grams of meat. How long will it take to roast a turkey that weighs 1,2 kg?
- A family on a weekend getaway travels at a speed of 120 km/h on the N2. If they are 160 km away from their destination, how long will it take to reach their destination? Give your answer in hours and minutes.
- Which face cream is the most expensive of the 3 brands sold at a pharmacy?
 - Smooth Delight (175 ml) for R89,99
 - Creamy Soft (180 ml) for R92,99
 - Silky Vanilla (150 ml) for R70,99

UNIT

7

PERCENTAGES

✓ gr 12

What is percentage (%)?

A percentage shows the size of a value in relation to a whole, where the whole is defined as being 100. A percentage is always expressed as a fraction out of 100.

Calculating percentages

- To calculate a percentage (%), we always determine the fraction of 100, i.e. take the fraction and multiply it by 100.

e.g. Write $\frac{4}{9}$ as a percentage.

$$\therefore \frac{4}{9} \times 100 = 44,44\%$$

e.g. Write 1,47 as a percentage.

$$\therefore 1,47 \times 100 = 147\%$$

Always round off to 2 decimal places; unless stated otherwise.

Increase a value by a certain percentage



There are two methods that can be used to increase an amount by a certain percentage:

e.g. Increase R450 by 12%

METHOD 1 Calculate the increase and add it to the original amount

$$\begin{aligned}\therefore \text{Increase} &= 12\% \text{ of R450} \\ &= \frac{12}{100} \times \text{R450} \\ &= \text{R54}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total amount} &= \text{original amount} + \text{increase} \\ &= \text{R450} + \text{R54} \\ &= \text{R504}\end{aligned}$$

METHOD 2 Multiply the original amount by (100% + % increase)

$$\begin{aligned}\text{Total} &= \text{Original amount} \times (100\% + \% \text{ Increase}) \\ &= \text{R450} \times (100\% + 12\%) \\ &= \text{R450} \times 112\% \\ &= \text{R450} \times \frac{112}{100} \\ &= \text{R504}\end{aligned}$$

NOTE!

The 100% represents the original amount.



Decrease a value by a certain percentage



There are two methods that can be used to decrease an amount by a certain percentage:

e.g. Decrease R450 by 12%

METHOD 1 Calculate the decrease and subtract it from the original amount

$$\begin{aligned}\therefore \text{Decrease} &= 12\% \text{ of R450} \\ &= \frac{12}{100} \times \text{R450} \\ &= \text{R54}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total amount} &= \text{original amount} - \text{decrease} \\ &= \text{R450} - \text{R54} \\ &= \text{R396}\end{aligned}$$

METHOD 2 Multiply the original amount by (100% - % decrease)

$$\begin{aligned}\text{Total} &= \text{original amount} \times (100\% - \% \text{ decrease}) \\ &= \text{R450} \times (100\% - 12\%) \\ &= \text{R450} \times 88\% \\ &= \text{R450} \times \frac{88}{100} \\ &= \text{R396}\end{aligned}$$

Worked Examples



- Josephine scored 94 out of 150 for her prelim exam. What percentage did she achieve?

$$\frac{94}{150} \times 100 = 62,67\%$$

REMEMBER!

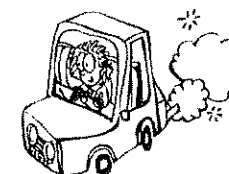
Always round off to 2 decimal places unless stated otherwise.



- A car costs R220 000 in 2014. If the price of cars increases by 8,5% p.a., determine the cost of the car in 2015.

$$\begin{aligned}\text{METHOD 1} \quad \text{Increase} &= 8,5\% \text{ of R220 000} \\ &= \frac{8,5}{100} \times \text{R220 000} \\ &= \text{R18 700}\end{aligned}$$

$$\therefore \text{New total cost} = \text{R220 000} + \text{R18 700} = \text{R238 700}$$



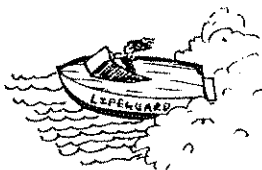
$$\begin{aligned}\text{METHOD 2} \quad \text{New total cost} &= \text{R220 000} \times (100\% + 8,5\%) \\ &= \text{R220 000} \times 108,5\% \\ &= \text{R220 000} \times \frac{108,5}{100} \\ &= \text{R238 700}\end{aligned}$$

3. The value of a R460 000 boat decreases by 4,8% p.a.
Determine the value of the boat after 1 year.

METHOD 1

$$\begin{aligned}\text{Decrease} &= 4,8\% \text{ of R460 000} \\ &= \frac{4,8}{100} \times \text{R460 000} \\ &= \text{R22 080}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total amount} &= \text{R460 000} - \text{R22 080} \\ &= \text{R437 920}\end{aligned}$$



METHOD 2

$$\begin{aligned}\text{Total} &= \text{R460 000} \times (100\% - 4,8\%) \\ &= \text{R460 000} \times 95,2\% \\ &= \text{R460 000} \times \frac{95,2}{100} \\ &= \text{R437 920}\end{aligned}$$

Calculating a percentage increase

- Calculate the percentage by which a quantity has increased, using the following general formula:

$$\% \text{ Increase} = \frac{\text{increase}}{\text{starting value}} \times 100$$

e.g. Determine the percentage increase if a fizz pop cost R2,50 in 2014 and R3,50 in 2015.

$$\begin{aligned}\therefore \% \text{ Increase} &= \frac{\text{increase}}{\text{starting value}} \times 100 \\ &= \frac{\text{R3,50} - \text{R2,50}}{\text{R2,50}} \times 100 \\ &= \frac{\text{R1,00}}{\text{R2,50}} \times 100 \\ &= 40\%\end{aligned}$$



Calculating a percentage decrease

- Calculate the percentage by which a quantity has decreased, using the following general formula:

$$\% \text{ Decrease} = \frac{\text{decrease}}{\text{starting value}} \times 100$$

e.g. The population of rhino in 2005 was 12 800, but had decreased to 8 700 in 2006. Determine the percentage decrease in the rhino population.

$$\begin{aligned}\therefore \% \text{ Decrease} &= \frac{\text{decrease}}{\text{starting value}} \times 100 \\ &= \frac{12\,800 - 8\,700}{12\,800} \times 100 \\ &= \frac{4\,100}{12\,800} \times 100 = 32,03\%\end{aligned}$$



Worked Examples

- What is the percentage mark-up on a belt, if the cost price is R38 and the selling price is R120?

$$\begin{aligned}\% \text{ Increase} &= \frac{\text{increase}}{\text{starting value}} \times 100 \\ &= \frac{120 - 38}{38} \times 100 \\ &= \frac{82}{38} \times 100 = 215,79\%\end{aligned}$$

$$\therefore \% \text{ Mark-up} = 215,79\%$$

$\therefore \% \text{ Mark-up}$ is the equivalent to $\% \text{ increase!}$



- A plot of land was bought for R275 000. If the owner could only sell the plot of land for R220 000, determine the owner's percentage loss.

$$\begin{aligned}\% \text{ Decrease} &= \frac{\text{decrease}}{\text{starting value}} \times 100 \\ &= \frac{275\,000 - 220\,000}{275\,000} \times 100 \\ &= \frac{55\,000}{275\,000} \times 100 = 20\%\end{aligned}$$

$$\therefore \% \text{ Loss} = 20\%$$

$\therefore \% \text{ Loss}$ is the equivalent to $\% \text{ decrease!}$



Calculations regarding discount



Calculating the discount

- Determine the discount percentage value of the original price

e.g. A pair of winter boots, that would retail for R1 500, are advertised as 15% off the original price. Determine the discount.

$$\begin{aligned}\therefore \text{Discount} &= 15\% \text{ of R1 500} \\ &= \frac{15}{100} \times \text{R1 500} \\ &= \text{R225}\end{aligned}$$



Calculating the sale price

- Decrease the original price by the percentage discount.
- There are two methods (see p. 18) to calculate the discount:

e.g. A pair of winter boots, that usually retail for R1 500, are advertised as 15% off the original price. Determine the sale price.

METHOD 1

$$\begin{aligned}\text{Discount} &= 15\% \text{ of R1 500} \\ &= \frac{15}{100} \times \text{R1 500} \\ &= \text{R225} \\ \therefore \text{Sale price} &= \text{R1 500} - \text{R225} \\ &= \text{R1 275}\end{aligned}$$

METHOD 2

$$\begin{aligned}\text{Sale price} &= \text{R1 500} \times (100\% - 15\%) \\ &= \text{R1 500} \times 85\% \\ &= \text{R1 000} \times \frac{85}{100} \\ &= \text{R1 275}\end{aligned}$$



REMEMBER!
100% represents the original price;
so 85% represents the sale price after the discount percentage has been subtracted.

Calculating the original price

e.g. A pair of winter boots cost R1 275 on sale. If the boots were on a 15% sale, determine the original price.

- There are two methods:

METHOD 1

Ratio method

$$\begin{aligned}\text{sale price} &: \text{original price} \\ \dots \% &: 100\% \\ \text{R} \dots &: \text{R} ?\end{aligned}$$

$$\begin{aligned}\text{sale price} &: \text{original price} & \text{Original price} &\approx 100\% \\ 85\% &: 100\% & \therefore \text{Sale price} & \\ \text{R1 275} &: \text{R} ? & = 100\% - 15\% & \\ & & = 85\% &\end{aligned}$$

$$\begin{array}{ccc} \div 85 & \therefore 85 : 100 & \div 85 \\ \times 1\,275 & 1 : \dots & \times 1\,275 \\ & 1\,275 : ? & \end{array}$$

$$\begin{aligned}\therefore \text{Original Price} &= (100 \div 85) \times 1\,275 \\ &= \text{R1 500}\end{aligned}$$

METHOD 2

Equation method

$$\begin{aligned}\text{original price} \times (\text{original \%} - \text{discount \%}) &= \text{sale price} \\ \therefore \text{original price} &= \text{sale price} \div (\text{original \%} - \text{discount \%}) \\ &= \text{R1 275} \div (100\% - 15\%) \\ &= \text{R1 275} \div 85\% \\ &= \text{R1 275} \div \frac{85}{100} \\ &= \text{R1 500}\end{aligned}$$



Worked Examples



1. A nursery sells a tray of seedlings for R12,99. If the nursery has a '10%-off' sale; determine the:

1.1 Discount

$$\begin{aligned} 1.1 \text{ Discount} &= 10\% \text{ of } R12,99 \\ &= \frac{10}{100} \times R12,99 \\ &= R1,30 \end{aligned}$$

1.2 Sale Price

NOTE!

R1,299 needs to be rounded off to 2 decimal places when working with rand and cents.



1.2 **METHOD 1**

Sale Price

$$\begin{aligned} &= \text{original price} - \text{discount} \\ &= R12,99 - R1,30 \\ &= R11,69 \end{aligned}$$

METHOD 2

Sale Price

$$\begin{aligned} &= \text{original price} \times (\text{original \%} - \text{discount \%}) \\ &= R12,99 \times (100\% - 10\%) \\ &= R12,99 \times 90\% = R11,69 \end{aligned}$$

2. If a lavender bush was on sale for R52,99, on the '10%-off' sale; determine the original price of the lavender bush.

METHOD 1

$$\begin{array}{rcl} \text{sale price : original price} & & \\ + 90 & 90\% : 100\% & + 90 \\ \times 52,99 & 1 : \dots & \times 52,99 \\ \hline & R52,99 : ? & \end{array}$$

$$\therefore \text{Original price} = (100 \div 90) \times 52,99 = R58,88$$

METHOD 2

$$\text{original price} \times (\text{original \%} - \text{discount \%}) = \text{sale price}$$

$$\begin{aligned} \therefore \text{Original price} &= \text{sale price} \div (\text{original \%} - \text{discount \%}) \\ &= R52,99 \div (100\% - 10\%) \\ &= R52,99 \div 90\% \\ &= R52,99 \div \frac{90}{100} \\ &= R58,88 \end{aligned}$$

Test Your Understanding



1. Which person pays higher bank fees, if the following transaction formulae apply to a deposit of R760:

- Person A pays R2,50 + 0,75% of amount deposited
- Person B pays R1,50 + 0,85% of amount deposited



2. Increase 1 255 kg by 16%.

3. A pottery store is closing down, so they decide to decrease the price on their statues by 75%. Determine the sale price of the statues, if they cost R350 each.

4. The number of South Africans living with HIV/AIDS in 2011 was 5 600 000 and 6 100 000 in 2012. Calculate the percentage increase in South Africans living with HIV/AIDS.

5. The total income generated by South African exports was R314 927 million in 2002 and R273 127 million in 2003. Determine the percentage decrease in total income generated by South African exports.

6. A batch of floor tiles was sold for R1 800, after a 20% sale discount. Determine the original price of the batch of floor tiles.

7. If a laptop worth R6 999 decreases in value by 4% p.a.; determine the value of the laptop after 1 year.

8. Calculate Poliso's percentage profit if it cost her R1 150 to make a matric dance dress; and she sold it for R2 400.

9. A painter's normal labour charge is R35,90 per hour. On Saturdays he charges 50% more than the normal rate. On Friday and Saturday he works for six hours each day. His invoice showed that he was owed R638,50 for his work. Verify whether the invoiced amount was correct.

10. According to the Agricultural Research Council, 80% of South Africa's land surface area is used for farming. However, only 11% of the farming land is suitable for planting crops. If the total land surface of South Africa is 1 219 090 km²; how much of the total land surface area is suitable for planting crops?

2 PATTERNS, RELATIONSHIPS AND REPRESENTATIONS

UNIT

1

MAKING SENSE OF GRAPHS THAT TELL A STORY

It is important to be able to read and understand any graph that is commonly found in newspapers, magazines and online etc. By investigating different features of a graph in context, we are able to make sense of the story behind the graph.

Features of a Graph in Context

Axes

- the quantity plotted on the **x-axis** is the **independent variable**, e.g. time
- the quantity plotted on the **y-axis** is the **dependent variable**, e.g. distance

Shape of the graph

- a **positive straight line** graph represents an increasing function, e.g. travelling up in a lift
- a **negative straight line** graph represents a decreasing function, e.g. returning home from work
- a **horizontal straight line** often represents **no movement**, e.g. waiting at a red robot in traffic
- a **curved line** (e.g. a parabola) often represents **motion** of an object, e.g. a ball being thrown

Slope of a straight line graph

- the **steeper the slope** of the straight line, the **quicker the change** in the dependent variable (y) vs the independent variable (x)
- in a graph of distance vs time, the **slope** of the graph represents the **speed** (since $\text{speed} = \frac{\text{distance}}{\text{time}}$).

Critical values of the graph

- identify the **maximum** and **minimum** values of the graph
- identify the **zero values** of the graph, i.e. where the graph **intersects** the x- and/or y-axis.

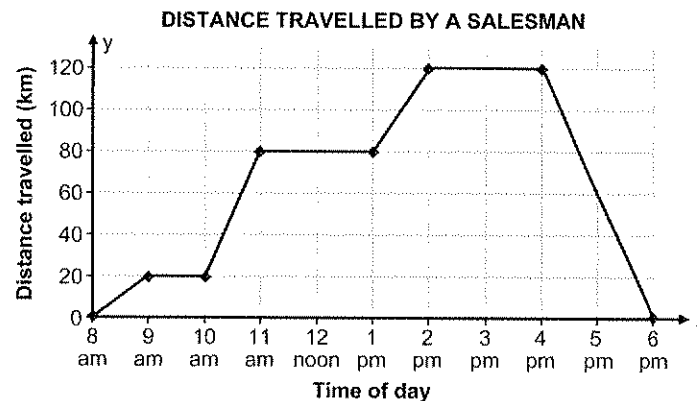


Refer to p. 25 for the **mathematical features** that are used to interpret and understand graphs.



Worked Examples

- Below is the graph of the distance travelled by a salesman:



- At what time did the salesman leave home?
8 am
- How far did the salesman travel to his first client?
20 km
- How long did the salesman spend at his first client?
1 hour



- 1.4 How many clients did the salesman visit in a day?

3 clients

- 1.5 How many kilometres did the salesman travel from the first client to his second client?

$$80 - 20 = 60 \text{ km}$$

- 1.6 Describe what the salesman did from 4 pm to 6 pm.

Salesman travelled back home.

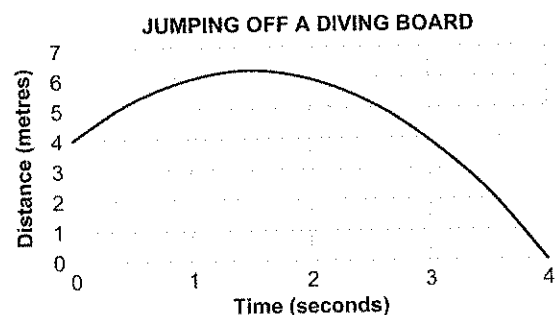
- 1.7 Determine the average speed at which the salesman was travelling from 4 pm to 6 pm.

$$\text{Average Speed} = \frac{\text{change in distance}}{\text{change in time}}$$

$$\text{Average speed} = \frac{120 \text{ km}}{2 \text{ h}} = 60 \text{ km/h}$$



2. The graph below represents Ursula's distance from the water over time, as she jumps off a diving board:



- 2.1 State the dependent and independent variables in this graph.

Dependent: Distance ; Independent: Time

- 2.2 What is the height (above the water) of the diving board?

4 m

- 2.3 How long does it take Ursula to reach 6 metres above the water?

1 second

- 2.4 Why are there two time values for the height 6 metres?

Ursula passes the 6 m mark twice when she jumps off the diving board up into the air and again when she dives down into the water.

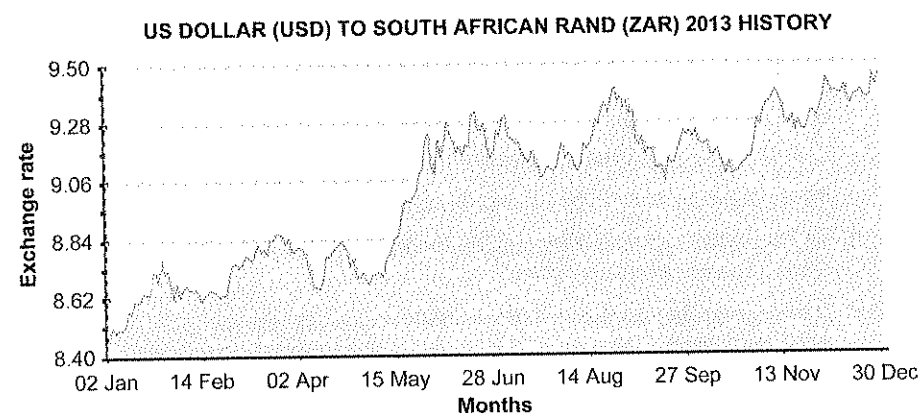
- 2.5 Estimate the maximum height Ursula achieves. After how many seconds does this occur?

$\pm 6,2 \text{ m}$ after 1,5 seconds

- 2.6 How many seconds after Ursula jumped off the diving board did she enter the water?

4 seconds

3. The graph below shows the exchange rate history of the US Dollar (USD) to the South African Rand (ZAR) for 2013:



- 3.1 What was the exchange rate on 15 May 2013?

8,84

- 3.2 On what date was the exchange rate approximately 8,60?

14 February

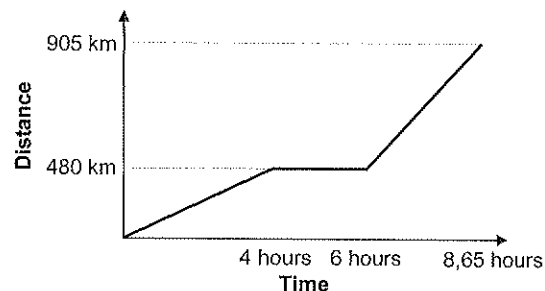
- 3.3 Describe the overall trend in this graph of the history of the US Dollar to South African Rand in 2013.

Overall, the exchange rate between the US Dollar and the South African Rand fluctuated between 8,40 and 8,84 between January and May. However, from 15 May, the exchange rate steadily climbed and ended on approximately 9,48 on 30 December.



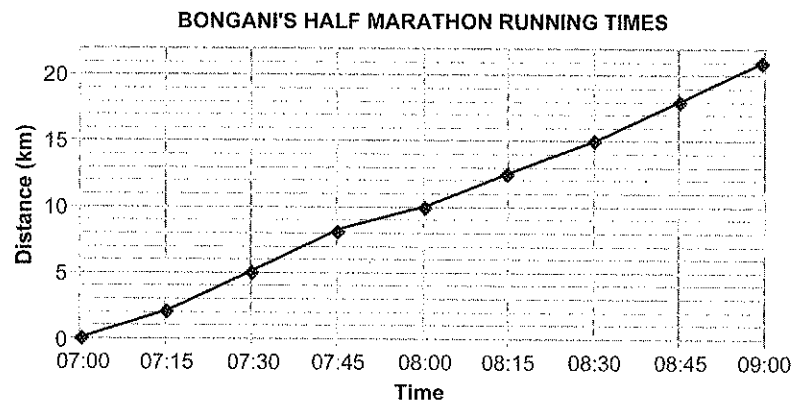
Test Your Understanding

1. The graph below shows the distance the North family travelled on their way from Johannesburg to Cape Town:



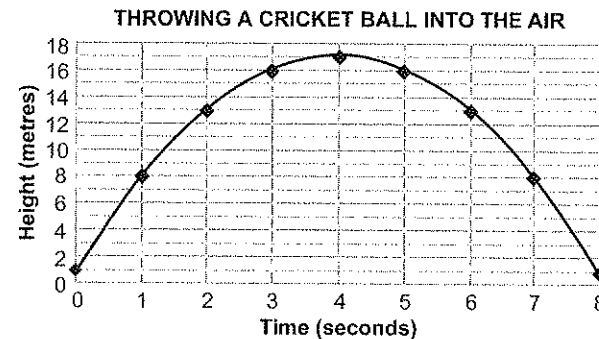
- 1.1 Identify the independent and dependent variables.
- 1.2 What does the flat (horizontal) part of the graph represent?
- 1.3 During which part of the trip did the family travel the fastest? Give a reason for your answer.
- 1.4 If the speed limit on the road was mostly 120 km/h, do you think the North family was speeding during the last 2,65 hours? Substantiate your answer by means of calculations.

2. The graph below shows the distance Bongani covered (in 15 minute intervals) during the Grape Run half marathon (21,1 km). Use the graph to answer the questions that follow:



- 2.1 Identify the independent variable.
- 2.2 What distance did Bongani cover during the first 15 minutes?
- 2.3 How long did it take Bongani to cover 12,5 km? Give your answer in minutes.

- 2.4 Did Bongani stop to rest anytime during the half marathon? Give a reason for your answer.
 - 2.5 How does his speed (in km/h) for the first 15 minutes compare to his speed from 07:15 - 09:00? Show by means of calculations and give a possible reason for your findings.
3. Richard threw a cricket ball into the air and then chased after the ball trying to catch it before it hit the ground.



- 3.1 At 0 seconds the height of the ball is 1 metre from the ground. What does this mean?
- 3.2 How long did it take for the ball to reach its maximum height? What was the maximum height?
- 3.3 How long did the ball stay in the air?
- 3.4 Did Richard catch the ball or did it drop to the ground? Explain your answer.

UNIT

2

PATTERNS AND RELATIONSHIPS

In this unit, we will be investigating 5 different patterns and relationships:

- A Constant/Fixed relationship
- B Linear relationship (including in **direct proportion** and **not in direct proportion**)
- C Indirect/Inverse Proportion relationship
- D Constant Ratio relationship
- E Combination of the different types of relationships.

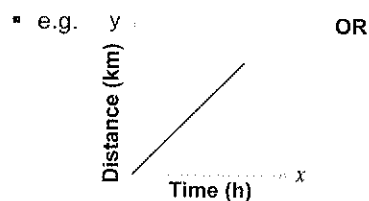
To understand these relationships, we must know the mathematical features of a graph.

Mathematical Features of a Graph



Variables

- the **independent** variable is the quantity you **choose**; and is usually represented on the x -axis of a graph.
- the **dependent** variable is the quantity which **depends** on the independent/chosen variable; and is usually represented on the y -axis of a graph.



OR

Time (h)	0	1	2	3
Distance (km)	0	60	120	360

$\therefore$ Time is the independent variable.
Distance is the dependent variable.

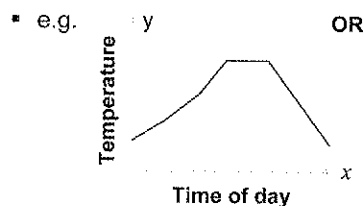
NOTE! The independent variable is always written in the first row of a table!



Types of variables

- discrete** variables have values that are only **whole numbers**, e.g. number of children, number of tins of paint, etc.
- continuous** variables have values that can be **any type of number** (i.e. whole numbers, fractions, decimals) e.g. height of a man, the mass of a rock, etc.

NOTE! Data is classified as **discrete** or **continuous**, based on the **independent variable**.



Number of candles	1	2	3	4
Total cost (R)	52	104	156	208

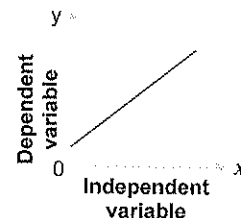
$\therefore$ Data is **continuous**, as the **independent variable** (time) is **continuous**.

$\therefore$ Data is **discrete**, as the **independent variable** (number of candles) is **discrete**.

Types of relationships

- An **increasing relationship** occurs when the dependent variable **increases** as the independent variable **increases**.
- A **decreasing relationship** occurs when the dependent variable **decreases** as the independent variable **increases**.

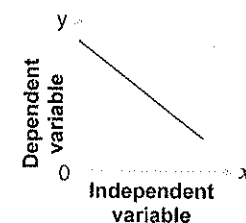
e.g.



$\therefore$ An **increasing function**, because as x gets bigger, y also gets bigger.



NOTE: The straight line has a **positive gradient**.



$\therefore$ A **decreasing function**, because as x gets bigger, y gets smaller.



NOTE: The straight line has a **negative gradient**.

Critical Values

- the **minimum** is the **smallest** value on a graph.
- the **maximum** is the **biggest** value on a graph.



NOTE! In a **constant/fixed relationship**, the **minimum** and **maximum** values are the same.



- the **zero** values is when either x or y equal zero.

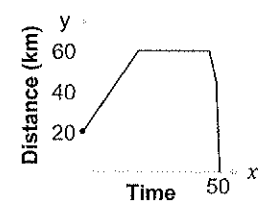


NOTE! An **indirect/inverse proportional graph** will not have a zero value.

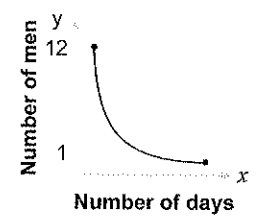
Refer to p. 22 for the **features of a graph in context** that are used to interpret and understand graphs.



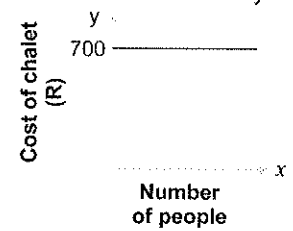
Examples of critical values on graphs



∴ Minimum value = 20
Maximum value = 60
Zero values: when $x = 0$; $y = 20$
and
when $y = 0$; $x = 50$



∴ Minimum value = 1
Maximum value = 12
No zero values



∴ Minimum value = 700
Maximum value = 700
If $x = 0$, then $y = 700$



Worked Examples

1. Alongside is a graph of the income generated from selling trees:

1.1 Name the dependent variable.

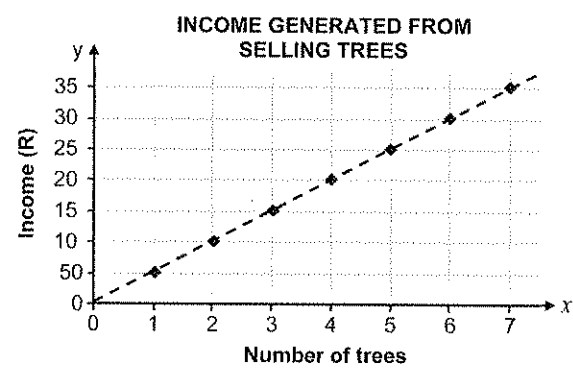
Income

1.2 Is this graph an increasing or decreasing function?

Increasing function

1.3 Is the data discrete or continuous?

Discrete data



NOTE! Although discrete data should be represented as a dotted line graph, it may be given as a solid line, but you only read **whole numbers** from the graph.

1.4 List the zero values.

When $x = 0$; $y = 0$

REMEMBER!

Data is classified based on the independent variable, i.e. number of trees.



2. Below is a table showing the relationship between the number of hours of sleep required by growing babies:

Age of babies (weeks)	1	2	3	4	5	6
Number of hours of sleep	20	19	18	17	16	15

2.1 Name the independent variable.

Age of babies

2.2 Is the data classified as discrete or continuous?

Continuous data

2.3 Is this an increasing or decreasing function?

Decreasing function

2.4 What are the maximum and minimum values?

Maximum = 20 hours of sleep

Minimum = 15 hours of sleep



REMEMBER!

The **independent variable** is always written on the **top row of a table**.



Formal Relationships

The 5 formal patterns and relationships are:

- A** Constant/fixed relationship
- B** Linear relationship (including in direct proportion and NOT in direct proportion)
- C** Indirect/Inverse proportion relationship
- D** Constant Ratio relationship
- E** Combination of all the different types of relationships



Representations of Formal Relationships

Formal relationships can be represented in **tables**, **equations/formulae** or **graphs**.

For the formal relationships **A**, **B** and **C** mentioned above, we will learn how to:

- 1 construct a table from a given scenario
- 2 determine a formula from a table to represent the relationship
- 3 use the formula to find missing values in a table
- 4 use the values in the table to draw a graph



Also, if a graph is given, we need to be able to:

- 5 determine the **type of relationship** represented by the graph
- 6 determine the **formula** from the graph
- 7 read off **values** from the graph

These skills will be illustrated by means of two worked examples for each of the three different formal relationships: **A**, **B** and **C**.

A Constant/Fixed Relationship



A **constant/fixed relationship** occurs when the dependent variable (y) remains constant, while the independent variable (x) continues to increase.

EXAMPLE 1 of a constant/fixed relationship:

Peter investigates the cost of hiring a night club for the night (considered 8 hours); in order to celebrate his 21st birthday.

He gets three quotes from the following nightclubs:

- Night Live: R12 000
- Disco Lounge: R5 800
- Smooth Vibes: R8 200



1 Constructing a table



- ▶ A table will always consist of a minimum of 2 rows: the **first row** is for the **independent variable** and the **second row** is for the **dependent variable**. If there is a number of different dependent variables, then a row is added. This is then followed by enough columns to show the pattern clearly.
- ▶ Always plot zero (0) and one (1) for the independent variable, followed by numbers that you select, which will clearly show a trend.
- ▶ Use the independent variable (x) to calculate the dependent variables (y), based on the information given.



The costs of hiring a night club for Peter's 21st birthday party can be illustrated in a table as follows:

Number of hours	0	1	2	4	6	8	independent variable
Cost of Night Live (R)	12 000	12 000	12 000	12 000	12 000	12 000	different dependent variables
Cost of Disco Lounge (R)	5 800	5 800	5 800	5 800	5 800	5 800	
Cost of Smooth Vibes (R)	8 200	8 200	8 200	8 200	8 200	8 200	

NOTE! In a constant/fixed relationship, the concept of adding the '0' column often results in a 'silly' scenario, as you wouldn't hire a night club for 0 hours! However, we include the '0' column, as it illustrates the trend. In this example, it can be thought to represent Peter hiring the club for less than an hour.

2 Determining a formula from a table



- ▶ When deriving a formula from a table, first assess the type of pattern or relationship that is present.
- ▶ Then relate the independent and dependent variables in a formula/equation:
dependent variable =
- ▶ The general formula for a constant relationship will be:



What must you do to your independent variable to give you your dependent variable?

Dependent Variable = constant/fixed value

The costs of hiring a night club for Peter's 21st birthday party can be represented by the following formulae:

- Cost of hiring Night Live = R12 000
- Cost of hiring Disco Lounge = R5 800
- Cost of hiring Smooth Vibes = R8 200



3 Finding missing values in a table



- ▶ Missing values in a table may be found by using the information in the given scenario, to determine the relationship between the independent and dependent variables.
- ▶ Alternatively, the missing values can also be found by substituting into a formula.



In the example of the costs of hiring the night club for Peter's 21st birthday party, find the missing values in the table below.

Number of hours	0	1	2	3	4	B	6	7	8
Cost of Night Live (R)	12 000	12 000	12 000	A	12 000	12 000	12 000	12 000	12 000
Cost of Disco Lounge (R)	5 800	5 800	5 800	5 800	5 800	5 800	5 800	5 800	5 800
Cost of Smooth Vibes (R)	8 200	8 200	8 200	8 200	8 200	8 200	8 200	C	8 200

There are 2 approaches to finding the missing values.

BY INSPECTION

From the given scenario, it is clear that the cost of hiring the night club will always be a constant for each of the night clubs respectively.

$$\therefore A = R12\ 000$$

Since the number of hours (up to a maximum of 8 hours in this scenario) does not affect the cost of hiring the night club the value of 'B' can be any number

$$\therefore B = 4,5 \text{ hours OR } 5 \text{ hours OR } 5,5 \text{ hours, etc.}$$

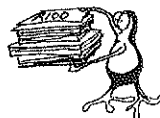
BY SUBSTITUTION

If the same scenario was given and the formula:
Cost of Smooth Vibes = R8 200

then we could determine the missing value by substituting into the formula.

$$\therefore \text{Cost of Smooth Vibes} = R8\ 200$$

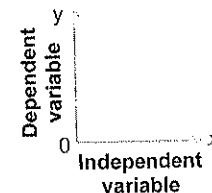
$$\therefore C = R8\ 200$$



4 Drawing graphs from tables



- ▶ On the Cartesian plane it is always the case that the **independent variables** are displayed on the **horizontal axis** (the x -axis) and the **dependent variables** are displayed on the **vertical-axis** (the y -axis).
- ▶ In a point, notated by $(x; y)$, the x -coordinate is always the first coordinate while the y -coordinate is always the second coordinate.



$(x; y)$
(independent variable ; dependent variable)

- ▶ The graph of a constant relationship will always be a straight line parallel to the horizontal axis (x -axis), at the point of the dependent variable.



In the example of the costs of hiring the night club for Peter's 21st birthday party, the graphs on p. 29 can be drawn, based on the information from the table below.

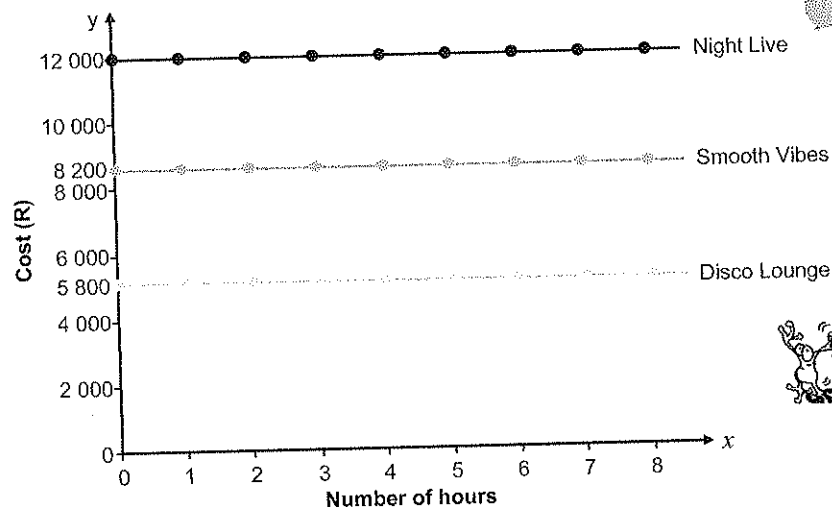
Number of hours	0	1	2	4	6	8	independent variable
Cost of Night Live (R)	12 000	12 000	12 000	12 000	12 000	12 000	different dependent variables
Cost of Disco Lounge (R)	5 800	5 800	5 800	5 800	5 800	5 800	
Cost of Smooth Vibes (R)	8 200	8 200	8 200	8 200	8 200	8 200	

Since there are 3 different dependent variables, there will be 3 different graphs to plot.

Therefore, the following points will be plotted per graph:

- Graph of the Cost of Night Live:
(0; 12 000); (1; 12 000); (2; 12 000); (4; 12 000); (6; 12 000); (8; 12 000)
- Graph of the Cost of Disco Lounge:
(0; 5 800); (1; 5 800); (2; 5 800); (4; 5 800); (6; 5 800); (8; 5 800)
- Graph of the Cost of Smooth Vibes:
(0; 8 200); (1; 8 200); (2; 8 200); (4; 8 200); (6; 8 200); (8; 8 200)

THE COSTS OF HIRING DIFFERENT NIGHT CLUBS



We join the plotted points with a **solid line** since the data is **continuous** (i.e. independent variable = number of hours is continuous data).

EXAMPLE 2 of a constant/fixed relationship:

Mountain High School has decided to rent a party-bus for their matric learners, in order to transport them from the dance to their after-party. Mountain High School got the following 3 quotes. (The party-bus caters for a maximum of 50 people)

- ▶ Safe Travels: R3 900
- ▶ Quick Transfers: R6 600
- ▶ Happy Bus: R4 500

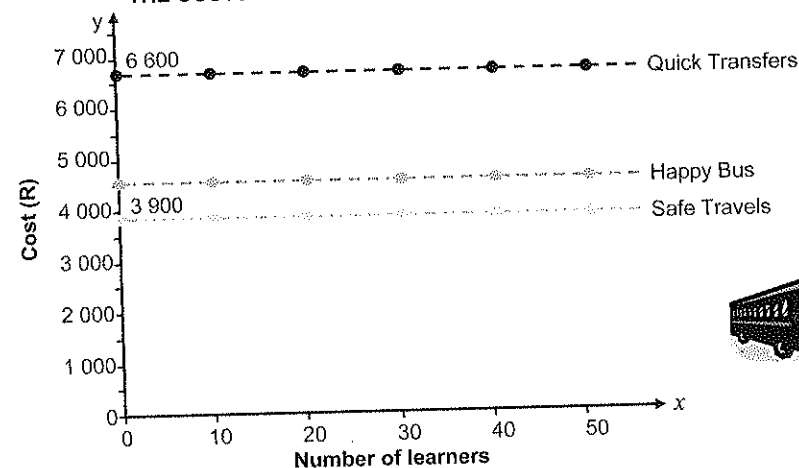


5 Determine the type of relationship from a graph

- ▶ Identify the shape of the graph, in order to determine the type of relationship (i.e. a constant relationship; a linear relationship that is or isn't directly proportional; an indirect/inverse relationship; a constant ratio relationship; or a combination of all the different types of relationships).

In the example of the costs of hiring a party-bus for Mountain High School's matric learners, the following graph would be given:

THE COSTS OF HIRING DIFFERENT PARTY BUSES



We join the plotted points with a **dotted line**, since the data is **discrete** (i.e. independent variable = number of learners, is classified as discrete data).

If we look at each graph, we see that the **dependent variable** (i.e. cost) is **constant** (for each respective graph), while the **independent variable** (i.e. the number of learners) **increases**.

∴ This is a constant/fixed relationship.

6 Determining the formula from a graph

- ▶ In order to determine the formula from the graph, the type of relationship must first be determined.
- ▶ Once the type of relationship has been decided on, a formula can be determined by evaluating the relationship between the independent and dependent variables.

The general formula for constant/fixed relationships is:

$$\text{Dependent Variable} = \text{constant/fixed value}$$

In the graph above of the costs of hiring different party buses; it can be seen that a constant/fixed relationship exists (as the dependent variable remains constant while the independent variable increases).

Therefore, the costs of hiring different party buses for Mountain High's matric learners can be represented by the following formulae:

- Cost of Safe Travels = R3 900
- Cost of Quick Transfers = R6 600
- Cost of Happy Bus = R4 500





Reading off values from a graph

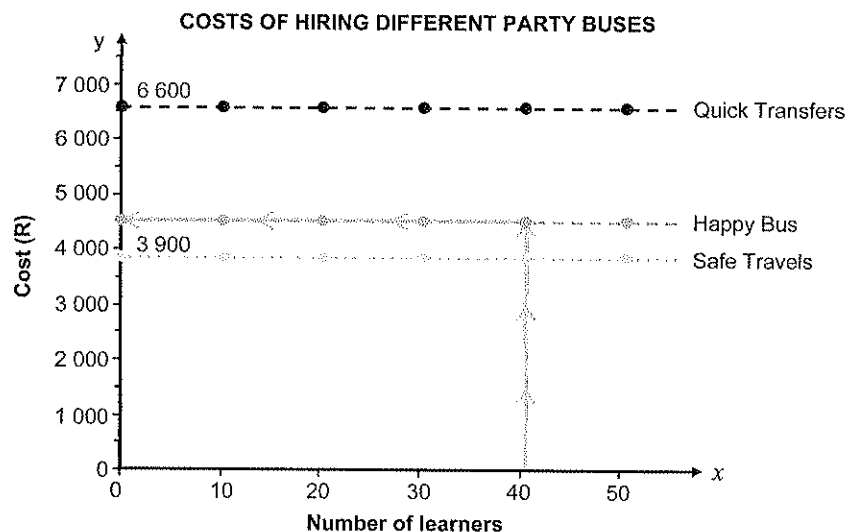


- ▶ If the question is asking for the corresponding dependent variable, locate the given independent variable on the x -axis and travel vertically upwards until you hit the graph. Now at that point, travel horizontally across until you reach the y -axis. Now read off the dependent variable value.
- ▶ If the question is asking for the corresponding independent variable, locate the given dependent variable on the y -axis and travel horizontally across until you hit the graph. Now at that point, travel vertically downwards until you reach the x -axis. Now read off the independent variable value.

Refer to the graph of the costs of hiring different party buses (see p. 29); and answer the following questions:

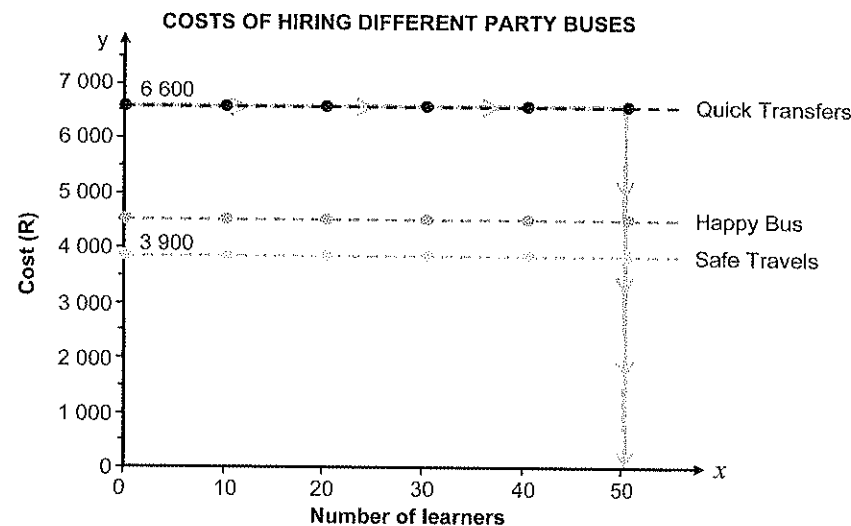
Q: What would it cost to hire the Happy Bus party bus for 40 learners?

A: R4 500



Q: What is the maximum number of learners that can be transported if Quick Transfers party bus was hired?

A: 50 learners



Test Your Understanding



1. Lucy investigates the bank charges of three different banks:

- ▶ Grow Rich Bank charges R5
- ▶ Bucks Bank charges R8
- ▶ Green Bank charges R4



All three banks charge those rates given above, up to a maximum of 10 transactions.

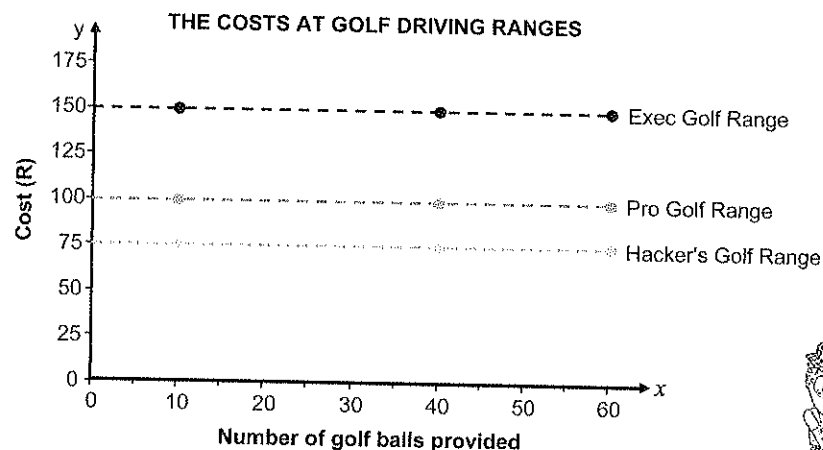
1.1 Given the table below, find the missing values:

	Number of transactions					
Total costs (R)	1	2	5	7	D	10
Grow Rich Bank	5	A	5	5	5	5
Bucks Bank	8	8	B	8	8	8
Green Bank	4	4	4	C	4	4

1.2 Determine the formulae to represent the bank charges charged by the three different banks.

1.3 Draw a graph of all three banks' charges on the same set of axes.

2. Below is a graph indicating the different costs of practising golf at different driving ranges:



- 2.1 What type of relationship is illustrated in the graph above?
- 2.2 Determine a formula for the cost of practising golf at Hacker's Golf Range.
- 2.3 What is the cost of hitting 30 golf balls at Pro Golf Range?
- 2.4 What is the maximum number of balls that you can hit at Exec Golf Range?
3. Phumba, Neil and Jan are barmen at three different pubs.
Phumba earns R120 per shift; Neil R100 per shift; and Jan R140 per shift regardless of how many drinks they serve.
- 3.1 Determine a formula for Neil's income for a shift.
- 3.2 Construct a table illustrating the barmen's income per shift, and the number of drinks they serve during a shift.
- 3.3 Using the table you constructed in Question 3.2; draw the graph illustrating the barmen's income, based on the number of drinks they serve during a shift.
- 3.4 How much will Phumba earn in a shift if he serves 10 drinks?
- 3.5 How much will Jan earn in a shift if he serves 1 drink?



Linear Relationship

Relationships are **linear** when there is a **constant difference** between consecutive values within a set of variables.

Linear Relationship in Direct Proportion

These relationships will **start at zero** and **increase/decrease** at a **constant rate**. Therefore, there will be a **constant ratio** between the independent and dependent variable.

Example:

Independent variable	0	1	2	3	4	5	6
Dependent variable	0	5	10	15	20	25	30

Starts at zero

$$\text{constant ratio} = \frac{\text{dependent variable}}{\text{independent variable}} = 5$$

$$\text{i.e. } \frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} = \frac{25}{5} = \frac{30}{6}$$

EXAMPLE 1 of a linear relationship in direct proportion:

Maggie earns a living by making and selling candles.

She makes a profit of R80 on her square candles;
R120 profit on her cylindrical candles; and
R60 profit on her candle sticks.



1 Constructing a table



Maggie's profit on her three different types of candles, can be illustrated in the table below:



Number of candles	0	1	2	3	4
Profit on square candles	0	80	160	240	320
Profit on cylindrical candles	0	120	240	360	480
Profit on candle sticks	0	60	120	180	240



Note the following:

- ▶ The relationships illustrated by the profits made on the different types of candles is linear, as there is a constant difference between consecutive values within a set of variables

i.e. Profit on square candles: constant difference = 80
 Profit on cylindrical candles: constant difference = 120
 Profit on candle sticks: constant difference = 60

- ▶ These linear relationships are in **direct proportion**, as they **start at zero** and have a **constant ratio**

i.e. Profit on square candles: constant ratio = $\frac{80}{1} = \frac{160}{2} = \frac{240}{3} = 80$
 Profit on cylindrical candles: constant ratio = $\frac{120}{1} = \frac{240}{2} = \frac{360}{3} = 120$
 Profit on candle sticks: constant ratio = $\frac{60}{1} = \frac{120}{2} = \frac{180}{3} = 60$

2 Determining a formula from a table



- ▶ For a **direct proportion linear relationship**, both sets of variables will start at 0 and a constant ratio will exist between the sets.
- ▶ The general formula for a direct proportion relationship will be:

$$\text{Dependent Variable} = \text{Constant Ratio} \times \text{Independent Variable}$$



Maggie's profit on the three different types of candles can be represented by the following formulae:

- Profit on square candles = $R80 \times \text{number of candles}$
- Profit on cylindrical candles = $R120 \times \text{number of candles}$
- Profit on candle sticks = $R60 \times \text{number of candles}$



3 Finding missing values in a table



In the example of Maggie's profit on her three different types of candles, find the missing values in the table below:

Number of candles	0	1	2	3	4	6	B	10
Profit on square candles	0	80	160	240	320	A	640	800
Profit on cylindrical candles	0	120	240	360	480	720	960	1 200
Profit on candle sticks	0	60	120	180	240	360	480	C



There are 2 approaches to finding the missing values.

BY INSPECTION

Establish the pattern and then determine **A** by continuing the pattern:

- ▶ Profit on square candles: constant ratio = $\frac{80}{1} = \frac{160}{2} = \frac{240}{3} = 80$
 $\therefore A = 6 \times 80 = 480$

BY SUBSTITUTION

If the formula was given, substitute to find the missing values of **B** and **C**:

- ▶ Profit on cylindrical candles = $R120 \times \text{number of candles}$
 $\therefore 960 = R120 \times B$
 $\therefore B = 960 \div 120$
 $B = 8 \text{ candles}$
- ▶ Profit on candle sticks = $R60 \times \text{number of candles}$
 $\therefore C = R60 \times 10 = R600$



4 Drawing graphs from tables



The graph of a linear relationship that is in direct proportion, will always start at the origin (zero) and be an ascending/increasing straight line graph.

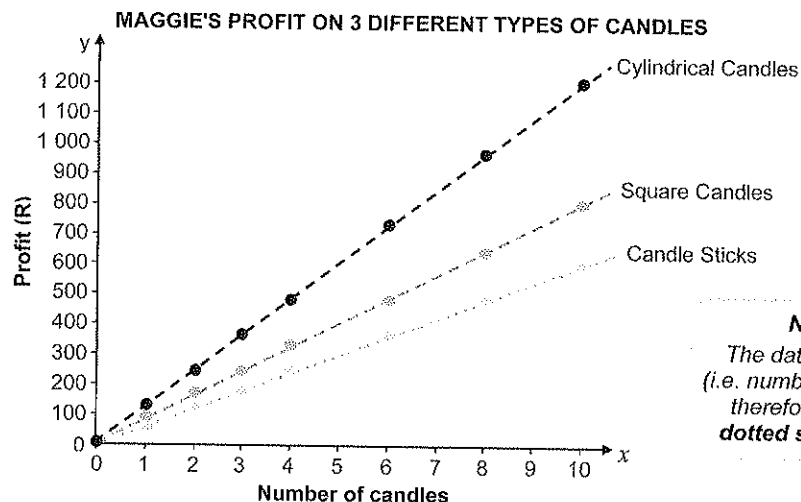
In the example of Maggie's profit on her three different types of candles, the graph on p. 33 can be drawn, based on the information from the table below:

Number of candles	0	1	2	3	4	6	8	10	independent variable
Profit on square candles	0	80	160	240	320	480	640	800	different dependent variables
Profit on cylindrical candles	0	120	240	360	480	720	960	1 200	
Profit on candle sticks	0	60	120	180	240	360	480	600	

Since there are 3 different dependent variables, there will be 3 different graphs to plot.

Therefore, the following points will be plotted per graph:

- ▶ Graph of the Profit on Square candles:
 $(0; 0); (1; 80); (2; 160); (3; 240); (4; 320); (6; 480); (8; 640)$ and $(10; 800)$
- ▶ Graph of the Profit on Cylindrical candles:
 $(0; 0); (1; 120); (2; 240); (3; 360); (4; 480); (6; 720); (8; 960)$ and $(10; 1 200)$
- ▶ Graph of the Profit on Candle sticks:
 $(0; 0); (1; 60); (2; 120); (3; 180); (4; 240); (6; 360); (8; 480)$ and $(10; 600)$



NOTE!
The data is **discrete**
(i.e. number of candles),
therefore we use a
dotted straight line.

EXAMPLE 2 of a linear relationship in direct proportion:

Thrill Rides theme park offers 3 types of entrance tickets:

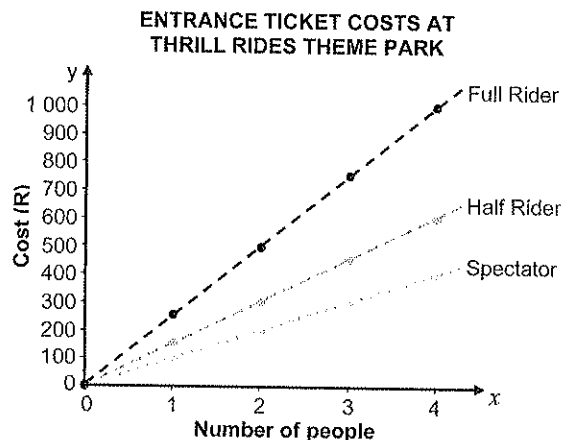
- Full Rider (adult) costs = R250 per person
- Half Rider (adult) costs = R150 per person
- Spectator costs = R100 per person

5 Determine the type of relationship from a graph

A linear graph that is in direct proportion will always be an increasing straight line that starts at the origin (zero).

The accompanying graph illustrates the cost at the Thrill Rides theme park.

Therefore, since these are increasing straight line graphs that start at the origin (0; 0), these graphs represent directly proportional linear relationships.



6 Determining the formula from a graph

- Once it has been established that the type of graph is a linear relationship in direct proportion, the following general formula can be used:

$$\text{Dependent Variable} = \text{constant ratio} \times \text{independent variable}$$

The ticket costs can be represented by the following formulae:

Full Rider

- the dependent variable is 'Full Rider ticket costs'
- the independent variable is 'Number of people'
- the constant ratio can either be established from the wording of the question; or a table can be drawn up:

i.e.	Number of people	0	1	2	3	4
	Full Rider ticket costs (R)	0	250	500	750	1 000
			250	500	750	1 000
			1	2	3	4

Formula for Full Rider Ticket Costs:

constant ratio = 250

$$\text{Full Rider Ticket Costs} = \text{R}250 \times \text{number of people}$$

Half Rider

- the dependent variable is 'Half Rider ticket costs'
- the independent variable is 'Number of people'
- the constant ratio, determined from a table:

i.e.	Number of people	0	1	2	3	4
	Half Rider ticket costs (R)	0	150	300	450	600
			150	300	450	600
			1	2	3	4

Formula for Half Rider Ticket Costs:

constant ratio = 150

$$\text{Half Rider ticket costs} = \text{R}150 \times \text{number of people}$$

Spectator

- the dependent variable is 'Spectator costs'
- the independent variable is 'Number of people'
- the constant ratio, determined from the wording is 100.

Formula for Spectator Ticket Costs:

$$\text{Spectator Ticket Costs} = \text{R}100 \times \text{number of people}$$

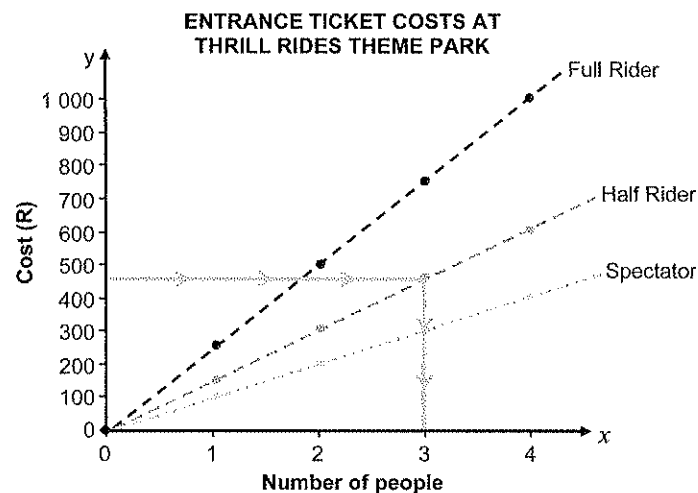
7 Reading off values from a graph



Refer to the graph of Entrance Ticket costs at Thrill Rides Theme Park on p. 33 and answer the following questions:

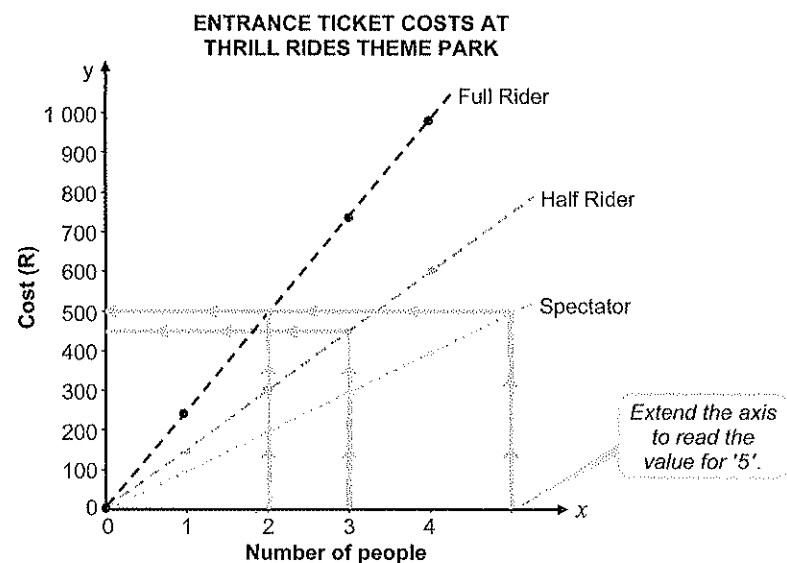
Q: If I spent R450 on entrance tickets, how many tickets and of what type did I buy?

A: 3 Half-Rider tickets



Q: Estimate the total cost of buying 2 Full Rider tickets, 3 Half rider tickets and 5 Spectator tickets.

A: Total cost $\approx$ R500 + R450 + R500 $\approx$ R1 450



Extend the axis to read the value for '5'.

Test Your Understanding



1. Ashley was baking cupcakes. She decides to sell them at school for R5.

1.1 Construct a table to show Ashley's income, based on the number of cupcakes she sells. Start at 0 and go up in increments of 10, up to 70 cupcakes.

1.2 Determine a formula to represent Ashley's income.

1.3 Determine Ashley's income if she sold 120 cupcakes.

1.4 Use the table you constructed in Question 1.1 to draw the graph of Ashley's income based on cupcake sales.



2. Inneke and Jan love cycling. On average they cycle 500 m per minute.

2.1 Determine a formula to represent the distance that Inneke or Jan can cover, based on the number of minutes they cycle.

2.2 Complete the table:

Time (minutes)	0	1	2	3	5	C	D	10
Distance (metres)	0	500	1 000	A	B	3 000	4 000	5 000



2.3 Use the table above to draw the graph of the distance cycled by Inneke or Jan.

2.4 Use your graph to estimate how far they would have cycled after $3\frac{1}{2}$ minutes.

3. The graph alongside can be used to convert ounces to grams.

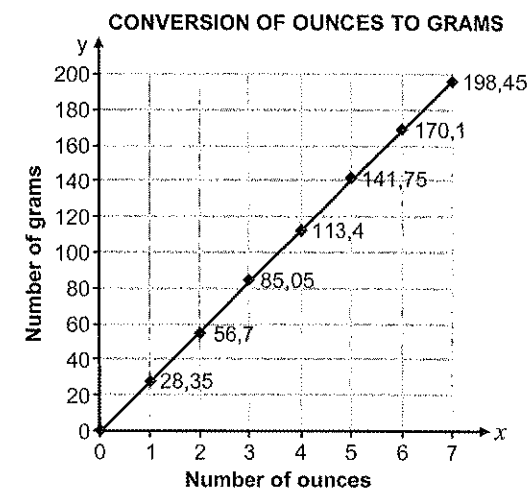
Study the graph and answer the questions that follow:

3.1 What type of relationship is represented by this graph?

3.2 Determine a formula to represent how to calculate the number of grams, given the number of ounces.

3.3 How many grams is equivalent to 4 ounces?

3.4 Approximately how many ounces are there in 170 grams?



Linear Relationship NOT in Direct Proportion

These relationships **will** have a **constant difference** between consecutive values within a set of variables. However, these graphs **will not start at zero**, *nor* will they have a constant ratio between the independent and dependent variable.

Example:

Independent variable	0	1	2	3	4
Dependent variable	200	250	300	350	400

Graph does **not** start at the origin - instead on a point on the y-axis.

+50 +50 +50 +50
constant difference but no constant ratio
 i.e. $\frac{200}{0} \neq \frac{250}{1} \neq \frac{300}{2}$ etc.

EXAMPLE 1 of a linear relationship NOT in direct proportion:

Ian is a photocopier salesman.

He earns a basic salary of R6 000 per month, plus commission, based on the number and type of photocopier sold:

- Small-scale photocopiers: 5% commission on the average price of R12 000 per photocopier.
- Medium-scale photocopiers: 7,5% commission on the average price of R18 000 per photocopier.
- Large-scale photocopiers: 12% commission on the average price of R30 000 per photocopier.

1 Constructing a table



Ian's total income, based on the number and type of photocopiers he sells, can be illustrated in the table below:

Number of photocopiers sold	0	1	2	3	10
Total income on small-scale photocopiers	6 000	6 600	7 200	7 800	12 000
Total income on medium-scale photocopiers	6 000	7 350	8 700	10 050	19 500
Total income on large-scale photocopiers	6 000	9 600	13 200	16 800	42 000

Note the following:

- The graphs **do not start at the origin** (0;0) but instead, at the point (0; 6 000) on the y-axis.
- The graphs will be linear, as there is a **constant difference** between the consecutive values within a set of variables.
 - Total income on small-scale photocopiers sold has a constant difference of 600 ($6\ 600 - 6\ 000 = 7\ 200 - 6\ 600$ etc.)
 - Total income on medium-scale photocopiers sold has a constant difference of 1 350 ($7\ 350 - 6\ 000 = 8\ 700 - 7\ 350$ etc.)
 - Total income on large-scale photocopiers sold has a constant difference of 3 600 ($9\ 600 - 6\ 000 = 13\ 200 - 9\ 600$ etc.)
- There is **no constant ratio** between the independent and dependent variables.

2 Determining a formula from a table



- For a **linear relationship that is not in direct proportion**, there will be a constant difference between consecutive variables. (Graphs will not start at the origin nor have a constant ratio between the dependent and independent variable.)
- The general formula for a linear relationship that is not in direct proportion:

$$\text{Dependent Variable} = (\text{constant difference} \times \text{independent variable}) + \text{starting value}$$



Ian's total income, based on the number and type of photocopiers he sells, can be represented by the following formulae:

$$\text{Total income on small-scale photocopiers sold} = (\text{R}600 \times \text{number of photocopiers sold}) + \text{R}6\ 000$$

$$\text{Total income on medium-scale photocopiers sold} = (\text{R}1\ 350 \times \text{number of photocopiers sold}) + \text{R}6\ 000$$

$$\text{Total income on large-scale photocopiers sold} = (\text{R}3\ 600 \times \text{number of photocopiers sold}) + \text{R}6\ 000$$

Starting value in this example is Ian's basic salary of R6 000 per month.



3 Finding missing values in a table



In the example of Ian's total income, based on the number and type of photocopiers he sells; find the missing values in the table below:

Number of photocopiers sold	0	1	2	3	4	D	10
Total income on small-scale photocopiers	6 000	6 600	7 200	7 800	A	10 200	12 000
Total income on medium-scale photocopiers	6 000	7 350	8 700	10 050	B	15 450	19 500
Total income on large-scale photocopiers	6 000	9 600	13 200	16 800	C	31 200	42 000

There are 2 approaches for finding the missing values:

BY INSPECTION



Establish the pattern and then determine the values of **A** and **B** by continuing the pattern:

- ▶ Total income on small-scale photocopiers: The constant difference = R600
 $\therefore A = R7\ 800 + R600 = R8\ 400$
- ▶ Total income on medium-scale photocopiers: The constant difference = R1 350
 $\therefore B = R10\ 050 + R1\ 350 = R11\ 400$

BY SUBSTITUTION

If the formulae were given, substitute to find the missing values of **C** and **D**:

- ▶ Total income on large-scale photocopiers sold:
 $= (R3\ 600 \times \text{number of photocopiers sold}) + R6\ 000$
 $\therefore C = (R3\ 600 \times 4) + R6\ 000$
 $= R20\ 400$

- ▶ Total income on small-scale photocopiers sold:
 $= (R600 \times \text{number of photocopiers sold}) + R6\ 000$
 $\therefore 10\ 200 = (R600 \times D) + R6\ 000$
 $4\ 200 = R600 \times D$
 $\frac{4\ 200}{6} = D$
 $\therefore 7 = D$



NOTE!
 Substitution into *any* of the 3 different photocopier formulae, would have yielded the same answer of 7 photocopiers.

4 Drawing graphs from tables



- ▶ The graph of a linear relationship that is not in direct proportion will be an increasing straight line graph that starts at a point plotted somewhere along the y-axis (i.e. not at the origin).

In the example of Ian's total income, based on the number and type of photocopiers he sells; the following graphs can be drawn, based on the information from the table below:

independent variable

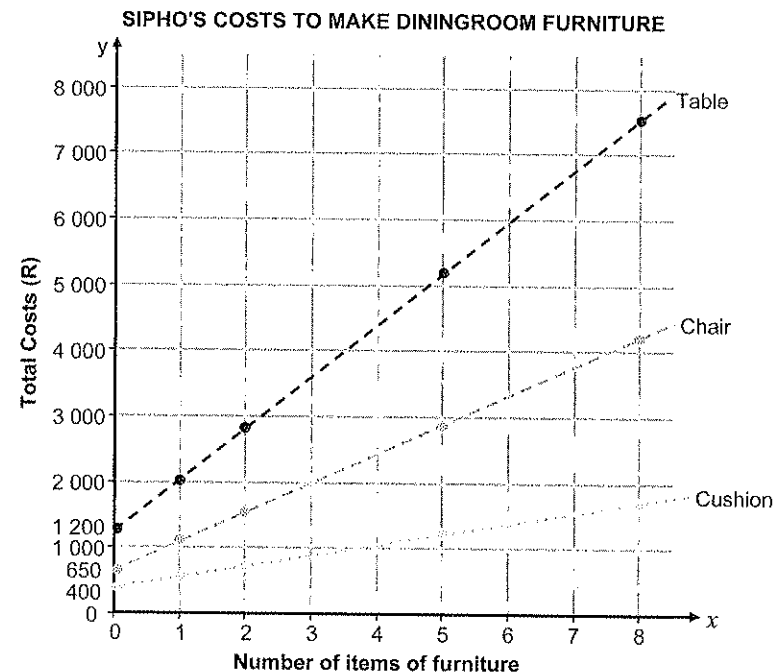
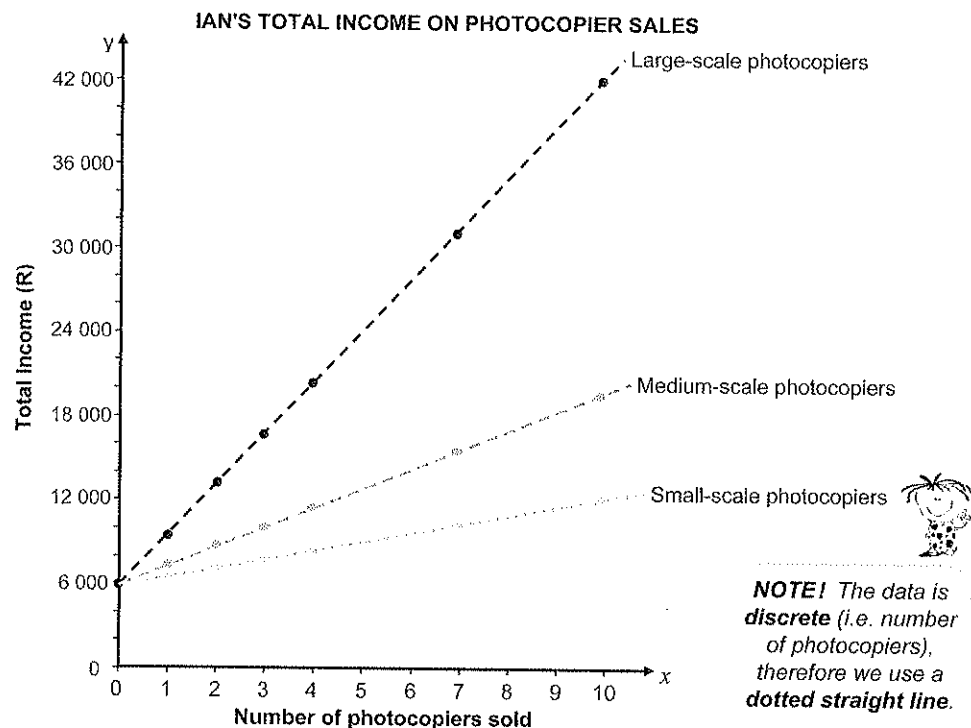
Number of photocopiers sold	0	1	2	3	4	7	10
Total income on small-scale photocopiers	6 000	6 600	7 200	7 800	8 400	10 200	12 000
Total income on medium-scale photocopiers	6 000	7 350	8 700	10 050	11 400	15 450	19 500
Total income on large-scale photocopiers	6 000	9 600	13 200	16 800	20 400	31 200	42 000

different dependent variables

Since there are 3 different dependent variables, there will be 3 different graphs to plot.

Therefore, the following points will be plotted per graph:

- Graph of the Total income on small-scale photocopiers sold:
 $(0; 6\ 000); (1; 6\ 600); (2; 7\ 200); (3; 7\ 800); (4; 8\ 400); \text{etc.} \dots$
- Graph of the Total income on medium-scale photocopiers sold:
 $(0; 6\ 000); (1; 7\ 350); (2; 8\ 700); (3; 10\ 050); (4; 11\ 400); \text{etc.} \dots$
- Graph of the Total income on large-scale photocopiers sold:
 $(0; 6\ 000); (1; 9\ 600); (2; 13\ 200); (3; 16\ 800); (4; 20\ 400); \text{etc.} \dots$



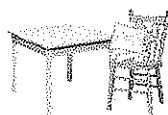
NOTE! The starting points of each graph represent the fixed costs of making the item of furniture.
i.e. Table → R1 200; Chair → R650; Cushion → R400

EXAMPLE 2 of a linear relationship NOT in direct proportion:

Sipho makes diningroom tables, chairs and cushions for a living. He has certain fixed and variable expenses to consider when pricing his goods.

Each table he makes costs him R1 200, plus an extra R800 in variable costs per table.

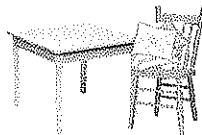
Each chair costs him R650, plus an extra R450 per chair.
Each cushion costs him R400, plus an extra R150 per cushion.



5 Determining the type of relationship from a graph

A linear graph that is not in direct proportion will be an increasing straight line graph that starts at a point somewhere along the y-axis (but *not* at the origin).

In the example of Siphos costs of making diningroom furniture the following graph would be given:



6 Determining the formula from a graph

- Once it has been established that the type of graph is a linear relationship that is not in direct proportion, the following general formula can be used:

$$\text{Dependent variable} = (\text{constant difference} \times \text{independent variable}) + \text{starting value}$$

Therefore, Siphos costs of making diningroom furniture can be represented by the following formulae:

Diningroom tables

- the dependent variable is 'Total costs'
- the independent variable is 'Number of items of furniture/tables'
- the starting value is R1 200 (i.e. the point on the y-axis)
- the constant difference can be determined in one of two ways:



METHOD 1 Draw up a table

Number of tables	0	1	2
Total costs (R)	1 200	2 000	2 800

starting value

+800 +800

 $\therefore$ constant difference = R800**METHOD 2** Calculation using point (1; . . .)

The coordinate at '1 item of furniture' for tables = (1; 2 000)

Constant difference = y-coordinate of point (1; . . .) - starting value
 = 2 000 - 1 200
 = R800

 $\therefore$ Formula for making diningroom tables is:

$$\text{Total costs} = (\text{R}800 \times \text{number of items of furniture/tables}) + \text{R}1\,200$$

▪ Diningroom chairs

- the dependent variable is 'Total costs'
- the independent variable is 'Number of items of furniture/chairs'
- the starting value is R650 (i.e. the point on the y-axis)
- If you used *Method 1* (table) the constant difference would be R450, i.e.



Number of chairs	0	1	2
Total costs (R)	650	1 100	1 550

starting value

+450 +450

 $\therefore$ Formula for making diningroom chairs is:

$$\text{Total costs} = (\text{R}450 \times \text{number of items of furniture/chairs}) + \text{R}650$$

▪ Diningroom cushions

- the dependent variable is 'Total costs'
- the independent variable is 'Number of items of furniture/cushions'
- the starting value is R400 (i.e. the point on the y-axis)
- if you used *Method 2* (point 1; . . .), the constant difference would be R150.

The coordinate at '1 item of furniture' for cushions = (1; 550)

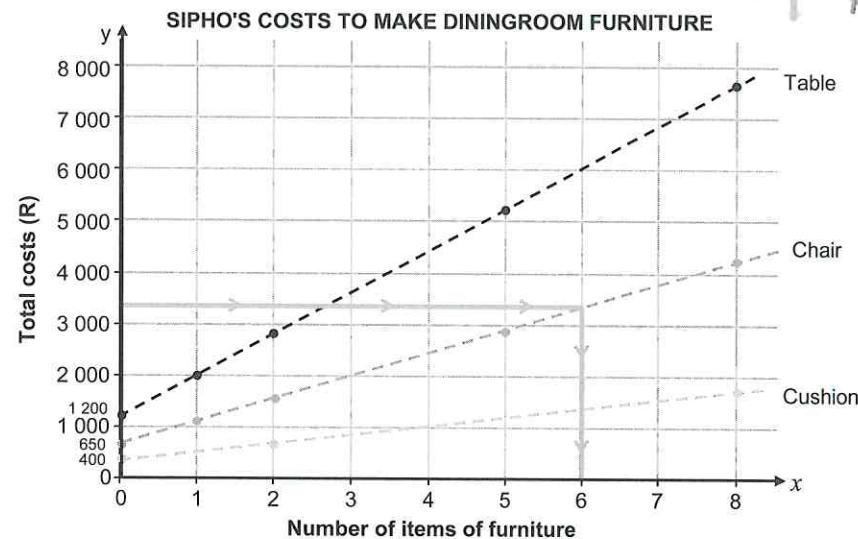
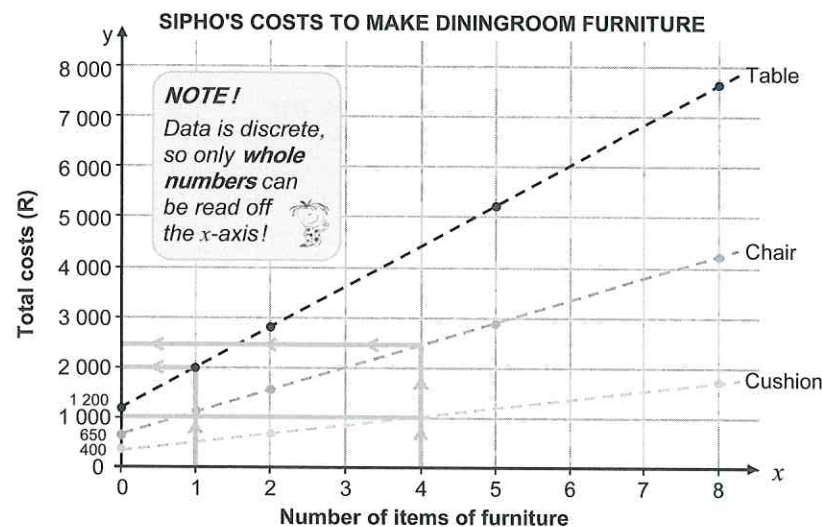
Constant difference = y-coordinate of point (1; . . .) - starting value
 = 550 - 400
 = R150

 $\therefore$ Formula for making diningroom cushions is:

$$\text{Total costs} = (\text{R}150 \times \text{number of items of furniture/cushions}) + \text{R}400$$

**7** Reading off values from a graph

Refer to the graph of Sipho's costs to make diningroom furniture (see p. 37):

Q: How many chairs can Sipho make if his costs total R3 350?**A:** 6 chairs**Q:** Estimate the total cost of making 1 diningroom table and 4 chairs, with 4 cushions.**A:** Total costs $\approx$ R2 000 + R2 500 + R1 050 $\approx$ R5 550



Test Your Understanding



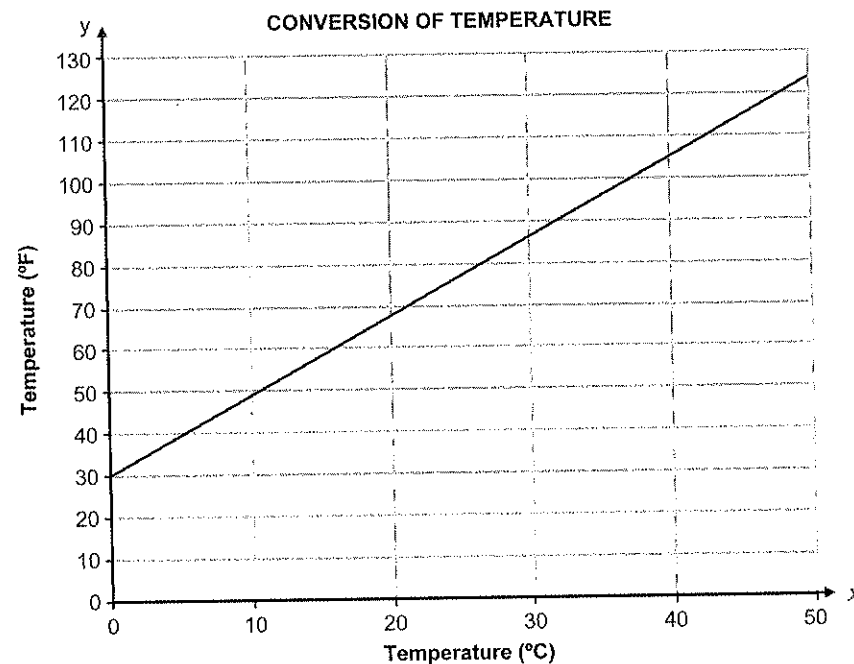
1. Michelle wants to hire Delicious Eats, a catering company, to cater for her 18th birthday party. The company charges a R500 deposit and an additional fee of R100 per hour.
 - 1.1 Construct a table to represent the total costs of hiring Delicious Eats per hour, up to a maximum of 10 hours.
 - 1.2 Determine a formula to represent the total costs per hour.
 - 1.3 Use your formula (from Question 1.2) to determine the costs of hiring Delicious Eats for 9 hours.
 - 1.4 Use your formula (from Question 1.2) to determine how many hours Delicious Eats could be hired for, if Michelle had a budget of R1 200.
 - 1.5 Use your table (from Question 1.1) to draw the graph of the costs of hiring Delicious Eats per hour.
2. Peter invests R20 000 in a savings account. The bank guarantees him a rate of 10% p.a. simple interest.
 - 2.1 Complete the table:

Year	0	1	2	8	C	20
Total value of investment (R)	A	22 000	B	36 000	50 000	60 000

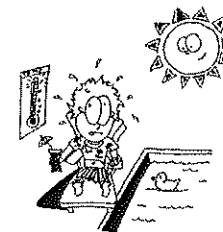
- 2.2 Determine a formula to represent the relationship between the number of years and the total value of Peter's investment.
- 2.3 Use the table in Question 2.1 to draw the graph of Peter's investment over time.



3. Mrs Smith is visiting South Africa for the Soccer World Cup. In her country, temperature is measured in degrees Fahrenheit. She used the graph below to easily convert temperatures between degrees Fahrenheit (°F) and degrees Celsius (°C).



- 3.1 What kind of relationship is illustrated by this graph?
- 3.2 Estimate the temperature in Fahrenheit, if the temperature is 40°C.
- 3.3 Estimate the temperature in Celsius, if the temperature is approximately 68°F.
- 3.4 If it is given that the constant difference is 1,8; determine a formula for calculating the temperature in Fahrenheit, if the temperature in Celsius is given.



Indirect/Inverse Proportion Relationship



An **indirect/inverse proportion relationship** occurs when one variable increases by a factor; while the other variable decreases by that same factor. This results in a **constant product** between the two sets of variables.

NOTE! Neither set of variables will ever contain a zero.



Example:

Independent variable	1	2	4	8
Dependent variable	8	4	2	1
	1×8 $= 8$	2×4 $= 8$	4×2 $= 8$	8×1 $= 8$

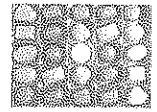
As the independent variable increases by a factor, so the dependent variable decreases by the same factor.

$\therefore$ constant product = 8

EXAMPLE 1 of an indirect/inverse proportion relationship:

Thandi has a box of 36 chocolates.

She decides to share her chocolates with her family members.



1 Constructing a table



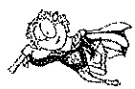
The number of chocolates per member of Thandi's family is illustrated in the table below:

Number of family members	1	2	3	4	6
Number of chocolates per family member	36	18	12	9	6

Note the following:

- ▶ The relationship illustrated by the table is an **indirect/inverse proportion relationship**, because as the number of family members increases, so the number of chocolates per family member decreases by the same factor.
- ▶ There is also a **constant product** between the two sets of variables, i.e. $1 \times 36 = 2 \times 18 = 3 \times 12 = 4 \times 9 = 6 \times 6 = 36$.

2 Determining a formula from a table



- ▶ For an indirect/inverse proportion relationship, as the one variable increases, so the other variable decreases by the same factor.
- ▶ There will always be a constant product between the two sets.
- ▶ The general formula for an indirect/inverse proportion relationship will be:

$$\text{Independent variable} \times \text{dependent variable} = \text{Constant Product}$$



The formula used to represent the number of chocolates per member of Thandi's family is:

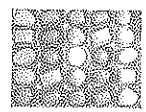
Number of family members $\times$ number of chocolates per family member = constant product.

$\therefore$ Number of family members $\times$ number of chocolates per family member = 36

NOTE! Only re-arrange the formula if the question asks for a particular variable as the **subject of the formula**:



e.g. Number of chocolates per family member
$$= \frac{36}{\text{number of family members}}$$



3 Finding missing values in a table



In the example of the number of chocolates per member of Thandi's family; find the missing values in the table below:

Number of family members	1	2	3	4	6	12	B	36
Number of chocolates per family member	36	18	12	9	6	A	2	1

There are 2 approaches for finding the missing values.

BY INSPECTION

Establish the pattern and then determine A by continuing the pattern:

$\therefore 12 \times A = 36$
 $\therefore A = 3$

$12 \times \text{'what?'}$

= constant product of 36.



NOTE!

- As the number of family members increase, so the number of chocolates per family member decreases by the same factor.
- The constant product
 $= 1 \times 36 = 2 \times 18 = \text{etc.} = 36$



BY SUBSTITUTION

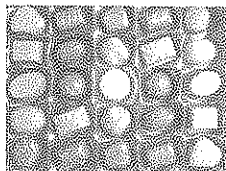
If the formula was given, substitute to find the missing value of **B**:

$$\therefore \text{Number of family members} \times \text{Number of chocolates per family member} = 36$$

$$B \times 2 = 36$$

$$\therefore B = 36 \div 2$$

$$B = 18$$



4 Drawing graphs from tables



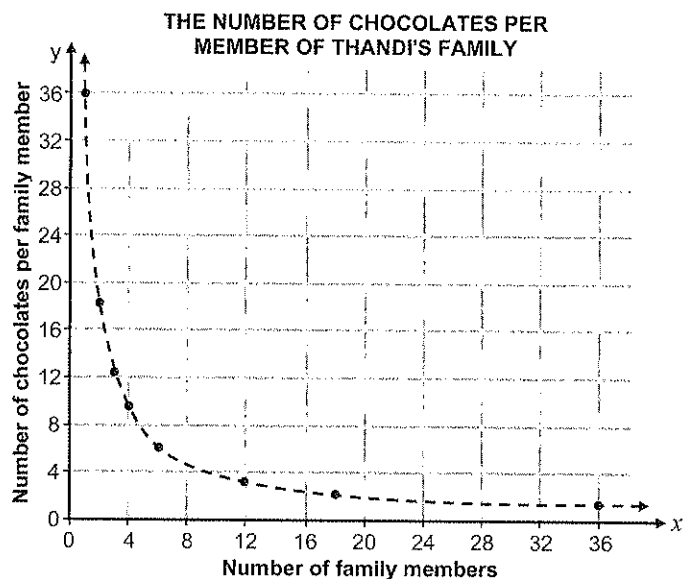
The graph of an indirect/inverse proportion relationship is a **decreasing curve line** (known as a **hyperbolic curve** in mathematical terms).

In the example of the number of chocolates per member of Thandi's family; the following graph can be drawn, based on the information from the table below:

Number of family members	1	2	3	4	6	12	18	36
Number of chocolates per family member	36	18	12	9	6	3	2	1

The following points will be plotted:

(1; 36); (2; 18); (3; 12); (4; 9); (6; 6); (12; 3); (18; 2); (36; 1)



NOTE!

Neither sets of variables will ever contain a zero. Therefore, this graph will not intersect the *x* nor *y*-axis.

Note the curved decreasing line (hyperbolic curve)

Data is discrete
 $\therefore$ a dotted line



EXAMPLE 2 of an indirect/inverse proportion relationship:

Michael owns a tiling business and has twelve tilers who work for him.

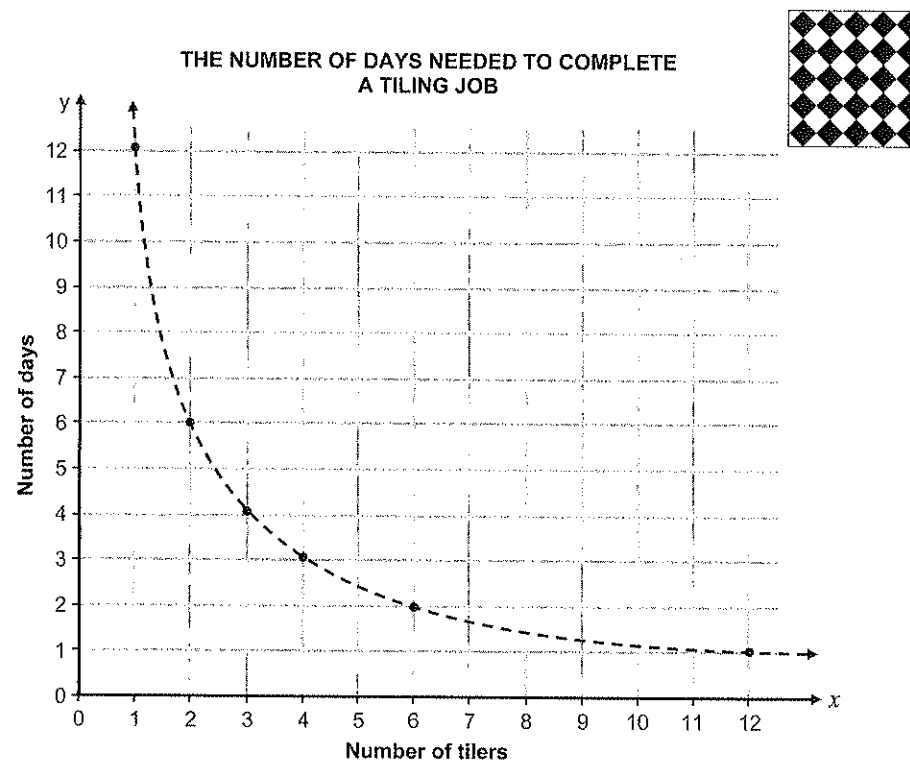


5 Determining the type of relationship from a graph



An indirect/inverse proportion graph will be a decreasing curved line (hyperbolic in nature) that will not intersect the *x*- nor *y*-axis. A constant product will always exist between the independent and dependent variables.

In the example of Michael's tiling business, the following graph would be given:



Therefore, since this is a hyperbolic curve with a constant product of 12, this graph represents an indirect/inverse proportion graph.

6 Determining the formula from a graph



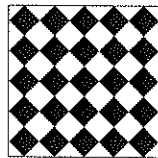
Once it has been established that the type of graph is an indirect/inverse proportion relationship, the following general formula can be used:

$$\text{Independent variable} \times \text{Dependent variable} = \text{Constant Product}$$



Therefore, the number of days needed to complete a tiling job can be represented by the following formula:

- ▶ the independent variable is 'Number of tilers'
- ▶ the dependent variable is 'Number of days'
- ▶ the constant product can be determined by drawing up a table:



Number of tilers	1	2	3	4	6	12
Number of days	12	6	4	3	2	1
	1×12	2×6	3×4	4×3	6×2	12×1
	$= 12$	$= 12$	$= 12$	$= 12$	$= 12$	$= 12$

$$\therefore \text{constant product} = 12$$

$\therefore$ Formula for determining the number of days needed to complete a tiling job is:
Number of tilers $\times$ Number of days = 12

If the question read:

'Determine the formula, in the form:

Number of days =';

then the answer would be:

Number of days = $12 \div \text{Number of tilers}$.



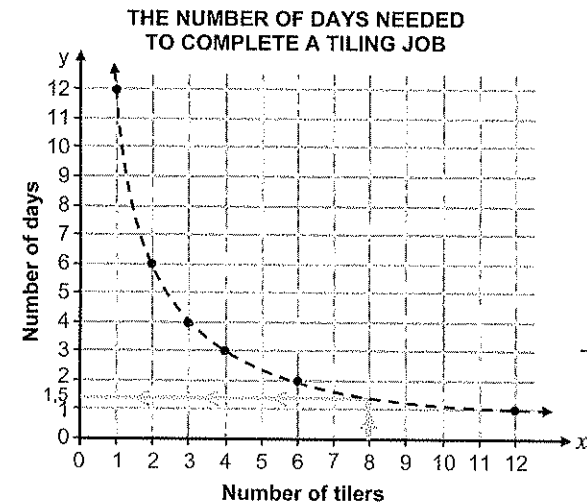
7 Reading off values from a graph



Refer to the graph of The Number Of Days Needed To Complete A Tiling Job (see p. 41) and answer the following questions:

Q: Estimate how many days it would take to complete the tiling job, if 8 tilers worked on the job.

A: 1,5 days

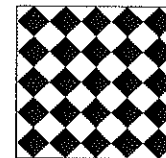
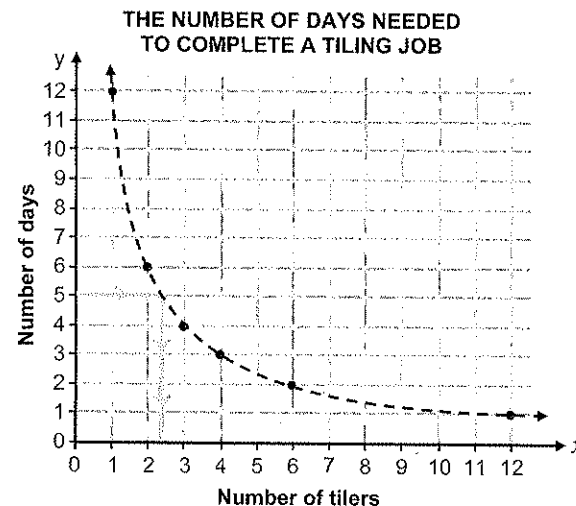


Q: If the client wanted the job done in 5 days, how many tilers must Michael get to work on the job?

A: According to the graph, one would need approximately 2,5 tilers. However, since we are working with discrete data, we need to round the answer up to 3 tilers.

If we rounded the answer down to 2 tilers, it would then take 6 days to complete the job - but this wouldn't meet the client's needs.

Therefore, Michael must employ another tiler (total 3 tilers) in order to ensure the job is completed in time.



Test Your Understanding



1. SUPERCLEAN is a company which runs a cleaning service. They are hired to clean a banquet hall. They have 48 cleaners who can complete the cleaning of the hall in 1 hour. The number of cleaners sent to clean the hall is dependent on the time between functions. If there is more than one function on a particular day, the hall may need to be cleaned in 1 hour.

1.1 Construct a table to clearly show the relationship between the number of cleaners and the number of hours needed to clean the hall.

1.2 Use your table from Question 1.1 to determine a formula that would represent the number of cleaners and number of hours needed to clean the hall.

Your formula must be in the form: **Number of cleaners** =

1.3 Use your table from Question 1.1 to draw the graph of the number of cleaners and number of hours needed to clean the hall.

1.4 Use your graph from Question 1.3 to determine the number of cleaners needed to clean the hall in 3 hours.

1.5 Use your graph from Question 1.3 to estimate how long it would take 20 cleaners to clean the hall.

1.6 What kind of relationship is represented by this example?



2. David and his family are travelling from Cape Town to Knysna for the Oyster Festival. The table below shows how long the journey would take them depending on the average speed at which they travelled.

Travel time (hours)	A	5,2	4	2,5
Speed (km/h)	65	100	B	208



2.1 Determine a formula to represent the relationship between the travel time and speed.

2.2 Determine the values of **A** and **B**.

2.3 Use the table given above to draw the graph of the travel time to Knysna, based on the speed.

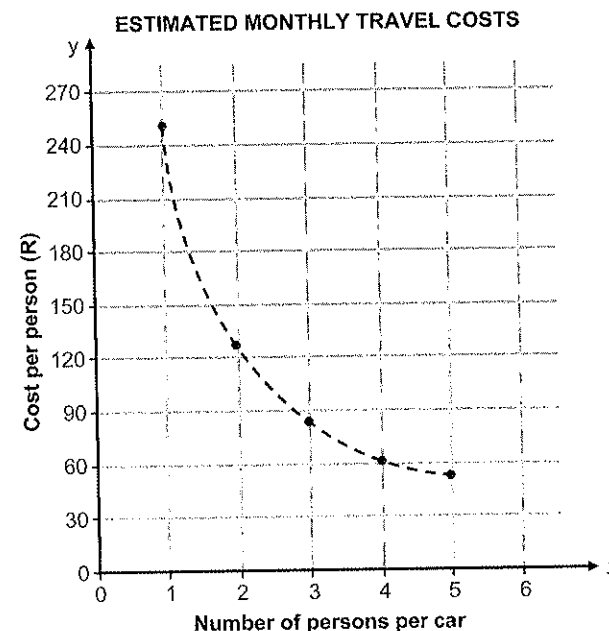
2.4 How far is Knysna from Cape Town? Use the formula: $\text{Speed} = \frac{\text{distance}}{\text{time}}$

3. A group of colleagues work for the same company. They want to organise a car pool so that they can travel together to help to reduce traffic congestion on the road. The estimated cost per person is shown in the graph below.

3.1 If each person paid R50 in travel costs per month; then how many people are in the car pool?

3.2 What are the travel costs per person if only 2 people travel together?

3.3 Determine a formula to represent the relationship between the number of people per car and the cost per person.



Representations of Relationships

Relationships can be represented in **tables**, **equations** or **graphs**.

For the relationships **D** and **E** (see p. 26); we will learn how to:



- 1 construct a table from equations/formulae or a given scenario
- 2 determine and extend the pattern in a table
- 3 use the values in the table to draw a graph
- 4 read values from a graph

These skills will be illustrated by means of a worked example for each of the relationships in: **D** and **E**

D Constant Ratio Relationship



A **constant ratio relationship** exists when there is a **constant ratio** between **consecutive terms** of the pattern.

EXAMPLE

Term:	1	2	3	4	5
	100	110	121	133,10	146,41 ; ...
Constant Ratio =	$\frac{T_2}{T_1}$	$= \frac{T_3}{T_2}$	$= \frac{T_4}{T_3}$	$= \frac{T_5}{T_4}$	
Constant Ratio =	$\frac{110}{100}$	$= \frac{121}{110}$	$= \frac{133,10}{121}$	$= \frac{146,41}{133,10}$	; 1.1



1 Constructing a table



This could be asked by expecting you to:

- substitute into a **given formula**; OR
- use the **given context** to formulate your own equation or approach.

EXAMPLE

Donald decides to invest R6 000 in a long-term savings account for his son. The bank guarantees him a fixed interest rate of 4% p.a. compounded annually. Construct a table to illustrate the total value of the investment after 5 years; using the formula:

Current value = (% × previous value) + previous value



The value of Donald's savings account investment over five years can be illustrated in the table below:

Year	1	2	3	4	5
Total value of investment (R)	6 000	6 240	6 489,60	6 749,18	7 019,15

- i.e.
- Year 1 = starting value
= R6 000
 - Year 2 = (4% × year 1) + year 1
= $\left(\frac{4}{100} \times 6\,000\right) + R6\,000$
= R6 240
 - Year 3 = (4% × year 2) + year 2
= $\left(\frac{4}{100} \times 6\,240\right) + R6\,240$
= R6 489,60 etc.

NOTE! To find the current year's value: calculate the % of the previous year and add it to the previous year's total.



2 Determining and extending the pattern



- First determine the constant ratio, by checking that $\frac{\text{term 3}}{\text{term 2}} = \frac{\text{term 2}}{\text{term 1}}$
- Once the constant ratio has been determined; use this to extend the pattern, by applying this general formula/approach:

Next term = previous term × constant ratio



Use the table below to determine the value of Donald's savings account in 8 year's time:

Year	1	2	3	4	5
Total value of investment (R)	6 000	6 240	6 489,60	6 749,18	7 019,15

$$\begin{aligned}\text{Constant Ratio} &= \frac{\text{year 2}}{\text{year 1}} \\ &= \frac{6\,240}{6\,000} \\ &= 1,04\end{aligned}$$

$$\begin{aligned}\text{Constant Ratio} &= \frac{\text{year 3}}{\text{year 2}} \\ &= \frac{6\,489,60}{6\,240} \\ &= 1,04\end{aligned}$$

Always check that the ratio is constant, by calculating the ratio of $\frac{\text{term 3}}{\text{term 2}}$!



- $\therefore$ • Year 6 = year 5 $\times$ constant ratio
 $= 7\,019,15 \times 1,04$
 $= R7\,299,92$
- Year 7 = year 6 $\times$ constant ratio
 $= 7\,299,92 \times 1,04$
 $= R7\,591,92$
- Year 8 = year 7 $\times$ constant ratio
 $= 7\,591,92 \times 1,04$
 $= R7\,895,60$
- REMEMBER!** Always round off to two decimal places for rand and cents!

$\therefore$ The value of Donald's savings account will be R7 895,60 in 8 years' time.

3 Drawing a graph

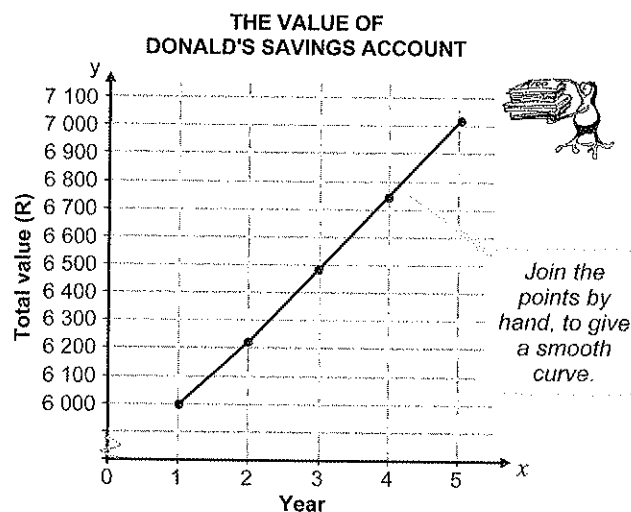
- Use the values in the table to plot the coordinates of the graph.

The table below shows the value of Donald's savings account:

Year	1	2	3	4	5
Total value of investment (R)	6 000	6 240	6 489,60	6 749,18	7 019,15

Therefore, the following points would be plotted:

(1; 6 000); (2; 6 240); (3; 6 489,60); (4; 6 749,18); (5; 7 019,15)



NOTE! The y-axis shows that it has been broken. We do this so that we can easily 'zoom' into the area of the graph which is relevant to values in the table. This helps us to see the shape of the curve.

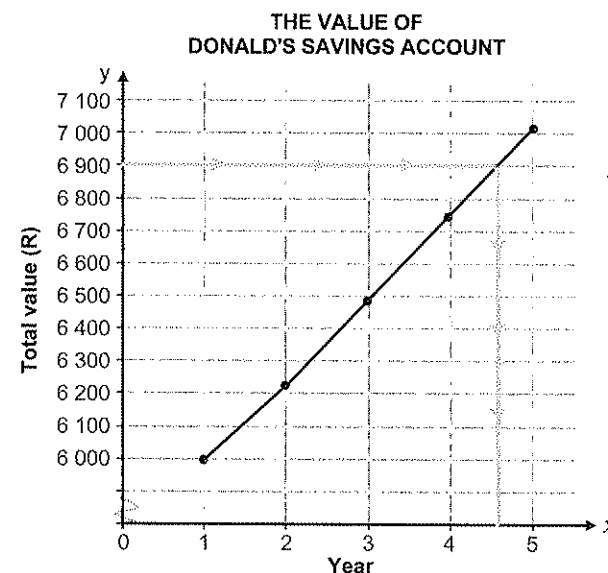
4 Reading values off a graph

- Estimate the values from the graph by being as accurate as possible, using a ruler and the scale on the axes to guide you.

Refer to the previous graph of the value of Donald's savings account and answer the following question:

Q: Estimate after how many months Donald's investment will be worth approximately R6 900.

A: $\approx$ 54 months



NOTE! Question asked for months:
 4,5 years
 $= 4,5 \times 12$
 $= 54$ months

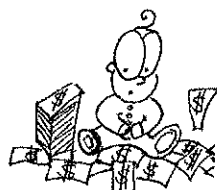
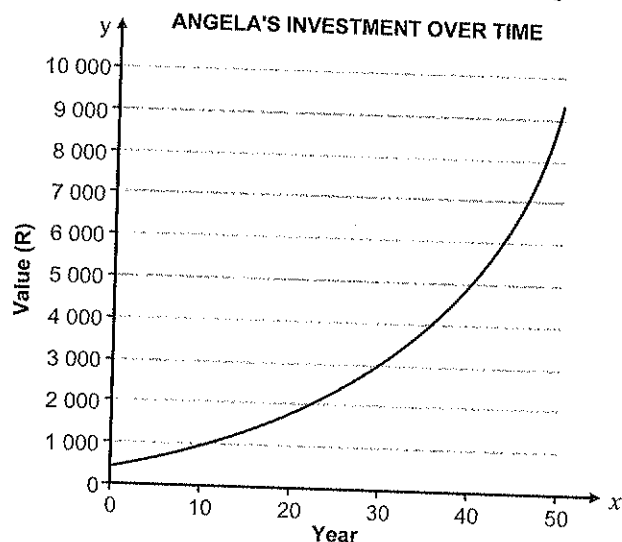
Test Your Understanding



- A marketing company investigated the change in the price of butter over a period of six years. If the starting price in Year 1 was R5,20; determine the following, based on inflation set at 7,1% p.a.:
 - Construct a table to show the price of butter over six years, due to inflation.
 - Determine the constant ratio between consecutive terms.
 - Use your constant ratio you found in Question 1.2 to determine the inflated price in 9 years' time.
 - Use your table from Question 1.1 to draw the graph of the cost of butter over the years.



- The graph below represents an investment that Angela has.



- How much did Angela initially invest?
- By how much did her investment grow after 30 years?
- After how many years was her investment worth approximately R4 000?
- After 40 years, Angela's investment will be worth approximately R5 100,00. Calculate the percentage growth on the investment over these 40 years.

% Growth $\equiv$ % Increase
Revision on % increase (see p. 19)



Combination of the different types of relationships



These relationships may incorporate some of the features of the different types of relationships, namely:

- CONSTANT/FIXED RELATIONSHIP** - where the **dependent variable (y)** remains **constant**, while the independent variable (x) increases.
- LINEAR RELATIONSHIP IN DIRECT PROPORTION** - where the relationship **starts at zero**; has a **constant difference** between consecutive values; and a **constant ratio** between the independent and dependent variable.
- LINEAR RELATIONSHIP NOT IN DIRECT PROPORTION** - where there is a **constant difference** between consecutive values; where the relationship **does not start at zero**; nor have a **constant ratio** between the independent and dependent variable.
- INDIRECT/INVERSE PROPORTION RELATIONSHIP** - where one **variable increases by a factor** and the **other variable decreases by the same factor**; where there is a **constant product**; and where **neither set of variables contains a zero**.
- CONSTANT RATIO RELATIONSHIP** - where there is a **constant ratio** between consecutive terms.

1 Constructing a table



EXAMPLE 1

Mountain View Lodge is a self-catering chalet that can accommodate a maximum of 12 people. The chalet costs R2 000 per night for up to 4 people; and an extra R250 per person per night for any additional people.



The total cost of the Mountain View Lodge chalet per night can be illustrated in the table below:

Number of people	1	2	3	4	5	6	7	8	9	10
Total cost per night (R)	2 000	2 000	2 000	2 000	2 250	2 500	2 750	3 000	3 250	3 500

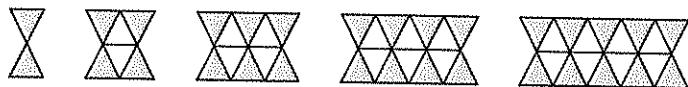
i.e. A flat cost of R2 000 per night for up to 4 people. +250 +250 +250 +250 +250 +250

i.e. A flat cost of R2 000 PLUS an extra R250 per person per night.



EXAMPLE 2

Bryan is building a model bridge and would like to use the following pattern of triangles in his bridge:



The number of triangles per term in the above illustrated pattern can be represented in the table below:

Term	1	2	3	4	5
Number of triangles	2	6	10	14	18

2 Determining and extending the pattern



- Once the type of relationship has been determined (see summary on p. 50); use the distinguishing features to extend the pattern.

EXAMPLE 1

Use the table given on p. 46 to determine the cost of renting Mountain View Lodge chalet if it accommodated the maximum number of 12 people per night:



Since there is a constant difference of R250 between consecutive terms from 4 people onwards, we can calculate the cost of 12 people as follows:

$$\begin{aligned} 11 \text{ people} &= 10 \text{ people} + R250 \\ &= R3\,500 + R250 \\ &= R3\,750 \end{aligned}$$

$$\begin{aligned} 12 \text{ people} &= 11 \text{ people} + R250 \\ &= R3\,750 + R250 \\ &= R4\,000 \end{aligned}$$

$\therefore$ The cost for 12 people is R4 000 per night.

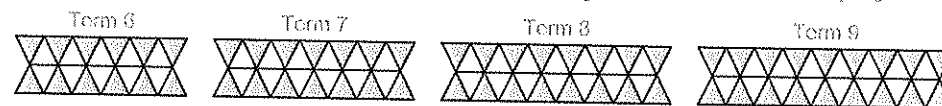
EXAMPLE 2

Use the pattern or table given above to determine the number of triangles in the 9th term of Bryan's model bridge:



- If we evaluate the pattern: each term has an extra 4 small triangles.

Therefore, if we extend the pattern, we would get:



$\therefore$ There would be 34 triangles in the 9th term.

- If we evaluate the table, we see that there is a constant difference of 4:

Term	1	2	3	4	5	6	7	8	9
Number of triangles	2	6	10	14	18	22	26	30	34
		+4	+4	+4	+4				

$\therefore$ There would be 34 triangles in the 9th term.

3 Drawing a graph



EXAMPLE 1

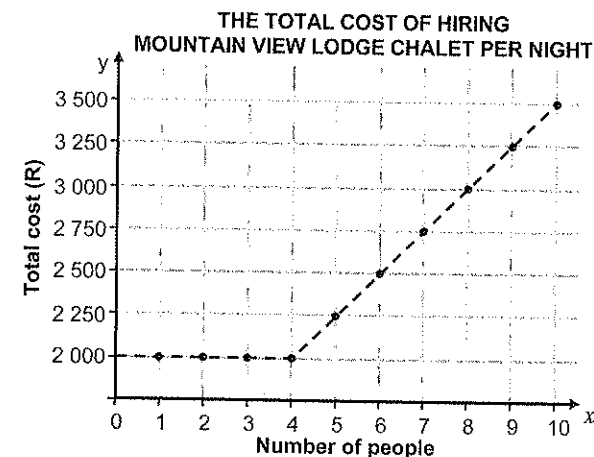
The table below shows the total cost per night for hiring Mountain View Lodge; based on the number of people per night:

Number of people	1	2	3	4	5	6	7	8	9	10
Total cost per night (R)	2 000	2 000	2 000	2 000	2 250	2 500	2 750	3 000	3 250	3 500

Therefore, the following points would be plotted:

(1; 2 000); (2; 2 000);
(3; 2 000); (4; 2 000);
(5; 2 250); (6; 2 500); etc.

NOTE: The first part of the graph (1 - 4 people) is a **constant/fixed** relationship; and thereafter (4 - 10 people), the graph is a **linear** relationship.



EXAMPLE 2

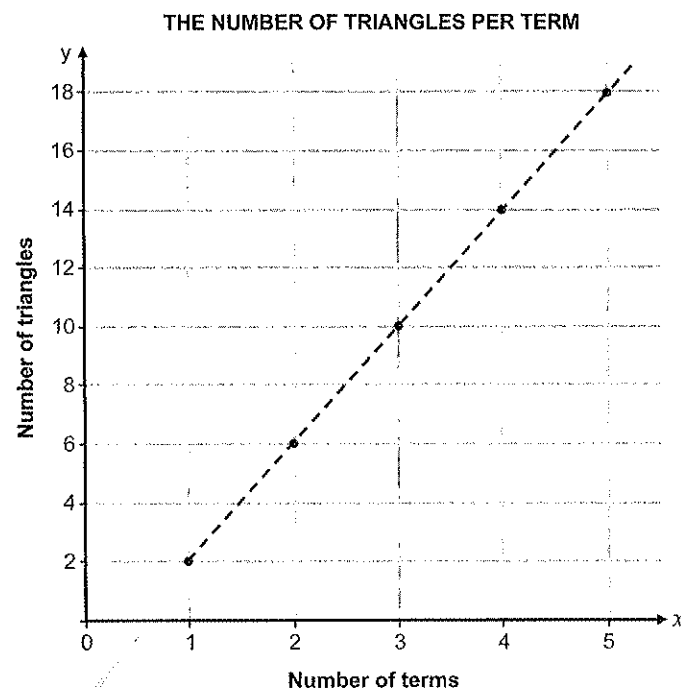
The table below shows the number of triangles per term in the given pattern of Bryan's model bridge.



Term	1	2	3	4	5
Number of triangles	2	6	10	14	18

Therefore, the following points would be plotted:

(1; 2); (2; 6); (3; 10); (4; 14); (5; 18)



The line is dotted as the data is discrete.



NOTE! The graph cannot be extended back to 0; as we wouldn't have a pattern with 0 terms!



4 Reading values off a graph



EXAMPLE 1

Refer to the graph of the total cost of hiring Mountain View Lodge chalet per night on p. 47 and answer the following question:

Q: Determine Mountain View's Lodge's total income if they have the following bookings for the chalet:

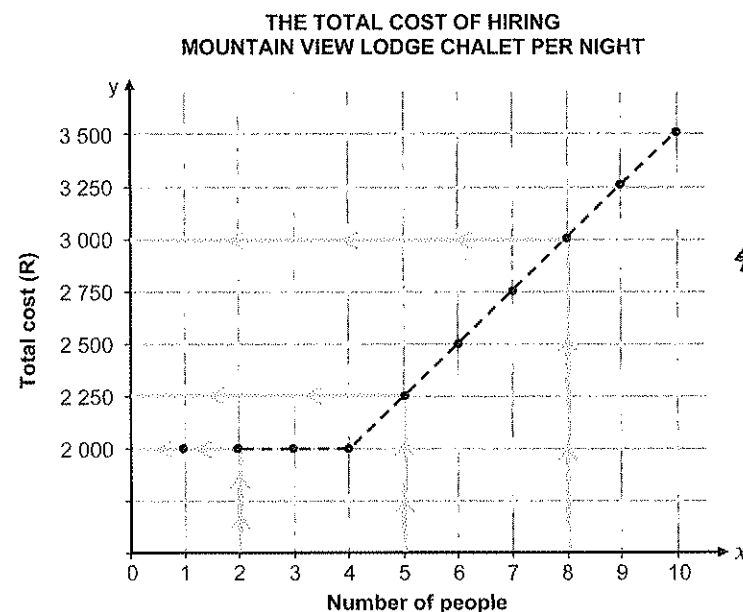
- 5 people for 2 nights
- 2 people for 1 night
- 8 people for 3 nights



$$\begin{aligned}
 \text{A: Total} &= (2\,250 \times 2) + (2\,000 \times 1) + (3\,000 \times 3) \\
 &= 4\,500 + 2\,000 + 9\,000 \\
 &= \text{R}15\,500
 \end{aligned}$$

REMEMBER!

The cost on the graph is per night, so $\therefore$ multiply the cost by the number of nights.



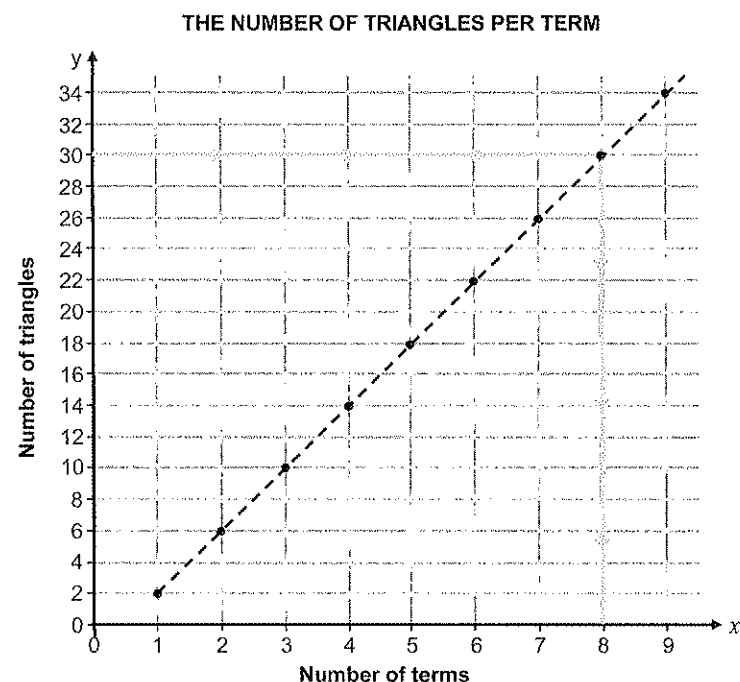
EXAMPLE 2

Refer to the graph provided of the number of triangles per term of Bryan's model bridge:

Q: Which term has 30 triangles in it?



A: The 8th term has 30 triangles



Test Your Understanding

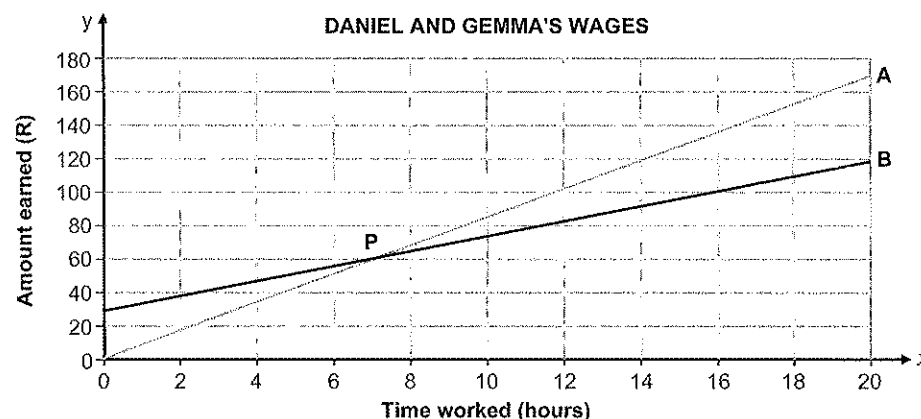
1. Robert wants to hire a Mercedes SLK Cabriolet for his wedding day, but he is not sure if this will fit his budget. He does an internet search and finds the following two packages:

Package 1: Voyager Car Hire charges R1 000 per day for a Mercedes SLK 200 Cabriolet, including insurance and free mileage.

Package 2: Exclusive Cape Cars charges R900 per day for the same car and includes insurance but mileage is charged at R0,80 per kilometre.



- 1.1 Construct a table which illustrates the costs of Package 1 and Package 2 for 0 - 200 kilometres. Your table should go up in increments of 25 km.
 - 1.2 Use your table from Question 1.1 to draw the graphs of the costs for Package 1 and Package 2, on the same set of axes.
 - 1.3 Determine the cost of Package 2 if Robert drove 250 km.
 - 1.4 How far must Robert drive for the cost of both packages to be the same?
2. Gemma and Daniel both work at restaurants to earn pocket money. Gemma works at Pizza 4 You. She gets R30 a week basic pay and an additional R4,50 per hour. Daniel works at Pizza Palace. He gets no basic pay, but gets paid R8,50 per hour. The graph below shows Daniel and Gemma's wages for one week.



- 2.1 For both graphs A and B, state:
 - 2.1.1 the type of relationship illustrated by the graph
 - 2.1.2 whether it represents Gemma or Daniel's income
- 2.2 Use the graph above to determine whether Gemma or Daniel earns more money, if they both work for 4 hours.
- 2.3 What does the point labelled P on the graph represent?
- 2.4 Read the approximate coordinates of P from the graph.
- 2.5 Use 'w' to represent wages and 't' to represent the time worked and write two equations to represent the wages of Daniel and Gemma respectively. Use the form $w =$
- 2.6 Use your equations from Question 2.5 to determine who earned more and by how much, if both Daniel and Gemma worked 15 hours.

SUMMARY TABLE OF THE DIFFERENT TYPES OF FORMAL RELATIONSHIPS

FORMAL RELATIONSHIP	CHARACTERISTICS	GENERAL EQUATION	GRAPH
A Constant/Fixed relationship	The dependent variable (y) remains constant (fixed), while the independent variable (x) increases.	Dependent variable = constant/fixed value	
B Linear relationship	A constant difference exists between variables.		
B1 Direct proportion	- A constant ratio exists between variables. - Both variables start at zero (the origin).	Dependent variable = constant ratio $\times$ independent variable	
B2 Not Direct proportion	No constant ratio exists between variables at all times. - Both sets of values do not start at zero.	Dependent variable = (constant difference $\times$ independent variable) + starting value	
C Indirect/Inverse proportion relationship	- There is a constant product between the sets of variables. - One variable will increase by the same factor that the other variable decreases by. - Neither variables will start at zero.	Independent variable $\times$ dependent variable = constant product	
D Constant ratio relationship	There is a constant ratio between consecutive terms of the pattern.	Next term = previous term $\times$ constant ratio	



1. When converting miles to kilometres, the following conversion is used:
1 mile = 1,61 km

1.1 Use the information above to complete the table below:

Miles (x)	0	10	20	30	40	50	100
Kms (y)	0	16,1					

- 1.2 Use the table from Question 1.1 to draw the graph of converting miles to kilometres.
1.3 What type of relationship is illustrated by the graph in Question 1.2?
1.4 Determine a formula that could be used to convert miles to kilometres.
1.5 Calculate the number of kilometres in 205 miles.

2. Puppies need to be fed at regular intervals. A two-month-old Golden Retriever puppy needs to be fed 330 grams of food per day spread out over a number of intervals.

2.1 Find the missing values in the table below: ✖

Number of servings per day	1	2	3	4	B
Serving size (grams)	330	A	110	82,5	55



- 2.2 Name the independent variable.
2.3 Is this data continuous or discrete? Give a reason for your answer.
2.4 What kind of relationship is illustrated in the table above?
2.5 Determine a formula that could be used to represent the relationship between the number of servings per day and the serving size.
2.6 Use the data from Question 2.1 to draw the graph of the number of servings vs serving size for a Golden Retriever puppy.
3. Cindy collects Barbie dolls and has a goal of increasing the size of her collection by 25% every year. Cindy started off with 12 dolls in her collection.
3.1 Construct a table to illustrate the size of her Barbie doll collection over a period of 5 years.

REMEMBER! Round up your answer since we are working with discrete data.



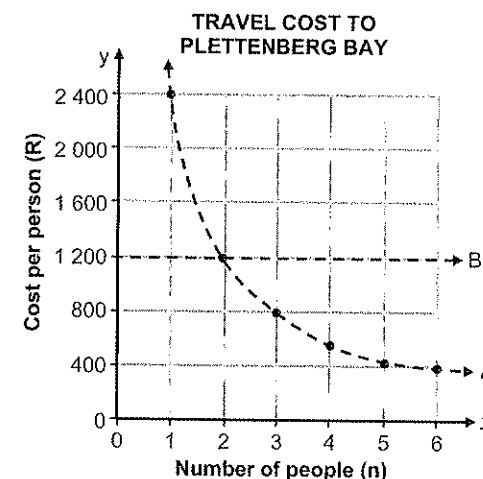
- 3.2 What kind of relationship is illustrated in this example?
3.3 Draw the graph of the number of Barbie dolls Cindy has in her collection over the years.

4. ✖ A group of friends would like to travel to Plettenberg Bay for a holiday.
Durban

They have two options:

- hire a minivan that can accommodate a maximum of 6 people for a flat rate of R1 200 per person; **OR**
- squeeze into a bakkie that can also accommodate up to six people, with a total cost of R2 400.

This is illustrated in the graph alongside:



- 4.1 For both graph A and B, state:
4.1.1 the type of relationship illustrated by the graph
4.1.2 whether the graph represents the minivan hire or the bakkie
4.1.3 a formula that could be used to illustrate the cost per person based on the number of people travelling.
4.2 Determine the cost per person if 3 friends travelled to Plettenberg Bay, by:
4.2.1 minivan
4.2.2 bakkie
4.3 Give the coordinates of the point at which the costs of using either the minivan or the bakkie is the same.
4.4 How many friends should travel together to make the cost per person more economical if they travel by bakkie?

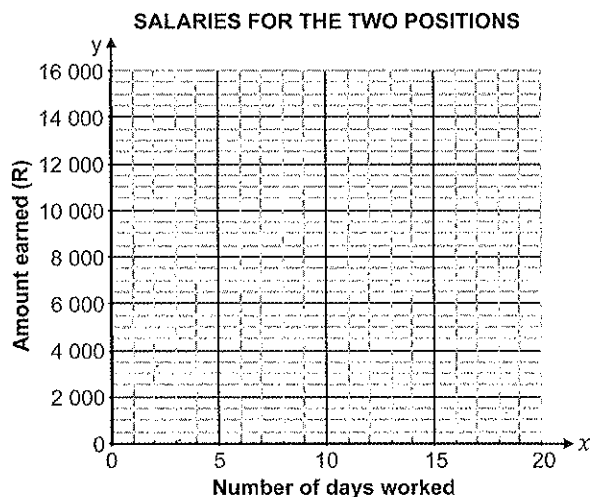
5. Timothy is a newly qualified marketing graduate. He has been offered two positions, one as a medical sales representative for Meds SA and the other as a tobacco sales representative for ABC Cigs.

The formula for calculating the monthly salary for a medical sales representative is:
Salary = R3 000 + R500 × number of days worked.

As a tobacco sales representative, he will earn a salary of R750 per day for each day worked in a month. He will only receive a salary if he works for one or more days in a month.

- 5.1 Write down a formula that can be used to calculate the monthly salary of a tobacco sales representative.

- 5.2 Copy the grid below and draw TWO line graphs to represent the monthly salaries for both the positions of medical and tobacco sales representatives. Clearly label each graph.



- 5.3 Use the graphs drawn to answer the following:

- 5.3.1 After how many working days will the two salaries be the same?
- 5.3.2 Suppose Timothy worked at Meds SA for 18 days. How many days would he have to work at ABC Cigs to earn the same salary?

6. The local child welfare organisation is planning a fundraising dinner-dance and intends selling 200 tickets at R150,00 each.

The organisers have to choose between the Impala Conference Centre and the Community Hall as a venue.

A musical group has agreed to provide the music for free.

The Impala Conference Centre has no basic fee, but charges R120 per person.

The Community Hally charges a basic fee of R3 000 and then charges R90 per person.

This information can be represented in the tables below:

TABLE 1: THE COST OF HIRING THE IMPALA CONFERENCE CENTRE

Number of tickets sold (n)	0	50	100	160	200
Cost (C) in Rand	0	6 000	12 000	A	24 000

TABLE 2: THE COST OF HIRING THE COMMUNITY HALL

Number of tickets sold (n)	0	50	B	140	200
Cost (C) in Rand	3 000	7 500	12 000	15 600	21 000

TABLE 3: INCOME FROM TICKET SALES

Number of tickets sold (n)	0	50	100	150	200
Cost (C) in Rand	0	7 500	15 000	C	30 000

- 6.1 Find the missing values A, B and C in the given tables above.
- 6.2 Use the values in TABLE 1 and TABLE 2 to write down the number of tickets sold when the cost for the two venues is the same.
- 6.3 Suppose all the tickets are sold. Use TABLE 1 and TABLE 2 to write down the ratio of the cost for 200 people of hiring the Impala conference Centre to the cost of hiring the Community Hall.
- 6.4 Write down the dependent variable in TABLE 1.
- 6.5 Is the data continuous or discrete in TABLE 2?
- 6.6 What type of relationship is illustrated in TABLE 3? Give reasons for your answer.
- 6.7 Use the data from TABLE 2 and TABLE 3, to draw two graphs on the same set of axes. Label the graphs clearly.
- 6.8 Use your graph from Question 6.7 to answer the following questions:
- 6.8.1 How many tickets must be sold for the income to equal the cost?
- 6.8.2 How many tickets must be sold for the income generated to total R18 000?
- 6.8.3 Determine the cost if 190 tickets were sold.



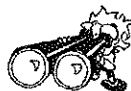
3 FINANCE

UNIT 1

FINANCIAL DOCUMENTS

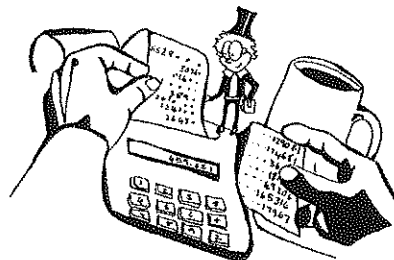
Interpreting and understanding financial documents is an important life skill, as it enables us to make sound financial decisions.

In this unit, we will investigate some everyday financial documents, including: till slips, account statements, household accounts/bills, invoices, payslips, travel allowances and loan agreements.



Other financial documents will be explored in greater detail in the following units:

- Unit 3: Income, Expenditure, Profit/Loss, Income-and-Expenditure Statements and Budgets (see p. 68)
 - Budgets
 - Income-and-Expenditure Statements
- Unit 7: Banking, Loans and Investments (see p. 82)
 - Bank fee tables
 - Monthly repayment factor tables
 - Bank Statements
- Unit 9: Taxation (see p. 98)
 - Income tax tables
 - IRP5 tax forms



Till slips



A till slip is a receipt which serves as proof of payment for goods purchased. It is important to keep till slips, should you wish to return damaged or unwanted goods.

The important features of a till slip are illustrated below:

Pick n Pay
Inspired by you

PHILIPPI PARK
TELEPHONE NO. 021-912 4400

YOUR CASHIER IS THANDEKA GAMA

CARRY BAG 24L	2 @ 0.36	0.72
S/BER PEDALBIN BAGS30EA		16.99
SANDWICH BAGS SMALL		12.99
TUFFY BLACK BAG ROLL 20		29.99
SUPERIOR BEST OF BOTH		11.69
CHIPS TOMATO SAUCE		7.49
WILLARDS FLINGS 150GR		7.99
SPRITE REG COOLDRINK 2L		14.88
POTATOES WASHED	VIT	21.99 #
NN EGGS JUMBO	18'S VIT	25.89 #
TOOTHPICKS HOLDER		16.99
ESK STREAKY BACON 250G		23.49
1 GRABOUW BOEREWORS		24.00
1 BULK T-BONE /CLUB ST		59.04
NN SMOKED VIENNAS 500GR		19.49
	DUE VAT INCL	293.64
	BANK CARDS	293.60
	ROUNDING	-0.04
	TOTAL ITEMS	16

----- TAX INVOICE -----

VAT INCLUDED @ 14.0% 30.18

NON SUPPLY @ ZERO RATED #

DISCOVERY VITALITY VIT

----- VAT REG NO. 40801065688 -----

PLEASE RETAIN AS YOUR GUARANTEE

CUSTOMER CARE LINE 0800 11 22 88

WEBSITE www.picknpay.co.za

THANK YOU FOR SHOPPING WITH US

25.07.12 10.35

Name of shop

Name of branch - many shops have several branches all over the country

Contact details of the store

Name of cashier

VAT is not charged on certain items. VAT-exempted items are indicated by #. See Unit 9 for detailed VAT calculations.

Total amount that must be paid (including VAT)

Number of items bought

Total VAT amount of 14% (R30,18) which is included in the cost of purchases.

Company's VAT registration number

Date and time

You will notice that the total comes to R92,64. In South Africa we don't have 1 cent or 2 cent pieces to make up 4 cents. It is against the law to charge more than the displayed price so 64 cents cannot be rounded up to 65 cents. So, the store has to give a 4 cent discount. You will notice that there is a rounding of -0,04.

Account statements

When you buy goods on account at a shop, you buy them on credit and then pay for the goods at a later date or over a period of time. An account statement is:

- sent to clients on a **regular basis**, normally once a month.
- a **record of transactions** showing purchases bought on credit, interest charged, amount due as well as all payments that have been made.
- A **receipt** for money paid by you.



Terminology used in an Account Statement



- **Opening balance** - amount owed at the beginning of the period
- **Closing balance** - amount owed at the end of the period
- **Amount due** - minimum amount to be paid (often a percentage of the total amount owed)
- **Debit** - the amount of money that is owed by the client
- **Credit** - payment(s) made by the client



Every time you buy something on your account you will receive an **invoice**.

On the statement opposite, the client would have received invoices on 13th and 24th June as well as 8th July for goods purchased on credit.



The important features of an account statement are illustrated below:

EXAMPLE OF AN ACCOUNT STATEMENT

Statement number, statement date, account number, the amount due and the date payment is due.

The client's name and address

Kwane Clothing		STATEMENT	
TO Ms Z. Palucci 115 Pluto Place Peace Town Polokwane 0699		STATEMENT NUMBER STATEMENT DATE ACCOUNT NUMBER AMOUNT DUE PAYMENT DUE DATE	23 10 July 2014 7659 000 R 298,24 31 July 2014
DATE	DESCRIPTION	CHARGES	CREDIT BALANCE FORWARD
12 June 2014	Balance		R1 243,50
13 June 2014	Men's Clothing	R328,25	R1 571,75
24 June 2014	Women's Clothing	R101,20	R1 672,95
29 June 2014	Payment - Thank you		R972,95
8 July 2014	Woman's Shoes	R220,00	R1 192,95
			Closing Balance

Credit refers to payments made by the client.

Please detach portion below and send with your remittance

FROM Ms Z. Palucci 115 Pluto Place Peace Town Polokwane 0699	STATEMENT NUMBER STATEMENT DATE ACCOUNT NUMBER AMOUNT DUE AMOUNT ENCLOSED	23 31 July 2014 7659 000 R 298,24 R
--	---	---

MAKE ALL CHEQUES PAYABLE TO
Kwane Clothing
BANK DETAILS
Kwane Clothing
FNB
Branch: 101 400
a/c: 727 333 0000

THANK YOU FOR YOUR BUSINESS!

Bank details of the store so that the customer is able to transfer money from their personal account to that of 'Kwane Clothing'.

Please pay 25% of last amount in Balance column

ADDRESS QUESTIONS TO Lerato Mampuru 015-600 4242	= 1 243,50 + R328,25 + R101,20 - R700,00 + R220,00 = R1 192,95
--	---

Contact details of the person who can be contacted in the event that the client has any queries regarding the account.



Household accounts/bills

A household bill is an account sent to you for services used by your household, such as water, electricity, telephone and cellphone.

The following information can often be found on a household bill:

- **Name** of the person using the service
- **Address** where services are used
- **Time period** for services used
- **Opening balance** - the amount owed at the beginning of the period
- **Closing balance** - the final amount at the end of the period

The closing balance can either be a:

- **Credit balance** - the amount paid on a previous account was more than the amount owed. Therefore, there is a surplus of money in the account. Credit amounts are often indicated by the abbreviation CR.
- **Debit balance** - the amount paid on a previous account was less than the amount owed. Therefore, you still owe money on the previous account.

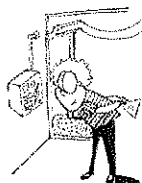
NOTE: Interest is charged on a debit balance for overdue.

- **Due date** - date by which the account must be paid

If not paid on time, the service can be suspended or additional charges can be added to your account.

- **Tariff** - the rate charged for the service
- **VAT** - 14% of the total charges

The closing balance includes 14% VAT.



The important features of a household account can be illustrated in this cellphone account below:

Name of
account holder



L9243867-2
OLIVER MICHAELS
407 MONTFRERE
1 CLAIR STREET
WESTDENE
BLOEMFONTEIN
65233

Billing address of
account holder



Account
number

Tax invoice

Account number: L9243867-2
Date: 03/07/2012

Your VAT registration number:

Date of statement

Account summary:

Date	Description	Item number	Reference	Amount	Total
04/06/2012	Balance Brought Forward			99.00	99.00
02/07/2012	Payment	SCZ1399863	159019663	-99.00	0.00
03/07/2012	Invoice	B227108833	726371238	99.00	99.00

Invoice summary:

Cellular number: 0731456720
Invoice number: B227108838
Due date: 31/07/2012

Due date of
when payment
must be made

-99 indicates a credit,
i.e. a payment was made

Description	Amount	VAT	Total
Subscription Services			
Data Promotion - Top Lip MyGig1	86.84	12.16	99.00
HSDPA Voice Tariff	0.00	0.00	99.00
VAS - Balance Notification	5.70	0.80	6.50
VAS - Free Balance Notification	-5.70	-0.80	-6.50
Total Subscription Services	86.84	12.16	99.00
Subtotal	86.84	12.16	99.00
This invoice amount	86.84	12.16	99.00

Vat-exclusive
total

Total VAT on
the invoice

VAT-inclusive
total

> > > 9 2060 1903 149 721 9

Invoice Total

99.00

Closing balance

Page 1 of 1

Your bank account will be debited with the full outstanding balance as reflected on this statement on the 1 August 2012.
Vodacom (Pty) Ltd. Registered office PO Box 3306 Cramerview 2060 Company Registration No. 1993/003367/07 VAT Registration No. 4010139121

Invoices



An invoice (also commonly known as a bill) is a **list of goods or services** rendered; together with a **statement** of the amount due.

It is important to note that often a 'Unit price' or 'Quantity' is given, which would then need to be multiplied by the number of items purchased, in order to get the total price. VAT is noted separately on the invoice, in order to allow VAT-registered companies to claim VAT back from the government.

The important characteristics of an invoice are illustrated below:

INVOICE

Italian Holidays

Name of service provider

Name of customer
Mrs G Choice

Invoice number
00001050

Date of invoice
January 12th, 2015

Your reference:
Email enquiry from Latroba

List of services provided
Rental of villa in Tuscany. Full payment is required 10 days before arrival. Thank you for your deposit of 15 000 rand to secure your booking. The deposit for damage and breakages will be refunded within 10 days of your departure, after our inspection.

Unit Price i.e. Price per single item/service

DESCRIPTION	UNIT PRICE	TOTAL	Total Price = Unit Price x No. of items/services e.g. bicycles = R750 x 5 = R3 750
Rental of villa for 7 nights arriving after 2:00 pm on June 24 th , 2015 and leaving by noon on 1 st July 2015	4 950.00	34 650.00	
Hire of bicycles for 5 days	750.00	3 750.00	
Final cleaning	1 045.00	1 045.00	
	Subtotal	R39 445.00	
	VAT @ 14%	R5 522.30	
	TOTAL DUE:	R44 967.30	

Banking details of service provider
Acc. Holder: Mr Amalfera
Acc. Number: 4621930469
Branch: 059621
Bank: FNB
Total Balance due = R39 445 + R5 522,30 = R44 967,30

We wish you a wonderful Italian Holiday!

Mr and Mrs L Amalfera Beech Acres, Spring Lane, Twizzelworth, W. Countyshire, PC1 2XZ.
Phone: +44 (0) 123 456 7890 Mobile: +44 (0) 123 456 7891

Payslips



A payslip is a **record** of information about **remuneration**. The employer is expected by law to provide their employees with a payslip containing the following information:

- **Name and address of employer**
- **Name and ID number of employee**; together with the employee's work number
- **Occupation/job title**
- **Pay period**
- **Basic wage**, including both ordinary hours and overtime worked; together with the rate and number of hours worked.
- **Deductions**
 - PAYE (Pay-as-you-earn)
 - UIF (Unemployment Insurance Fund)
 - SITE (Standard Income Tax on Employees)
 - Union Contributions
 - Medical Aid contributions
 - Pension Fund contributions and any others
- **Payment method and details**

UIF is usually calculated as 1% of gross income

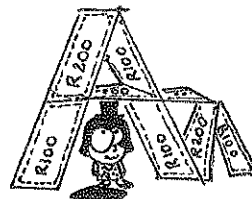
NOTE!

- Gross income is the income received before any deductions have been subtracted.
- Nett income is the income received after deductions have been subtracted
∴ Nett income = gross income - deductions

Salary or Wage?

A **salary** is a fixed payment in a pay period, based on an annual total. It is not based on the number of hours worked.

A **wage** is a payment for work based on an hourly/daily/weekly/piece-work rate.



The important characteristics of a payslip are illustrated below:

Employee's details **Employer's details** **Pay date**

FNB

Payslip number: 35
 Employer name: MPA South Africa, 22 Simon Street, Westlake, Cape Town
 Occupation/Title: Bookkeeper
 Employee number: 329041
 Employee name: Potter, Lisa
 Cost code: Finance 329
 Date engaged: 10-Mar-2014
 Identity number: 730112 0936 087
 Date of birth: 12-01-73
 Sex: F
 Marital status: M
 Week engaged: Pay point: Tax status/Children:
 Rate of pay or package: 14,500.00
 Tax reference number: Address: 61 Weiden Road, Simons Town, Cape Town

Basic rate

Cash Component	14 500.00	Tax Deducted	2 223.33
----------------	-----------	--------------	----------

any bar *Medical Unit*

Total of Tax Deducted based on the TAX tables issued by SARS; together with TAX deduction such as UIF and PAYE

Payment method

Payment details: EFT Cheque 32005	4913668421	12 276.67
-----------------------------------	------------	-----------

Income Taxable (3601)	141 206.05	UVE Annual (Balance)	4.50
Annual Payment (3605)	18 000.00		
SITE (4101)	2 039.03		
PAYE (4102)	24 152.75		
Total Tax (4103)	26 191.78		

Total tax deductions Year-To-Date (YTD)



Worked Example

Study the payslip given alongside and answer the following questions:

- What is the name of the employee?
Nosi Zulu
- Does Nosi earn a wage or a salary?
Give a reason for your answer.
A salary - as she receives a fixed payment per month.
- What is Nosi's rate for ordinary hours worked?
 $R6\ 500 \div 130\ h$
 $= R50\ \text{per hour}$
- How many hours of overtime did Nosi work?
 $R900 \div R150 = 6\ \text{hours}$
- Calculate how much Nosi contributes towards UIF, if UIF is calculated as 1% of her gross income.

$$\text{Gross income} = R6\ 500,00 + R900,00 = R7\ 400,00$$

$$\text{UIF} = \frac{1}{100} \times R7\ 400,00 = R74$$

- Determine Nosi's net income.

$$\text{Total deductions} = R1\ 332,00 + R74,00 + R33,00 = R1\ 439,00$$

$$\therefore \text{Net income} = R7\ 400,00 - R1\ 439,00 = R5\ 961,00$$

More examples on UIF and income tax in Unit 9: (see p. 98)



PAYSIP

NAME OF EMPLOYER: SPRAY ART INC.
ADDRESS: 6A Lamb Street
 BLACKHEATH
 7421
NAME OF EMPLOYEE: NOSI ZULU
ID NO: 9204270314087 **EMPLOYEE NUMBER:** 63004
OCCUPATION/GRADE: RECEPTIONIST
PAY PERIOD: FROM 1 March 2014 TO 31 March 2014
BASIC WAGE: R6 500.00
MANNER OF PAYMENT: ☐ PER HOUR ☐ PER DAY ☐ PER WEEK ☒ PER FORTNIGHT ☐ PER MONTH

	Rate	No of hours	Rand earned
Ordinary hours worked	R50	130	R6 500.00
Overtime worked	150.00	6	900.00
Sundays time worked			
Public holidays time worked			
Payment in kind			
Allowances (specify)			

TOTAL (GROSS) R 7400.00
DEDUCTIONS: P.A.Y.E. R1 332.00
 UIF 1% R74.00
 UNION R33.00
OTHER (Full details)
TOTAL R 5961.00
TOTAL AMOUNT DUE (NETT) R 5961.00

REMEMBER!

$$\text{Net income} = \text{Gross income} - \text{Deductions}$$



Travel Allowances



- A travel allowance is a set amount of money that is paid to the employee to cover the fixed cost, fuel and maintenance costs of the employee's vehicle, which is used for company purposes (e.g. a salesman).
- The South African Revenue Services (SARS) publishes a table used to determine the total running costs of a vehicle, based on the value of the vehicle, as shown below:

e.g. **SARS Vehicle Cost Table: 1 March 2013 - 28 February 2014**

Value of the vehicle (including VAT) (R)	Fixed cost (R p.a.)	Fuel cost (c/km)	Maintenance cost (c/km)
0 - 60 000	19 310	81,4	26,2
60 001 - 120 000	38 333	86,1	29,5
120 001 - 180 000	52 033	90,8	32,8
180 001 - 240 000	65 667	98,7	39,4
240 001 - 300 000	78 192	113,6	46,3
300 001 - 360 000	90 668	130,3	54,4
360 001 - 420 000	104 374	134,7	67,7
420 001 - 480 000	118 078	147,7	70,5
Exceeding 480 000	118 078	147,7	70,5

Worked Example

Nelson is a solar panel salesman. He travels on average, 45 000 km per annum. He drives a minivan valued at R250 000.
Calculate: Nelson's monthly average travel allowance.

Fixed cost of R250 000 vehicle = R78 192 per annum

Fuel costs = 45 000 km × 113,6 cents = 5 112 000c = R51 120

Maintenance costs = 45 000 km × 46,3 cents = 2 083 500c = R20 835

∴ Total annual vehicle costs = R78 192 + R51 120 + R20 835
= R150 147

∴ Monthly travel allowance = R150 147 ÷ 12
= R12 512,25

REMEMBER!
To convert
cents to rands
÷ by 100!



Loan Agreements



- A loan agreement is a contract between **two parties**; whereby the **lender** gives money to the **borrower**, with the expectation that the money will be repaid, together with **interest** within a fixed time period.
- The interest on the loan is calculated using either a fixed or variable interest rate; based either on simple or compound interest.
- The **monthly repayment** amount comprises of both paying off part of the original borrowed amount; plus a part of the interest.
- The fixed time period of the loan is often referred to as the **term of the loan**; and is usually stated in months.



The important characteristics of a loan agreement are illustrated below:

Lender		
HAPPY HOUSE HOME LOANS		
Mr & Mrs Apple	Borrower	Loan Acc No: 438291
23 Pear Street, Cape Town 8001		Loan Statement: 30 April 2014
Account Summary: 23 Pear Street, Cape Town, 8001		
Registered Bond Amount	R950 000	Value of the loan
Balance Outstanding	R800 000	
Registered Loan Term	240 months	House loans are typically 20 years
Remaining Loan Term	200 months	
Interest Rate	8.5% p.a.	Interest is compounded daily
Basic Repayment	R8 089,73	
Administration Fee	R57,00	Monthly repayment of loan and interest
Total Monthly Repayment	R8 146,73	

Worked Example



Study the loan table below and answer the following questions:

Loan term (No. of instalments)	Multi loan		9 months		23 months		35 months		49 months		84 months	
	Lowest	Highest	Lowest	Highest	Lowest	Highest	Lowest	Highest	Lowest	Highest	Lowest	Highest
Loan Amount	1 000	1 000	10 000	10 000	82 000	82 000	110 000	110 000	150 000	150 000	230 000	230 000
Monthly repayment	1 168	1 231	1 385	1 466	4 323	4 942	4 113	4 983	4 365	5 595	4 984	5 939
Initiation fee *	137	171	1 140	1 140	1 140	1 140	1 140	1 140	1 140	1 140	1 140	1 140
Monthly service fee *	n/a	n/a	57	57	57	57	57	57	57	57	57	57
Credit life insurance	n/a	n/a	FREE	FREE	FREE	FREE	FREE	FREE	FREE	FREE	FREE	FREE
Interest rate	32,00%	60,00%	17,10%	32,00%	17,10%	32,00%	17,10%	32,00%	17,10%	31,70%	18,50%	25,20%
Total repayment	1 168	1 231	12 465	13 197	99 431	113 663	143 957	174 413	213 880	274 157	418 696	498 913

* Fees include VAT. Instalments depend on the exact number of days of the loan. Terms and conditions apply

- What is the longest loan term, based on this table? Give your answer in years.
 $84 \text{ months} \div 12 = 7 \text{ years}$
- Give the range of your monthly repayment if you took out a loan for 23 months.
 $R4\ 323 - R4\ 942$
- How much is the initiation fee on a R150 000 loan?
 $R1\ 140$
- How much is the monthly service fee on an R82 000 loan?
 $R57$
- What is the interest rate range on an R10 000 loan?
 $17,10\% - 32,00\%$
- How much is the loan amount if the total repayment is R498 913?
 $R230\ 000$
- What is the total repayment on a R110 000 loan, if the lowest interest rate was used?
 $R143\ 957$

Test Your Understanding



- The till slip alongside shows the purchases made by Mrs Vosloo at Woolworths.

- Which Woolworths store did Mrs Vosloo visit?
- How are the VAT-inclusive items indicated on the till slip?
- What was the '24 LITRE VTC (FOODS)' at 40 cents?
- How did Mrs Vosloo pay for her purchases?
- If thick cut rump steak was R78,99 per kilogram, what was the weight of the steak that Mrs Vosloo bought?

- Determine the VAT-exclusive total for this till slip.
- If Mrs Vosloo paid cash, how would the total of R224,39 be rounded off?

WOOLWORTHS

Welcome to our Store
Parklands Junction
Tel no: (021) 521 4711
VAT REG no. 4550102216
Please retain as proof of purchase

LAST DAY FOR A FULL REFUND IS 04/09/2015

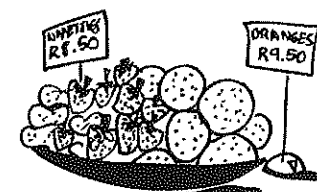
LEMONADE 275ml 6PK	A	R 37,45
THICK CUT RUMP STEAK	A	R 46,12
24 LITRE VTC (FOODS)	A	R 0,40
3L AYR LF MILK	C	R 25,95
ROSA TOMATOES 600G	C	R 24,99
ROASTING VEGETABLES	C	R 27,99
B.POTS/HERB	A	R 16,99
125G SOFT EAT FRUIT	A	R 10,95
125G SOFT EAT FRUIT	A	R 10,95
25G FR DROEWORS	A	R 9,95
60G FRUIT DAINTIES	A	R 7,65
THE ARGUS	A	R 5,00
***TOTAL		R224,39
ABSA		R224,39
ACCOUNT NUMBER *****3018		
AUTH: 662785		
CHANGE		R 0,00

-----TAX INVOICE-----

A - 14% VAT	R 17,86
VAT TOTAL	R 17,86

-----VALID VAT INVOICE-----

EPS No 7433
06/07/2015 17:26 0267 006 1030 7045694
CASHIER - Tokoza 7045694



2. Study the municipal account statement below and answer the questions that follow:



a world class African city



JOHANNESBURG
2019

(011) 375-5555

(011) 358 3408/9

joburgconnect@joburg.org.za

PO Box 5000

Johannesburg, 2000

Page 1 of 2

VAT NO. CITY OF JOHANNESBURG 4750117194 VAT NO. PIKITUP 4750191282
VAT NO. JOHANNESBURG WATER 4270191077 VAT NO. CITY POWER 4710191182

Date 2014/06/26

Statement for June 2014

Physical Address

Stand No./Portion

Township

Municipal Valuation

Market Value R2,820,000.00

Stand Size 920 m² Number of Dwellings 1 Date of Valuation 2008/07/01

Invoice Number: 34000562492
Client VAT Number

Deposit R0.00

Account Number:

Previous Account Balance	1 172.33
Less: Incoming Payment (Last Payment Made 2014/06/26)	-3 700.00
Sub Total	-2 527.67
Current Charges	1 646.76

60 DAYS +	60 DAYS	30 DAYS	CURRENT	INSTALLMENT PLAN	TOTAL AMOUNT OUTSTANDING
0.00	0.00	0.00	-880.91	0.00	-880.91

Total Due -R881.00

Due Date 2014/06/26

Introducing our new contact number. 0860 Joburg. Your City. Your Number -- Starts 1 June 2014



Remittance Advice:

This stub must accompany payment.
Please do not detach if paying at the post office.

Date 2014/06/26
Acc. No.



911152023957398

ABSA

City of Johannesburg Bank Acc No 405 439 8463
Branch Code 632005
Client Account No/Deposit Reference 202395739

0146202395739



Total Due -R881.00

Due Date 2014/06/26

- 2.1 What is the opening balance of this account statement?
- 2.2 How much extra did the customer pay towards their account?
- 2.3 Using the information given on the current charges and stand size, determine the charge per square metre.
- 2.4 Is this account in credit or debit? Give a reason for your answer.

3. Study the invoice below and answer the questions that follow:



Xin Cube Inc

380 Francisco St

San Francisco

CA 94122

US

Tel: (415) 989-1188 Fax: (415) 989-2288

Email: admin@xincube.com

Website: www.xincube.com

Invoice No: INV10100005

Currency: ZAR

Invoice

Bill To
John Smith
Synex Inc
128 Juanita Ave
Glenvista
South Africa

Ship To
John Smith
Synex Inc
128 Juanita Ave
Glenvista
South Africa

Phone:	Fax:	Phone:	Fax:		
Date	19-Oct-2014	Order No	Sales Person	Charles Woolen	
Shipping Date	19-Oct-2014	Shipping Terms	Terms	30 Days	
ID	SKU / Description	Unit Price	Qty	Amount	
PS.V860.05	AMD Athlon X2DC-7450, 2.4GHz/1GB/160GB/SMP-DVD/VB	580.00		3 480.00	
PS.V880.37	PDC-E5300 -- 2.6GHz/1GB/320GB/SMP-DVD/FDD/VB		4.00	6 580.00	
LC.V890.02	LG 19.5" WLCD	1 230.00	10.00	12 300.00	
HP.Q754.71	HP LaserJet 5200	7 100.00	9.00		

Note:
Thanks for your business!

Sub Total:
VAT:

Shipping:

Total:

Amount due:

- 3.1 What is the invoice number and date?
- 3.2 How many 'AMD Athlon' did John order?
- 3.3 What is the unit price of a 'PDC-ES300'?
- 3.4 Determine the total cost of the 'HP LaserJet'.
- 3.5 Determine the rand value of VAT due on this invoice.
- 3.6 Determine the cost of shipping, if it is calculated using the following formula:
Cost of shipping = 5,5% of subtotal + R1 520
- 3.7 Calculate the amount due on this invoice.

4. Study the payslip below and answer the questions that follow:

TESTCO3
0153077008

First Name: Alexandra
Cost Centre:
Project:

1 Main Rd
Tzaneen
0850
Limpopo
South Africa

the **tax** shop
tax, accounting, much more.

Identity Number: 7112170795187
Employee No.: 000004

Run January 2014

First Name: Alexandra
Birth Date: 17 Dec 1971
Period Start Date: 01 Jan 2014
Engagement Date: 02 Feb 2000
Cost Centre:
Ordinary Days: 22 00
Bank Name:

Last Name: Schoeman
Position:
Period End Date: 31 Jan 2014
Termination Date:
Tax Status: Standard Employment (Normal)
Ordinary Hours: 176 00
Branch Code:

Company Name	TESTCO3	Run	January 2014
First Name:	Alexandra	Last Name:	Schoeman
Birth Date:	17 Dec 1971	Position:	
Period Start Date:	01 Jan 2014	Period End Date:	31 Jan 2014
Engagement Date:	02 Feb 2000	Termination Date:	
Cost Centre:		Tax Status:	Standard Employment (Normal)
Ordinary Days:	22 00	Ordinary Hours:	176 00
Bank Name:		Branch Code:	

Allowances	Hours	Amount	MTD Amount	YTD Amount
Basic Pay	176 00	100 000 00	100 000 00	100 000 00
Total Earnings		100 000 00	100 000 00	100 000 00

Deductions	Balance	Amount	MTD Amount	YTD Amount
Tax Paid		10 410 00	10 410 00	10 410 00
UIF Employee		124 78	124 78	124 78
Total Deductions		10 534 78	10 534 78	10 534 78

Net Pay		R89,465,22	
Deductions	Amount	MTD Amount	YTD Amount
Skills Development Levy	1 000 00	1 000 00	1 000 00
UIF Employee	124 78	124 78	124 78
Total Company Contributions	1 124 78	1 124 78	1 124 78

Package Structure Breakdown	Amount
Basic Pay	100 000 00
Total Package	100 000 00

- 4.1 What is the name of the employee?
 - 4.2 How many hours did the employee work for?
 - 4.3 Calculate the hourly rate of the employee.
 - 4.4 How much tax did the employee pay?
 - 4.5 By how much has the employee been under-paid in terms of their UIF contributions; if UIF is calculated at 1% of the employee's gross income?
 - 4.6 Calculate the employee's correct net pay, based on the correct UIF contribution.
5. Use the SARS Vehicle Cost Table on pg. 58 to calculate the monthly travel allowance for a salesman who drives a R365 000 vehicle and travels, on average, 22 000 km per year.



6. Study the following loan agreement summary and answer the questions below:

INTEREST AND PAYMENT SUMMARY:

	First 78 Payments	Last 282 Payments
Principal amount and interest	761.78	761.78
Mortgage Insurance	82.35	----
Property Tax and insurance	206.13	----
Total Monthly Payment		967.91

- 6.1 What is the total length of this loan? Give your answer in years.
- 6.2 Calculate the total monthly payment for the first 78 payments.
- 6.3 Determine the property tax and insurance for the last 282 payments.
- 6.4 Calculate the total amount paid over this loan period.
- 6.5 If it was given that the loan amount was R150 000, then what percentage of the total amount paid back was interest?



UNIT

2

TARIFF SYSTEMS

Tariffs

- A tariff is the **charge** in rands **per measuring unit** for a specific service, such as electricity, water and telephone calls.

Water consumption is measured in kilolitres (kl), electricity consumption in kilowatt-hours (kWh) and cellphone airtime in minutes or seconds.

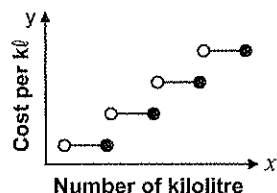
$$\text{Tariff (cost per unit)} = \frac{\text{total cost}}{\text{number of units}}$$

- Tariffs are not always constant, for example: the price of electricity/water per unit gets more expensive the more electricity/water you use.
- For cellphones and electricity there are two main tariff systems, namely **prepaid** and **contract/monthly billing**.



Water Tariffs

- Water usage is measured in kilolitres (kl). 1 kilolitre = 1 000 litres
- The amount that a person will pay for water each month is determined by the number of kl of water that they use during the month.
- Water is charged according to a '**sliding scale**' to avoid consumers paying the same rate regardless of the amount of water they use. The more water you use, the higher the rate charged.
- The graph of a water tariff table is usually a 'stepped' graph:



NOTE!

- The 'steps' are due to the sliding-scale.
- Open dots (○) indicate 'excluding' and closed dots (●) indicate 'including'.

Worked Example

Below is a table, which shows the costing structure of water:

Water Tariffs

These tariffs came into effect on July 1, 2010

Water usage in kilolitres	Tariff * (per kilolitre)
0 kl to 9 kl	Nil
From 9 kl to 25 kl	R9,27
From 26 kl to 30 kl	R12,36
From 31 kl to 45 kl	R19,06
More than 45 kl	R20,96

Fixed charges per month	Tariff *
0 kl to 9 kl	Nil
Greater than 9 kl	R83,43

* Tariffs excludes 14% VAT

In order to fix a crack in the side of the pool, 46 kl of water had to be drained from the pool. With the use of the water tariff table above, calculate the cost of refilling the pool.

Cost of water usage (excl. VAT):

$$\begin{aligned}
 &= (R0,00 \times 9 \text{ kl}) + (R9,27 \times 16 \text{ kl}) + (R12,36 \times 5 \text{ kl}) + \\
 &\quad (R19,06 \times 15 \text{ kl}) + (R20,96 \times 1 \text{ kl}) \\
 &= R0,00 + R148,32 + R61,80 + R285,90 + R20,96 \\
 &= R516,98
 \end{aligned}$$

Cost of fixed charges per month (excl. VAT):

$$= R83,43$$

Total costs (excl. VAT):

$$\begin{aligned}
 &= R516,98 + R83,43 \\
 &= R600,41
 \end{aligned}$$

VAT (at 14%) on total costs:

$$\begin{aligned}
 &= \frac{14}{100} \times R600,41 \\
 &= R84,06
 \end{aligned}$$

∴ Total cost (incl. VAT)

$$\begin{aligned}
 &= R600,41 + R84,06 \\
 &= R684,47
 \end{aligned}$$

NOTE!

If you add up all kl, you must get to the 46 kl in the question

$$(9 \text{ kl} + 16 \text{ kl} + 5 \text{ kl} + 15 \text{ kl} + 1 \text{ kl})$$

Cellphone Tariffs

The billing structure of cellphone networks can either be **prepaid** or on **contract**.

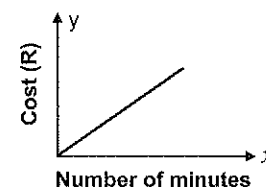
Prepaid tariff system

- General formula for the prepaid tariff system:

$$\text{Prepaid cost} = \text{tariff} \times \text{total number of units (minutes or seconds)}$$

- A prepaid tariff system graph is a straight line graph that starts at the origin.

i.e.



NOTE!

- If no calls are made (i.e. 0 minutes), there is no cost.
- The constant increase represents the cost per minute (e.g. R2/min).

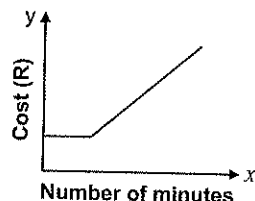
Contract tariff system:

- A cellphone contract is taken out from a service provider for a specific period of time (e.g. 2 years).
- A cellphone is included in the package.
- The cost per month includes:
 - Subscription fee** - pays off the cost of the cellphone, as well as administrative services and other options, e.g. sms bundles.
 - Cost for the calls** made during the month. Some contracts include 'free' minutes per month.
 - General formula for the contract tariff system:

$$\text{Contract cost} = \text{Subscription fee} + \text{tariff} \times (\text{total number of minutes} - \text{'free minutes'})$$

- A contract tariff system graph is a horizontal line (which does not start at the origin); followed by a straight line graph.

i.e.



NOTE!

- Starting point is not at the origin (subscription fee excluding calls).
- Horizontal line represents number of free minutes (subscription fee only during 'free minutes' time).
- Straight line graph represents constant cost per minute of calls beyond the free minutes.

Worked Example

Nosi was offered a bursary to study Film and Multimedia Studies at City Varsity in Cape Town. Since she does not have her own computer, she will need a cell phone with unlimited web-browsing. The table below illustrates the costs of 3 different deals from Vodacom.

Deal	Phone and Internet Costs	Call Costs
Deal 1	- R2 500 cash for phone - R50/month for internet	R2 per minute for calls
Deal 2	R250 per month over 24 months; including internet and phone	50 free minutes - R2,50 per minute for calls after free minutes used
Deal 3	R400 per month over 24 months; including internet and phone	120 free minutes - R1,50 per minute for calls after free minutes used

- Determine which deal (for phone, internet and calls) is the cheapest over 24 months (if Nosi makes 3,5 hours of calls per month). Show all calculations.

$$\begin{aligned} \text{Deal 1 total costs} &= R2\,500 + (50 \times 24) + (R2 \times 210 \times 24 \text{ months}) \\ &= R3\,700 + (R420 \times 24 \text{ months}) \\ &= R3\,700 + R10\,080 \\ &= R13\,780 \end{aligned}$$

$$\begin{aligned} 3,5 \text{ hours} \times 60 \text{ min} \\ &= 210 \text{ minutes} \end{aligned}$$



$$\begin{aligned} \text{Deal 2 total costs} &= R250 \times 24 + (R2,50 \times (210 - 50 \text{ min}) \times 24 \text{ months}) \\ &= R6\,000 + (R400 \times 24 \text{ months}) \\ &= R6\,000 + R9\,600 \\ &= R15\,600 \end{aligned}$$

REMEMBER!
Calculate the call charges for 24 months.

$$\begin{aligned} \text{Deal 3 total costs} &= R400 \times 24 + (R1,50 \times (210 - 120 \text{ min}) \times 24 \text{ months}) \\ &= R9\,600 + (R135 \times 24 \text{ months}) \\ &= R9\,600 + R3\,240 \\ &= R12\,840 \end{aligned}$$

Don't forget to subtract the free minutes!



∴ Deal 3 works out to be the cheapest option in total.

- Determine a formula for the total costs per month (excluding the phone) for:

REMEMBER! Contract costs =
Subscription fee + tariff × (total number of minutes - free minutes)

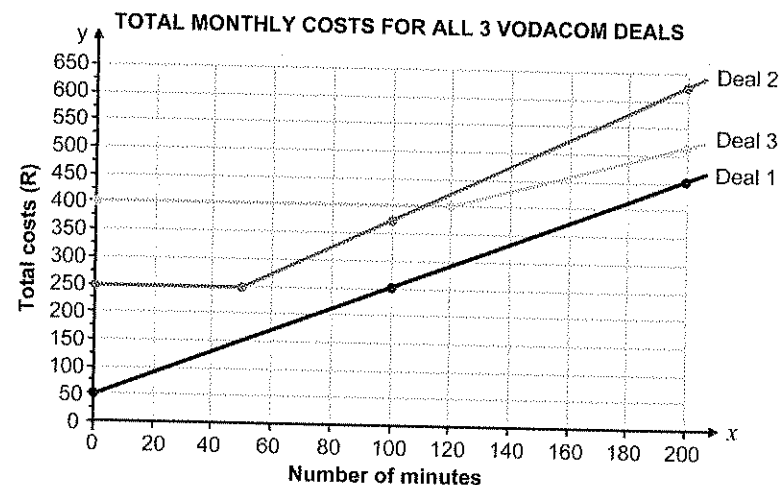
2.1 Deal 1:

$$\text{Total costs} = R50 + (R2 \times \text{number of minutes})$$

2.2 Deal 2:

$$\text{Total costs} = R250 + [R2,50 \times (\text{total number of minutes} - \text{free minutes})]$$

- Draw the graph of the total monthly costs for all 3 Vodacom deals.



REMEMBER! Always provide a title for your graph; labels for your axes and labels for each graph!



4. Using your graph from Question 3, determine the following:
 - 4.1 Which deal is the cheapest if Nosi speaks for 40 minutes per month?
Deal 1
 - 4.2 Which deal is the most expensive if Nosi speaks for 160 minutes per month?
Deal 2
 - 4.3 The point at which Deal 2 and Deal 3 cost the same amount of money.
110 minutes

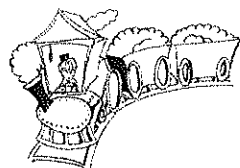
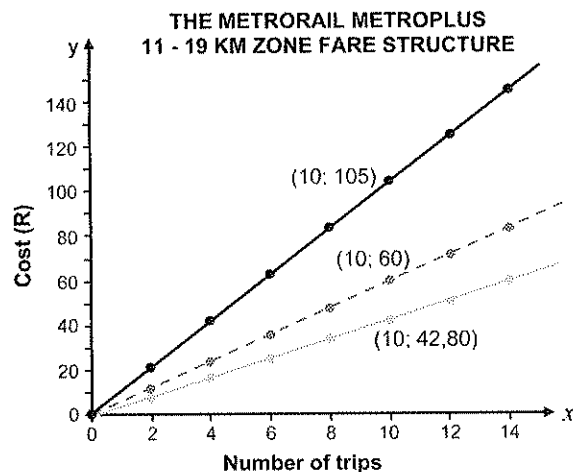
Transport Tariffs

- The most common means of public transport in South Africa are taxi, bus and train.
- Fares are based on the distance travelled; the cost of petrol and the general wear-and-tear on the vehicle.
- Single fares (one-way only); return trip fares (there-and-back); and weekly/monthly ticket fares can be purchased.
- People who use their private vehicles for transport, often have to pay for parking. These fees are usually based on the duration of parking.



Worked Examples

1. Study the Metrorail graph below of the fare structure for MetroPlus 11-19 km zone for a single, weekly or monthly ticket and answer the questions that follow:



- 1.1 Which fare structure is the most expensive? *Single fare structure.*
- 1.2 By how much is the weekly fare more expensive per trip than the monthly fare?

$$\text{Weekly fare} = R60 \div 10 = R6,00$$

$$\text{Monthly fare} = R42,80 \div 10 = R4,28$$

$$R6,00 - R4,28 = R1,72$$
- 1.3 How much would a person save travelling to and from work (Monday to Friday), if they purchased a weekly fare as opposed to single fares every day?

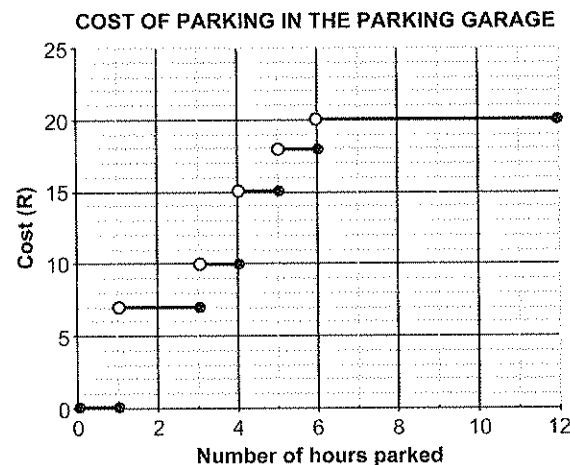
$$R105 - R60 = R45$$
- 1.4 Determine a formula to calculate the total cost of a certain number of trips, if single fare tickets were bought.

$$\text{Single fare} = R105 \div 10 = R10,50$$

$$\therefore \text{Total cost} = R10,50 \times \text{number of trips}$$

Travelling to-and-from work,
 Monday to Friday = 2 trips $\times$ 5 days
 = 10 trips.
 $\therefore$ To determine the savings, look at the cost of 10 trips for the single and weekly fare structure.

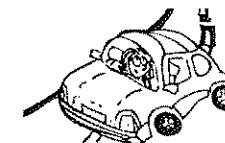
2. Study the graph below that shows the cost of parking at a parking garage that is open 12 hours daily; and answer the questions that follow:



This type of graph is called a 'stepped'-graph.



- 2.1 What is the maximum time that you can park at the parking garage for free? *1 hour*
- 2.2 If you paid R10,00 for parking, how long was your car at the parking garage? *3 - 4 hours*
- 2.3 How much would you pay if you parked your car at the parking garage for 5 hours 42 minutes? *R18*
- 2.4 How much would you pay if you parked for exactly 3 hours? *R7*
- 2.5 What is the maximum cost for parking your car at this parking garage? *R20*

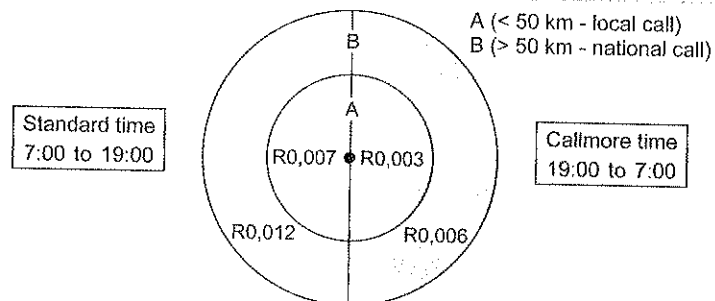


Test Your Understanding



1. To help their customers understand how the cost of calls made to other landline telephones is calculated, the telephone company supplies the following tariff structure.

THE CHATTERBOX TELEPHONE CO.
TARIFF STRUCTURE FOR CALLS TO LANDLINE TELEPHONES
 Miss Sinead Sneary Telephone Number: (033) 343 3300



All calls are charged per second.

[Source: Adapted from the Yellow Pages & Phone Book 2006/2007, Telkom - National Call Charging Structure, Page 19]

Location	Dialling Code	(032)	(058)	(039)	(036)	(033)	(039)	(035)
Location	Dialling Code	Ballito	Harrismith	Kokstad	Ladysmith	Pietermaritzburg	Port Shepstone	Richards Bay
Bloemfontein	(051)	B	B	B	B	B	B	B
Cape Town	(021)	B	B	B	B	B	B	B
Durban	(031)	A	B	B	B	B	B	B
East London	(043)	B	B	B	B	B	B	B
Johannesburg	(011)	B	B	B	B	B	B	B
Kimberley	(053)	B	B	B	B	B	B	B
Pietermaritzburg	(033)	B	B	B	B	A	B	B
Port Elizabeth	(041)	B	B	B	B	B	B	B
Pretoria	(012)	B	B	B	B	B	B	B
Worcester	(023)	B	B	B	B	B	B	B

- 1.1 What is the dialling code for Kimberley?
- 1.2 According to the cost matrix table, in which town does Sinead live?
- 1.3 How much does it cost per second to make a local call during standard time?
- 1.4 Write down the name of a town that is less than 50 km away from Durban.
- 1.5 Use the tariff structure to calculate how much it would cost Sinead to make a 12 minute 39 seconds call to an (021) number at 19:04.

- 1.6 Sinead made the following call on 9 May:

Number dialled: (033) 203 2480

Length of call: 11 minutes 35 seconds

Time of day: 18:53

Use the data provided to calculate the total cost of this call.



- 2.1 One of the water accounts that Solly received from the Mtutulu Municipality is provided below.

The Mtutulu Municipality Business (Unusual)

44 High Street
 P.O. Box 321781
 Tel: (098) 4328910

Street Address	Client Name	Invoice Number
Flat 3C, Ladswood Heights	Mr. S van Heerden	WEST - WAT231098

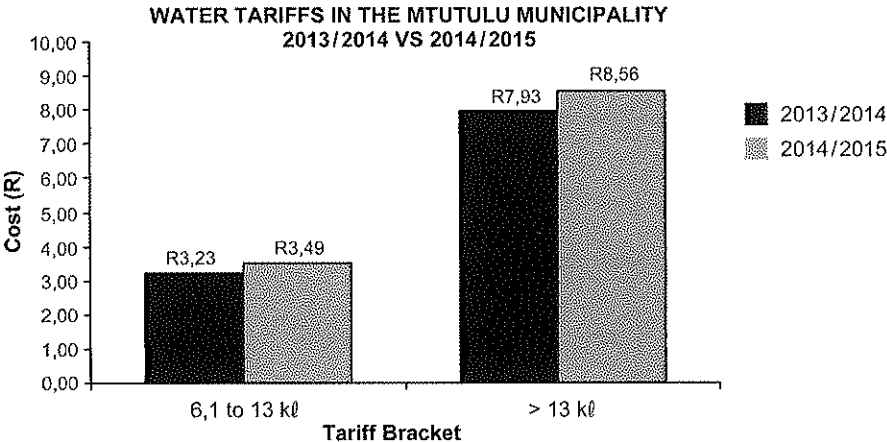
Date	Consumption Details	Charge	VAT (14%)	Grand Total Due
29/06	28 kℓ			

Kilolitre	Tariff (per kℓ)
0 to 6 kℓ	Free
6,1 to 13 kℓ	R3,49
> 13 kℓ	R8,56

Previous consumption statistics:	Jan	Feb	Mar	Apr	May
	35 kℓ	33 kℓ	32 kℓ	29 kℓ	30 kℓ

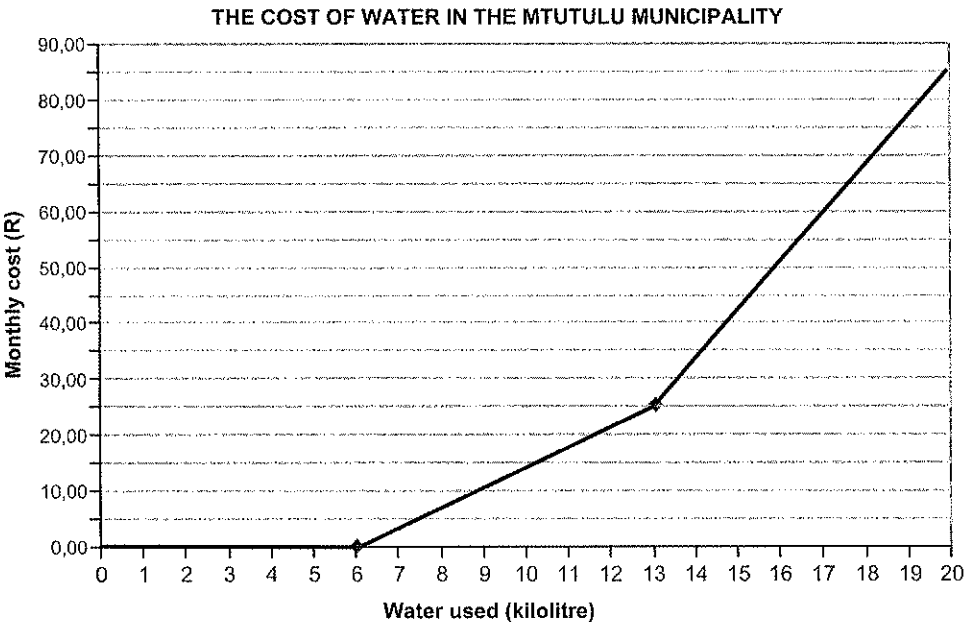
- 2.1.1 How many kℓ of water did Solly use in June?
- 2.1.2 Calculate the 'Charge' for the water Solly used in June.
- 2.1.3 Determine the 'Grand Total Due' for Solly's water bill.

2.2 The graph below shows a comparison of the water tariffs in 2013/2014 and the water tariffs in 2014/2015.



Which tariff bracket increased the most when the Mtutulu Municipality increased the water tariffs from 2013/2014 to 2014/2015? You must show all working.

2.3 The graph below shows the monthly cost of using different quantities of water (i.e. between 0 kℓ and 20 kℓ) in the Mtutulu Municipality.



Explain in detail why the graph is shaped in this way. In your explanation you must discuss why the graph has a horizontal portion and two other portions of differing steepness.

3. Elsie decides to compare costs of travelling by bus, taxi and train. She collects the following information :

- A bus trip from Bellville to Cape Town costs R20,40 one-way.
- A taxi from Bellville to Century City costs R9,50 and then another taxi from Century City to Cape Town costs R5,80.
- A return fare by train between Bellville and Cape Town costs R25,00.

3.1 Which mode of transport is the cheapest for a single trip from Bellville to Cape Town? Show all calculations.

3.2 Draw a graph to illustrate the cost per trip from Bellville to Cape Town, using the 3 different modes of transport. Show the costs of 5 trips.

4. Mr Benny Bunckle, a resident in the Molopi Municipality, has a *Flat Rate* electricity system in his house. He receives the following electricity bill for January.

ELECTRICITY BILL						
Type of System: 3 Part Flat Rate - 60 AMP				Molopi Municipality		
Account No.		Account Date	Enquiries		Fax	
00112371		31/01/2015	427-1914		427-1920	
Name		Street Address	Location		Type of Dwelling	
Mr Benny Bunckle		Hillside Road	Mpipi		Residential	
Date	Details		Charge/ Tariff	Cost	VAT (14%)	Amount Due
31/01	ELECTRICITY CONSUMPTION: Reading in December: 376 912 kWh Current reading: 377 449 kWh					
28/01	Consumption for January: 537 kWh		31,5 c/kWh	R169,16	R23,68	R192,84
28/01	ADDITIONAL CHARGES: Fixed Service Fee		R58,80 per month		R8,23	R67,03
28/01	Fixed Network Charge		R20,20 per month		R2,83	R23,03
Total Amount Due					R282,90	

At the end of February, the reading on Mr Bunckle's electricity meter is 377 957 kWh. Calculate the *Total Amount Due* by Mr Bunckle for his electricity consumption in February.

5. Kim parks her car in a parking garage attached to the office block.

The table alongside shows the costs for parking there for a given time period.

PARKING TARIFFS	
Number of hours	Cost
0 to 1 hours	FREE
1 to 3 hours	R4,00
3 to 5 hours	R6,00
5 to 7 hours	R8,00
7 to 9 hours	R10,00
More than 9 hours	R12,00
Saturdays before 13:00	R5,00
Sundays and public holidays	FREE

- 5.1 On Tuesday, Kim arrives at 08:01 and leaves at 08:45 for a meeting. She returns to work at 12:15 and leaves again at 17:30. How much does she pay in total for parking?
- 5.2 Kim is charged R8,00 for parking there from 08:45 to 13:25. Use the table of parking tariffs to determine if this charge is correct. Show ALL your calculations and give a reason for your answer.
6. Ma Ndlovu has a landline telephone. A service provider has offered her a choice of two different call packages.

CALL PACKAGE 1	CALL PACKAGE 2
• Monthly rental of R150	• Monthly rental of R300
• First 100 minutes are free	• First 500 minutes are free
• Calls cost R0,50 per minute	• Calls cost R0,50 per minute

The total cost for CALL PACKAGE 1 is given by the following formula:

$$\text{Total cost (in rand)} = R150 + R0,50 \times (\text{number of minutes more than 100})$$

- 6.1.1 Write down a formula which can be used to calculate the total cost (in rand) for CALL PACKAGE 2.
- 6.1.2 If CALL PACKAGE 2 is used, determine the total cost, in rand, if Ma Ndlovu and her family have made calls with a total duration of 510 minutes.
- 6.2.1 On the same system of axes, draw a line graph to illustrate the total cost for CALL PACKAGE 1 and 2.
- 6.2.2 At what point will the cost of call package 1 be the same as call package 2.
- 6.2.3 Which call package is cheaper if Ma Ndlovu makes 700 minutes worth of calls per month?
- 6.2.4 Which call package would you recommend Ma Ndlovu purchase if she makes only 300 minutes of calls per month?

7. Sbusiso's brother, Mpho, manages a golf club in Johannesburg which runs just like a business, falling into the <500 kVA bracket. Last month he received an electricity bill for R1 745,50. Although it is 'low season' and he agrees with the fact that they used 2 700 kWh of electricity last month, he is convinced that this total of R1 745,50 for the bill is incorrect. Use the tariff table given to show by means of calculations that the bill is in fact correct.

CURRENT TARIFFS		2013/2014							
SEGMENT	Supply Position	Service Charge	Network Charge	Maximum Demand		Energy Charge			
				Summer	Winter	TOU	c/kWh		
				R/kVA	R/kVA		Non-seasonal	Low Season	High Season
Large Customer - TOU	MV-kVA	5 379,15		57,84		Peak		35,23	81,69
						Standard		25,99	31,72
						Off-Peak		21,54	22,74
Large Customer - TOU	LV-kVA	4 374,00		63,89				39,17	90,83
								28,98	35,29
								23,94	25,29
Large Customer	MV-kVA	1 194,14		78,24	80,67				34,68
Large Customer	LV-kVA	597,08		77,63	81,52				37,17
Large Customer	c/kVAh						6,13		
Business < 50 kVA	400 V	149,92	66,00						
< 100 kVA		167,74	84,50						
< 500 kVA		228,95	109,85						
> 500 kVA		336,07	164,78						
0 to 500 kVA								47,96	74,98
501 to 1000 kVA								50,29	75,74
1001 to 2000 kVA								48,29	72,71
2001 to 3000 kVA								52,10	78,45
Above 3000 kVA								52,46	78,97
Business Prepaid < 50 kVA							63,92		
Business Prepaid > 50 kVA							63,92		

UNIT

3

INCOME, EXPENDITURE, PROFIT/LOSS, INCOME-AND-EXPENDITURE STATEMENTS AND BUDGETS

Income



- Income is **money coming into** a household/business/government.
- There are 3 types of income:

1 FIXED INCOME

constant income
of same rand value
per month

Personal	e.g. salary; wage; pocket money
Business	e.g. fixed term contract sales / services rendered; retainer income, rent
National / Global	e.g. fixed rates and taxes; levies; rent; fixed term contract services rendered; service charges

2 VARIABLE INCOME

a changing income
of a different rand value
per month

Personal	e.g. commission; interest on savings
Business	e.g. sales; services rendered; interest on savings
National / Global	e.g. taxes; levies; interest on savings; licenses; permits; fines

3 OCCASIONAL INCOME

a 'once-off' income;
or an unreliable form of
income per month

Personal	e.g. gift; bursary; inheritance; completing a job/project; bonus
Business	e.g. completing a job/project; bonus; donation; loan; grants
National / Global	e.g. completing a job/project; donation; grant; loan

Expenditure/Expenses



- Expenditure/Expenses is **money being spent** or money going out of the household/business/government.
- There are 3 types of expenditure:

1 FIXED EXPENDITURE

an expense that is the
same every month

Personal	e.g. car repayment; rent; school fees; insurance; cellphone contract; medical aid contribution; gym; savings; membership; magazine subscription etc.
Business	e.g. car repayments; loan repayments; insurance; rent; salaries/wages etc.
National / Global	e.g. loan repayments; insurance; rent; salaries/wages etc.

2 VARIABLE EXPENDITURE

an expense that
changes every month

Personal	e.g. electricity; water; telephone call costs; petrol; food; clothing; entertainment; savings
Business	e.g. electricity; water; telephone; petrol; repairs and maintenance
National / Global	e.g. electricity; water; telephone; transport; repairs and maintenance

3 OCCASIONAL EXPENDITURE

an expense that is
'once-off' or very
infrequent or
unexpected

Personal	e.g. burst geyser; broken car; school outing
Business	e.g. broken car; broken machinery
National / Global	e.g. strikes

Budgets vs Income-and-Expenditure Statements



- Both list the income and expenses for a month
- A **budget** is a list of **expected** income and expenditure
- An **income-and-expenditure** statement is a list of **actual** income and expenditure

e.g. BUDGET	vs	INCOME-AND-EXPENDITURE STATEMENT
Income R26 000 - Basic Salary R16 000 - Commission R10 000 Expenditure R17 670 - Rent R5 500 - Car repayment R4 000 - Insurance R800 - Medical Aid R670 - Food R1 500 - Water and Electricity R700 - Entertainment R1 000 - Petrol R1 500 - Savings R2 000		Income R24 500 - Basic Salary R16 000 - Commission R8 500 Expenditure R24 139 - Rent R5 500 - Car repayment R4 000 - Insurance R800 - Medical Aid R670 - Food R1 620 - Water and Electricity R649 - Entertainment R1 350 - Petrol R1 400 - Savings R2 000 - Geyser replacement R6 150

NOTE!

This is a list of **expected** income and expenditure.



NOTE!

This is a list of **actual** income and expenditure.



NOTE:

- The commission and $\therefore$ income is *less* than the budgeted amount
- The food and entertainment expenses are *more* than the budgeted amount
- The water and electricity and petrol expenses are *less* than the budgeted amount
- The geyser replacement expense was not budgeted for $\therefore$ it is called an unforeseen expense

Profit and Loss



- Profit** is when your income is bigger than your expenditure
i.e. **income > expenses**
 $\therefore$ you will have money left over at the end of the month / a surplus.
- Loss** is when your expenses are bigger than your income
i.e. **expenses > income**
 $\therefore$ you will not have enough money to cover your bills / you will be in debt / have a deficit of money.



NOTE! When your **income = expenses**, we call it **break-even**.
More detailed notes on break-even in Unit 5 (see p. 75).

- The general formula used to determine if a profit or loss was made is:

$$\text{Profit or Loss} = \text{Income} - \text{expenditure}$$



- if the answer is **positive**, a **profit** was made.
- if the answer is **negative**, a **loss** was made.

- When we want to compare the profit and income, we calculate the profit margin; as this provides a means for comparing business performance over different time intervals:

$$\text{Profit Margin} = \frac{\text{profit}}{\text{income}} \times 100$$



- e.g. In March, a business generated an income of R750,00 with a total profit of R135,00. In April, the business generated an income of R1 025,00 with a profit of R175,00.

Based on appearances it would seem that the business performed better in April because more profit was earned. However, if we compare, using percentage as the common unit, we will see that the business actually performed better in March.

$$\begin{aligned} \text{March: Profit Margin} &= \frac{\text{profit}}{\text{income}} \times 100\% \\ &= \frac{R135,00}{R750,00} \times 100\% \\ &= 18\% \end{aligned}$$

$$\begin{aligned} \text{April: Profit Margin} &= \frac{R175,00}{R1\,025,00} \times 100\% \\ &= 17,1\% \end{aligned}$$

As you can see, this business actually did better in March because more of their income was actual profit.

Worked Examples



1. Peter is a car salesman. He earns a basic salary of R8 500 per month, plus commission. In the month of February, he expects to make R26 000 in commission, plus a bonus of R7 000.

His estimated monthly expenses are rent (R8 200); petrol (R950); electricity (R2 000); gym membership (R350); car repayment (R4 200); insurance (R1 330); cellphone (R770); groceries (R1 800); entertainment (R2 300) and water (R250).

- 1.1 Draw up Peter's budget for the month of February.

Peter's budget for February

Income		Expenses	
Salary	R8 500	Rent	R8 200
Commission	R26 000	Petrol	R950
Bonus	R7 000	Electricity	R2 000
		Gym membership	R350
		Car repayment	R4 200
		Insurance	R1 330
		Cellphone	R770
		Groceries	R1 800
		Entertainment	R2 300
		Water	R250
TOTAL	R41 500	TOTAL	R22 150

- 1.2 Will Peter have enough money to cover an unforeseen expense such as a motor car accident to the value of R25 000? Give a reason for your answer by means of calculations.

$$\begin{aligned}
 \text{Peter's savings} &= \text{income} - \text{expenses} \\
 &= R41\,500 - R22\,150 \\
 &= R19\,350
 \end{aligned}$$

$\therefore$ No, Peter will not be able to cover the unforeseen expense of a car accident to the value of R25 000, as he only has R19 350 saved for the month of February.

2. Mrs Phumzile is starting a transport business. She owns one taxi and she employs Pieter as a taxi driver.

The table below shows a list of the income and expenses of Mrs Phumzile's business for the month of February 2014:

	INCOME	EXPENSES
Maintenance costs:		
(a) Fuel		R1 065,40
(b) Servicing and Repairs		R546,09
(c) Cleaning		R60,00
Insurance for the taxi		R305,45
Taxi license fee		R400,00
Taxi driver's salary		R3 500,00
Taxi association fee		R200,00
Fares collected	R7 842,00	
TOTAL	R7 842,00	R6 076,94

- 2.1 Is this a budget or an income-and-expenditure statement?

Income-and-expenditure statement

- 2.2 List the fixed and variable costs.

Fixed costs:

- Insurance for the taxi
- Taxi licence fee
- Taxi driver's salary
- Taxi association fee

NOTE: There are actual rand and cent values, e.g. R546,09 and R305,45. Budgets have rounded off values.



Variable costs:

- Maintenance costs (incl. fuel, servicing, repairs and cleaning)

- 2.3 Did Mrs Phumzile make a profit? Show all calculations.

$$\begin{aligned}
 \text{Profit/Loss} &= \text{income} - \text{expenses} \\
 &= R7\,842,00 - R6\,076,94 \\
 &= R1\,765,06
 \end{aligned}$$

$\therefore$ Mrs Phumzile made a profit.

- 2.4 Determine Mrs Phumzile's profit margin for the month of February.

$$\begin{aligned}
 \text{Profit margin} &= \frac{\text{profit}}{\text{income}} \times 100 \\
 &= \frac{R1\,765,06}{R7\,842,00} \times 100 \\
 &= 22,51\%
 \end{aligned}$$



- 2.5 Suggest a way in which Mrs Phumzile could increase her profits.
She could increase her taxi fares.

Test Your Understanding



1. Below is a copy of the shop's budget for the month of April:



THE BAGGY PANTS GOLF SHOP Budget for April 2014			
Income		Expenses	
Sales	R855 200,00	<u> A </u>	Expenses
		Rent	R20 200,00
		Advertising	<u> C </u>
		Salaries	R45 000,00
		Insurance	R2 800,00
		SUBTOTAL	
		<u> A </u>	Expenses
		Stock purchases	<u> D </u>
		Telephones	R4 432,00
		Electricity	R2 400,55
		Water	R1 800,45
		Incidentals	R11 867,00
		SUBTOTAL	592 700,00
TOTAL INCOME	R855 200,00	TOTAL EXPENSES	

- 1.1 Name the type of expenses, labelled **A** and **B**.
- 1.2 Calculate the advertising expense (**C**), if it is 20% less than the salary expense.
- 1.3 Determine the value of stock purchases (**D**).
- 1.4 Is The Baggy Pants Golf Shop expected to make a profit in April?
- 1.5 What is the expected profit margin for April?

2. The following table lists the income and expenses for Fatima's Transport Company for January 2014. Running costs include rental for the business premises, telephone, electricity and water.

INCOME		EXPENSES	
Payment received from customers	R682 500,00	Running costs	R325 400,00
		Salaries	R100 000,00
		Petrol	R50 000,00

- 2.1 Name different types of costs that could be classified as 'Running costs' in Fatima's Transport Company.
 - 2.2 What percentage of her total expenses is allocated to salaries?
 - 2.3 Did Fatima make a profit in January?
 - 2.4 If Fatima's income for February was 13,5% more than the income for January; determine Fatima's income for February.
 - 2.5 If Fatima's expenses for February total R490 200; determine which month had the higher profit margin.
3. The table on p. 72 gives a breakdown of the income of the South African Government for the 2005/2006 and 2006/2007 financial periods.

[Source: Clive Sharwood (ed), *Deloitte, Tax News - Budget 2006*, (page 5 - 6)]

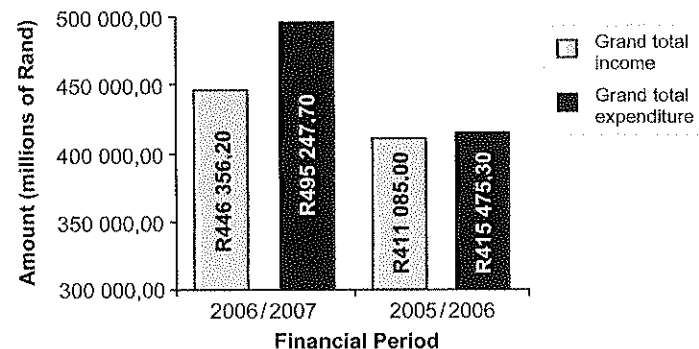
- 3.1.1 How much income did the South African Government receive from the *Skills Development Levy* during the 2005/2006 financial period?
- 3.1.2 By what percentage did income from *Fuel Levy* increase from 2005/2006 to 2006/2007?
- 3.1.3 Did the South African Government generate more income from *Transfer Duties* in 2005/2006 or in 2006/2007?
- 3.1.4 Calculate the *Total Income Tax* for the 2006/2007 financial period.
- 3.1.5 In the 2006/2007 financial year, the South African Government collected R23 194,2 million in *Customs Duties*. Write this amount as a full number.
- 3.1.6 Explain why the percentage change for *Tax on Retirement Funds* from 2005/2006 to 2006/2007 is a negative value.
- 3.1.7 Calculate the percentage increase/decrease in income from *Transfer Duties* from 2005/2006 to 2006/2007.
- 3.1.8 Calculate the *Total Customs and Excise* value for the 2005/2006 financial period.

South African Government Income

	2006/2007 Budget (Rand in millions)	2005/2006 Budget (Rand in millions)	% Increase/ Decrease
Income Tax			
• Individuals and Trusts	132 475	125 760	5,3%
• Companies	95 201	84 900	12,1%
• Other	1 890	1 720	9,9%
Total Income Tax		212 380	8,1%
Inland Revenue			
• Tax on Retirement Funds	2 400	4 500	-46,7%
• Secondary Tax on Companies	13 850	11 850	16,9%
• Value-added Tax (VAT)	131 200	115 000	14,1%
• Transfer Duty	6 210	8 700	
• Stamp Duty and Uncertified Securities Tax	3 034	2 670	13,6%
• Donations Tax and Estate Duties	642	620	3,5%
• Other Income	9 320	8 180	13,9%
• Skills Development Levy	5 600	5 000	12,0%
Total Inland Revenue	172 256	156 520	10,1%
Customs and Excise			
• Customs Duty	23 194,2	18 600	24,7%
• Excise Duty	17 955	15 799	13,6%
• Fuel Levy	21 800	20 700	5,3%
• Other	1 329	1 231	8,0%
• Less: Customs Union Agreement	19 744	14 145	39,6%
Total Customs and Excise	44 534,2		5,6%
Grand Total Income	446 356,2	411 085	8,6%

3.2 The graph below provides a summary of the total income and expenditure of the South African Government for the 2005/2006 and 2006/2007 financial periods.

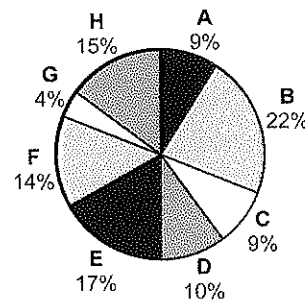
GRAND TOTAL INCOME AND GRAND TOTAL EXPENDITURE FOR THE 2005/2006 AND 2006/2007 FINANCIAL PERIODS



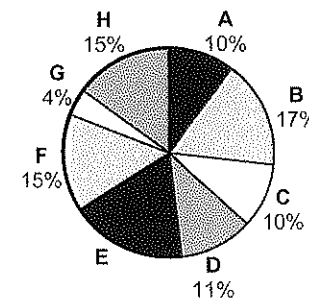
Calculate the profit or loss that the South African National Government generated during the 2006/2007 financial period.

4. Mr Riet wanted to show his colleagues that the government was spending more on education than on most other departments. The two graphs below show the budgeted government expenditure for the financial years 2012/2013 and 2013/2014.

BUDGETED GOVERNMENT EXPENDITURE 2012/2013



BUDGETED GOVERNMENT EXPENDITURE 2013/2014



1 billion = 1 000 million

KEY

- | | |
|------------------------------------|----------------------|
| A: Public order and safety | E: Education |
| B: Economic affairs | F: Social protection |
| C: Housing and community amenities | G: Defence |
| D: Health | H: Other |



NOTE: The names of the national government departments have since changed.

[Source: www.sats.gov.za]

- 4.1 Show that the difference between the amounts budgeted for education for the financial years 2012/2013 and 2013/2014 is more than R20 000 000 000.
- 4.2 Give TWO possible reasons why you think the South African government should increase its budgeted expenditure for education.
5. MT Trading is having a marketing day to generate sales and to advertise. They hire the hall from a local school for R500,00 and offer to rent space to other patrons for R80,00 per 1 m². They have 50 m² to rent out. In addition to their rental income, they have secured a sponsor who will donate money to the value of 15% of their total rental income. They also hire two Jumping Castles at R275,00 each and security guards which will cost them R1 100,00 in total. One of the big draw cards for the day is a local band that is coming to play for 1,5 hours. The band is charging MT Trading R280 per 30 minutes.

Draw up a budget for MT Trading and calculate whether their budget predicts a profit or loss for the marketing day.

UNIT

4

COST PRICE AND SELLING PRICE

Cost Price

- The cost price of a product is the cost of manufacturing or buying a product.
- The cost price is determined by the:
 - Production costs - the costs directly related to producing/manufacturing the product, e.g. raw materials, packaging, labour, equipment, etc.
 - Operating costs - the costs involved in operating the business, e.g. water, electricity, rent, telephone, etc.

- General formula:

$$\text{cost price} = \text{selling price} - \text{profit}$$

Selling Price

- The selling price is the price at which the product is sold.
- The selling price is determined by a number of factors, such as:
 - the cost price of the product
 - the competitor's prices
 - the profit required
 - the price the customer is prepared to pay

- General formula:

$$\text{Selling Price} = \text{cost price} + \text{profit}$$

Profit

- The profit is the difference between the income and expenses; or the difference between the selling price and cost price.
- General formula used to calculate profit are:

$$\text{Profit} = \text{selling price} - \text{cost price}$$

$$\text{Profit} = \text{income} - \text{expenses}$$

- To compare the relationship between the profit and cost price, we calculate the percentage profit:
- To compare the relationship between the profit and income, we calculate the profit margin:

$$\% \text{ Profit} = \frac{\text{profit}}{\text{cost price}} \times 100$$

$$\text{Profit margin} = \frac{\text{profit}}{\text{income}} \times 100$$

More examples and exercises on profit margin in Unit 3 (see p. 71).

Worked Examples

1. Adam grows cacti plants and sells them to the local nursery. He generally sells 20 cacti in a batch for R690 excluding VAT.
- His costs (with VAT) per batch include:
potting soil (R85); plant pots (R120); water (R40); labels (R62) and transport (R58).

- 1.1 Determine the selling price per cactus (including VAT).

$$\text{Selling price per cactus excl. VAT} = R690 \div 20 = R34,50$$

$$\text{Selling price per cactus incl. VAT} = R34,50 \times \frac{114}{100} = R39,33$$

REMEMBER! There are 2 ways of calculating VAT:

1. Total % of VAT-excl. price

$$\begin{aligned} \text{i.e. VAT-incl. price} &= \text{VAT-excl. price} \times (100\% \text{ product} + 14\% \text{ VAT}) \\ &= \text{VAT-excl. price} \times \frac{114}{100} \end{aligned}$$

2. Determine the % increase and add it to the total

$$\begin{aligned} \text{i.e. Increase} &= \text{VAT-excl. price} \times \% \text{ increase} \\ \therefore \text{VAT-incl. price} &= \text{VAT-excl. price} + \text{increase} \end{aligned}$$

More examples and exercises on VAT in Unit 9 (see p. 98).

- 1.2 Determine the cost price per cactus (incl. VAT).

$$\begin{aligned}\text{Total costs (incl. VAT) per batch} \\ &= R85 + R120 + R40 + R62 + R58 \\ &= R365\end{aligned}$$

$$\begin{aligned}\therefore \text{Total cost (incl. VAT) per cactus} \\ &= R365 \div 20 \\ &= R18,25\end{aligned}$$

- 1.3 Calculate Adam's profit per cactus.

$$\begin{aligned}\text{Profit} &= \text{selling price} - \text{cost price} \\ &= R39,33 - R18,25 \\ &= R21,08\end{aligned}$$

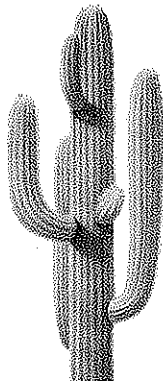
- 1.4 Determine the percentage profit Adam makes per cactus.

$$\begin{aligned}\% \text{ Profit} &= \frac{\text{profit}}{\text{cost price}} \times 100 \\ &= \frac{R21,08}{R18,25} \times 100 \\ &= 115,51\%\end{aligned}$$

2. If Adam generates an income of R3 600 per month; with expenses totalling R1 400; calculate his profit margin.

$$\begin{aligned}\text{Profit} &= \text{income} - \text{expenses} \\ &= R3\,600 - R1\,400 \\ &= R2\,200\end{aligned}$$

$$\begin{aligned}\therefore \text{Profit Margin} &= \frac{\text{profit}}{\text{income}} \times 100 \\ &= \frac{R2\,200}{R3\,600} \times 100 \\ &= 61,11\%\end{aligned}$$

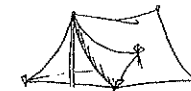


Test Your Understanding



- 1.1 The profit on a backpack is 30%. If the cost price is R1 200, calculate the selling price.

- 1.2 The selling price of a tent is R2 400 and the profit is R890. Calculate the cost price.



2. Sbusiso buys 14 sets golf clubs for R999,00 each. He plans to sell them for R2 999,00 per set.

- 2.1 How much profit will he make if he sells all 14 sets?

- 2.2 How many sets must be sold in order for the shop to break even with the sets?

- 2.3 Sbusiso wants a profit margin of 60% on the sale of the golf sets. If he offers regular customers a discount of 15%, will he still maintain a 60% profit margin? You must show all working.



3. Mr Francis buys key rings for R4,80 each and coffee mugs for R7,00 each.

He sells each key ring for R7,00 and each coffee mug for R10,00.

- 3.1 Which item has the highest percentage profit?

- 3.2 Mr Francis' income for the items sold is given in the table below:



ITEM SOLD	DAY OF WEEK						
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Key ring	R175	R385	R259	R231	R259	R259	R322
Coffee mug	R250	R350	R370	R380	R270	R350	R370

- 3.2.1 Which item was sold the most and by how much?

- 3.2.2 Which item had the biggest profit margin over the week?

BREAK-EVEN ANALYSIS

Break-even analysis is commonly used in business; in order to determine the exact point at which the business' **income equals** the business' **expense**. In this way, businesses are able to optimize their profits.

Terminology used in Break-even Analysis

- **Income** - money that comes into the business, usually from sales or services rendered.
- **Expenses** - money that is spent on producing the product or rendering the service, as well as operational costs for the business.

Expenses can be classified as:

① Fixed Expenses

expenses that stay the same every month;
e.g. rent, salaries/wages, loan repayments, etc.



More on production and operating costs in Unit 4 (see p. 73).

② Variable Expenses

expenses that change every month;
e.g. electricity, water, telephone, petrol etc.

More on income and expenses in Unit 3 (see p. 68).



- **Profit** - the surplus/extra money that is left over after all expenses have been covered, i.e. **Profit: Income > Expenses**.
- **Loss** - when there is not enough money to cover all the expenses, i.e. **Loss: Expenses > Income**.
- **Break-even** - the point at which the income equals the expenses; therefore the business is neither making a profit nor a loss, i.e. **Break-even: Income = Expenses**.



A typical **BREAK-EVEN GRAPH** includes:

① Total Income

- ▶ this straight line graph always **starts at the origin** (0; 0), because if no product is sold; then no income is made.

② Total Expenses

- ▶ this straight line graph **start at a point somewhere on the y-axis** - which represents the **fixed costs** of the business.
- ▶ the **constant increase** in the straight line graph represents the **variable costs**.

More on profit in Unit 3 (see p. 69) and Unit 4 (see p. 73).

③ Break-even Point

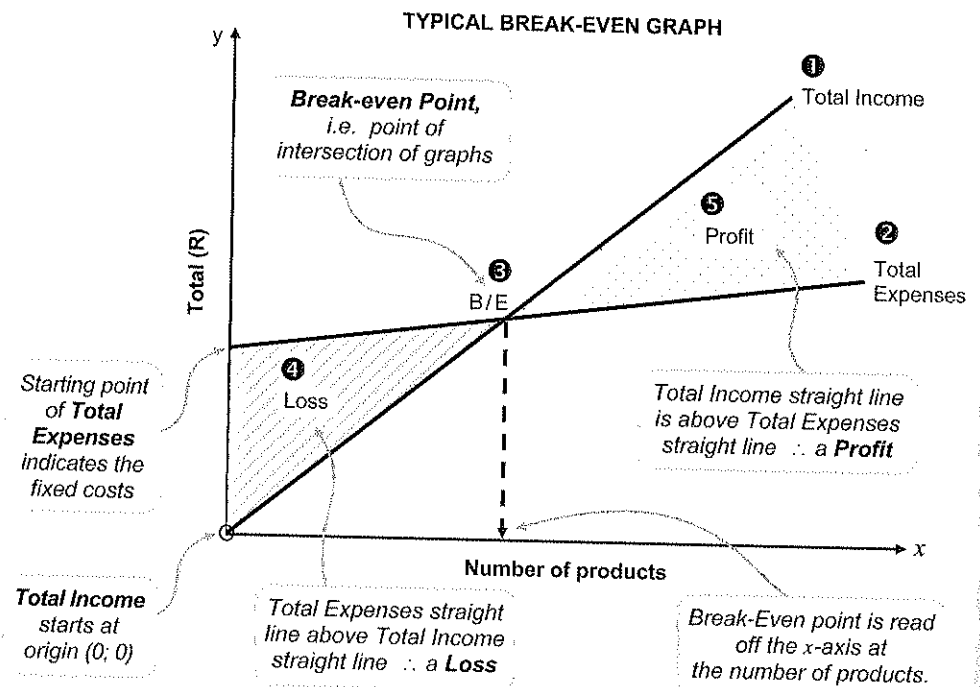
- ▶ this is the **point of intersection** of the Total Income and Total Expenses straight line graphs.
- ▶ the **break-even point** is read at the number of products needed, (i.e. the value on the x-axis).

④ Loss

- ▶ this is the **area between the Total Expenses and Total Income straight lines; before the break-even point**.
- ▶ before the break-even point, the Total Expenses straight line is above the Total Income straight line - showing a **loss**, i.e. **Expenses > Income**.

⑤ Profit

- ▶ this is the **area between the Total Income and Total Expenses straight lines; after the break-even point**.
- ▶ after the break-even point, the Total Income straight line is above the Total Expenses straight line - showing a **profit**, i.e. **Income > Expenses**.



Break-even analysis can also be applied to other similar situations to determine the exact point at which the two situations are equal in value, e.g. costs from two rental car companies; costs of two different cellphone packages; costs of prepaid versus flat-rate electricity etc.

In these scenarios, **break-even** is the point at which **two competing values are equal**.

Worked Examples

1. Xanthi bakes chocolate brownies for market day. The brownies cost R3,50 each to bake and she sells them for R5. Her mom charges her R50 for electricity and it costs her R100 to hire a stall at the market:

See Module 2 for more on equations, tables and straight line graphs.

- 1.1 Write down a formula to represent Xanthi's Total Income.

$$\text{Total Income} = R5 \times \text{number of brownies}$$

- 1.2 Draw up a table to represent Xanthi's Total Income, if she sold 180 brownies. [Go up in increments of 20]

Number of brownies	0	20	40	60	80	100	120	140	160	180
Total Income (R)	0	100	200	300	400	500	600	700	800	900

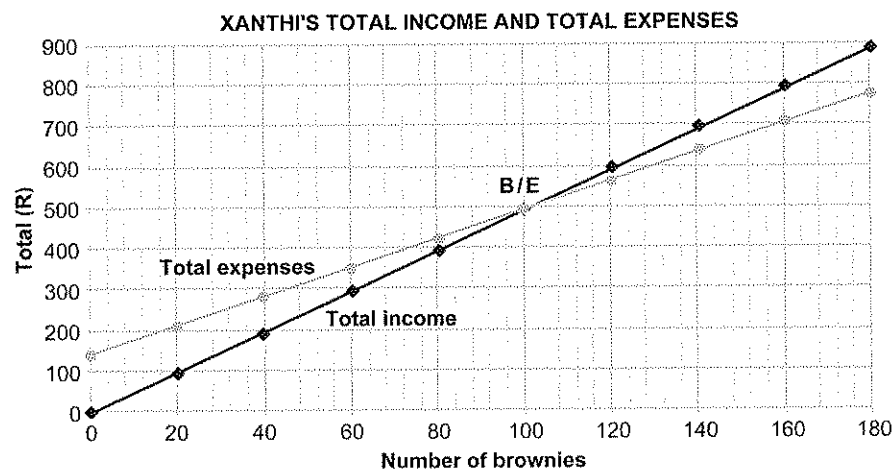
- 1.3 Write down a formula to represent Xanthi's total expenses.

$$\begin{aligned} \text{Total Expenses} &= \text{variable costs} + \text{fixed costs} \\ &= (R3,50 \times \text{number of brownies}) + (R50 + R100) \\ &= R3,50 \times \text{number of brownies} + R150 \end{aligned}$$

- 1.4 Draw up a table to represent Xanthi's Total Expenses, if she bakes 180 brownies. [Go up in increments of 20]

Number of brownies	0	20	40	60	80	100	120	140	160	180
Total Expenses (R)	150	220	290	360	430	500	570	640	710	780

- 1.5 Draw the graph of Xanthi's Total Income and Total Expenses for her Brownie Market Day and label the break-even point (B/E).



- 1.6 How many brownies must Xanthi make in order to break-even?

100

- 1.7 How many brownies must Xanthi sell in order to make a profit?

101

- 1.8 Determine the profit or loss Xanthi will make when she makes 40 brownies.

$$\begin{aligned} \text{income} - \text{expenses} \\ &= R200 - R290 \\ &= -R90 \end{aligned}$$

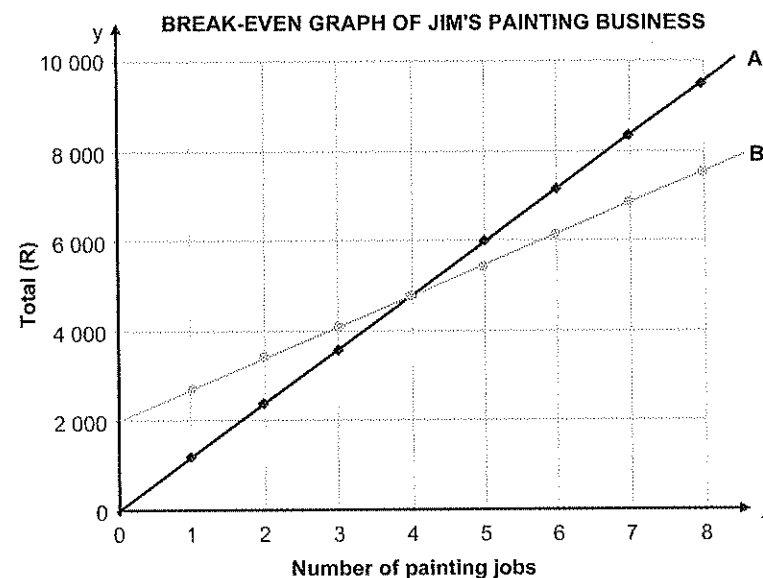
$\therefore$ Xanthi will make a loss of R90.

REMEMBER! At the break-even point of 100 brownies, the income = expenses.

$\therefore$ to make a profit, it is the first point after the B/E point.

$\therefore 100 + 1 = 101$ brownies

2. Jim has a painting business. He draws the break-even graph below:



- 2.1 Label the straight line graphs A and B.

A: Total Income

B: Total Expenses

- 2.2 How much are the fixed expenses for the painting business?

R2 000

The starting value of the Total Expenses graph represents the fixed expenses.

- 2.3 If the variable expenses of Jim's business amount to R700 per painting job, determine a formula for his Total Expenses.

$$\text{Total Expenses} = R700 \times \text{number of painting jobs} + R2\,000$$

- 2.4 How much income does Jim receive per painting job?

$$\text{Income} = R6\,000 \div 5 \text{ jobs} = R1\,200$$

- 2.5 How many painting jobs must Jim complete in order to:

- 2.5.1 break-even?

4 jobs

- 2.5.2 make a profit?

5 jobs



Break-even is the point of intersection of the 2 graphs.



- 2.6 Estimate the profit that Jim will make on 8 painting jobs.

$$\begin{aligned}\text{Profit} &= \text{income} - \text{expenses} \\ &= R9\,600 - R7\,600 \\ &= R2\,000\end{aligned}$$

Test Your Understanding



1. Amir wants to start a small business offering tour packages for the Rugby World Cup.

- 1.1 Amir lists the following monthly expenses:

Telephone and fax:	R20 (per tour package)
Equipment:	R3 450
Stationery:	R50 (per tour package)
Staff:	R1 000 (per tour package sold)
Rent:	R4 000
Travel agency fees:	R780 (per tour package sold)



- 1.1.1 Calculate Amir's total fixed costs.
- 1.1.2 Calculate Amir's total variable costs per tour package.
- 1.1.3 Write an equation for Amir's total costs.
- 1.1.4 Study the table given below. Write down the question number and missing values.

Number of tour packages	0	1	5	20
Total costs	a	b	c	d

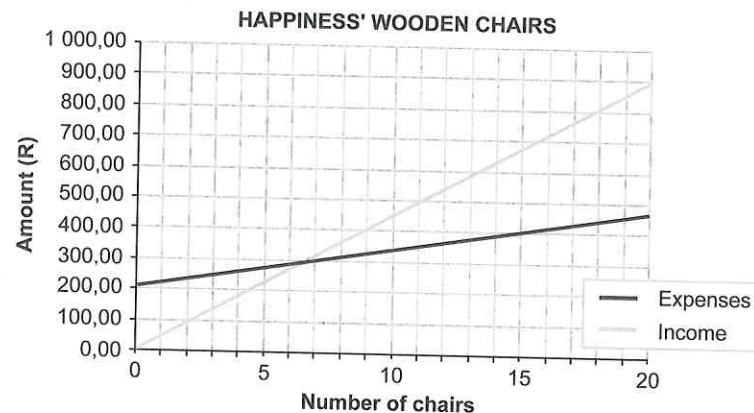
- 1.2 Amir plans to sell the tour packages for R6 500 each.

- 1.2.1 Write an equation for Amir's total income.

- 1.2.2 Study the table given below and write down the question number and missing values.

Number of tour packages	0	1	5	20
Total income	a	b	c	d

- 1.3.1 Use the information from your tables in Questions 1.1.4 and 1.2.2 to draw graphs of Amir's income and costs on the same set of axes. Label the break-even point.
- 1.3.2 Use your graph from Question 1.3.1 to determine how many tour packages Amir must sell a month in order to break even.
- 1.3.3 Use your graph from Question 1.3.1 to determine the approximate profit Amir will make if he sells 6 tour packages.
2. Happiness makes wooden chairs for a living. The graph below shows his income and expenses for a month.



- 2.1 Use the graphs to answer the following questions.
- 2.1.1 Estimate how much Happiness pays in fixed expenses.
- 2.1.2 Give TWO examples of fixed expenses that Happiness might have.
- 2.2 How much does Happiness sell his chairs for?
- 2.3 How many chairs does Happiness need to sell in order to break even?
- 2.4 How much income will Happiness receive if he sells 12 chairs?
- 2.5 How much profit will Happiness make if he sells 15 chairs?
- 2.6 Which graph(s) would be affected if Happiness increased the selling price of his chair? Explain your choice.

3. Joyce sells vegetables on the side of the road every day. She spends R70,00 buying her fresh vegetables every day, which she then repacks into smaller plastic packets. 280 packets cost R12,00 and last her 2 weeks (14 days). She pays R22,00 in transport fares every day. A permit to sell costs her R280,00 a year.

- 3.1 Calculate the total for Joyce's daily expenses.
- 3.2 If Joyce sells each packet for R15,00, what would her weekly income be if she sold 12 packets of vegetables per day?
- 3.3 If Joyce sells each packet for R15,00, how many packets must she sell each day to break even?
- 3.4 If Joyce sells 15 packets a day, for how much should she sell them if she wants to make R200,00 profit a day? [Round off your answer to the nearest whole number]

4. BDB Transports is a company that transports goods between Bloemfontein and Durban. They regularly dispatch trucks to cover the 600 km to Durban harbour when goods need to be shipped to many different destinations.

4.1 Benny leaves Bloemfontein at 06:00 in the morning and drives a full truck of goods to Durban at 80 km/h. Copy and complete the following table:

Number of hours	0	1	2	3	4	
Distance from Bloemfontein (km)	0					600

4.2 Dumi leaves Durban at 06:00 the same morning and drives an empty truck back to Bloemfontein at 100 km/h. Copy and complete the following table:

Number of hours	0	1	2	3	4	
Distance to Bloemfontein (km)	600					0

- 4.3 Draw the graph of Benny and Dumi's trip on the same set of axes, using the information from the tables in Questions 4.1 and 4.2.
- 4.4 After how many hours and minutes on the road did Benny pass Dumi?
- 4.5 What is the approximate time at which Benny passes Dumi on the road?



UNIT

6

INTEREST

Terminology used in Finance



- Interest is the **amount of money** that is either:
 - earned - this is the **income** that a person receives from a bank/ company, for depositing/investing money with them
 - charged - this is the **fee** that a bank/company charges a person for borrowing money from them
- Interest rate is the **percentage** used to calculate the amount of interest that is either earned or charged.
- Principal amount is the original amount of **money initially invested or borrowed**, i.e. the starting amount.
- Accumulated/total amount of the investment or loan is the final amount which is made up of the **principal amount plus interest**.
- Time period of the investment or loan is the length of time that the money is invested or borrowed for.

Simple vs Compound Interest



SIMPLE INTEREST

COMPOUND INTEREST

DEFINITION

Interest is **calculated only** on the **principal amount** of money that is invested/borrowed.

∴ Interest remains the same.

Interest is **calculated on** the **accumulated/total amount** of money, i.e. on the **principal amount plus** on any **interest** accrued (i.e. interest on interest)

∴ Interest constantly increases.

SIMPLE INTEREST

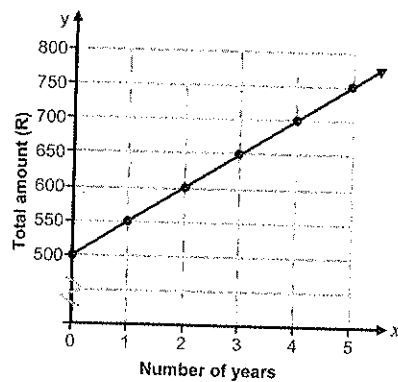
COMPOUND INTEREST

TABLE ILLUSTRATION

e.g. 10% p.a. simple interest on R500

Year	Starting Amount	Interest	Total Amount
1	R500	$\frac{10}{100} \times \text{R}500$ = R50	R500 + R50 = R550
2	R550	$\frac{10}{100} \times \text{R}500$ = R50	R550 + R50 = R600
3	R600	$\frac{10}{100} \times \text{R}500$ = R50	R600 + R50 = R650
4	R650	$\frac{10}{100} \times \text{R}500$ = R50	R650 + R50 = R700
5	R700	$\frac{10}{100} \times \text{R}500$ = R50	R700 + R50 = R750

Note how the interest is calculated at 10% of R500 every year. Therefore the **interest remains the same** at a R50 increase per year.

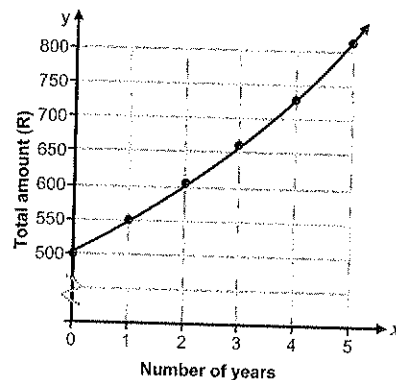


NOTE that a simple interest graph will always be a **straight line graph**, where the **constant increase** represents the **amount of interest** i.e. R50 interest added every year.

e.g. 10% p.a. compound interest on R500

Year	Starting Amount	Interest	Total Amount
1	R500	$\frac{10}{100} \times \text{R}500$ = R50	R500 + R50 = R550
2	R550	$\frac{10}{100} \times \text{R}550$ = R55	R550 + R55 = R605
3	R605	$\frac{10}{100} \times \text{R}605$ = R60,50	R605 + R60,50 = R665,50
4	R665,50	$\frac{10}{100} \times \text{R}665,50$ = R66,55	R665,50 + R66,55 = R732,05
5	R732,05	$\frac{10}{100} \times \text{R}732,05$ = R73,21	R732,05 + R73,21 = R805,26

Note how the interest is calculated at 10% of the **Total Amount** from the previous year. Therefore the **interest constantly increases** per year.



NOTE how a compound interest graph is **NOT** a straight line graph, but an **exponentially curved graph** as the **amount of interest** added every year **constantly increases**.

Worked Examples

SIMPLE INTEREST

1. Jan wants to buy himself a bicycle.

He approaches his uncle for a loan of R800 and promises to pay his uncle interest of 5% **simple interest** per month. His uncle agrees, but says his loan must be paid back within 3 months.

- 1.1 Determine Jan's total monthly interest repayments for the 3 months.

$$\text{Interest} = \frac{5}{100} \times \text{R}800$$

$$= \text{R}40$$

$$\therefore \text{Total interest} = \text{R}40 \times 3 \text{ months}$$

$$= \text{R}120$$



Simple interest remains the same, therefore multiply the interest by the number of months.



COMPOUND INTEREST

1. Jan wants to buy himself a bicycle.

He approaches his uncle for a loan of R800 and promises to pay his uncle interest of 5% **compound interest** per month. His uncle agrees, but says his loan must be paid back within 3 months.

- 1.1 Determine Jan's total monthly interest repayments for the 3 months.

Month 1 interest

$$= \frac{5}{100} \times \text{R}800$$

$$= \text{R}40$$

Month 2 interest

$$= \frac{5}{100} \times (\text{R}800 + \text{R}40)$$

$$= \frac{5}{100} \times \text{R}840$$

$$= \text{R}42$$

Month 3 interest

$$= \frac{5}{100} \times (\text{R}840 + \text{R}42)$$

$$= \frac{5}{100} \times \text{R}882$$

$$= \text{R}44,10$$

$$\therefore \text{Total interest} = \text{R}40 + \text{R}42 + \text{R}44,10$$

$$= \text{R}126,10$$



Interest is calculated on total amount of the previous month.



Total Value of Month 2 is used to calculate interest for Month 3.

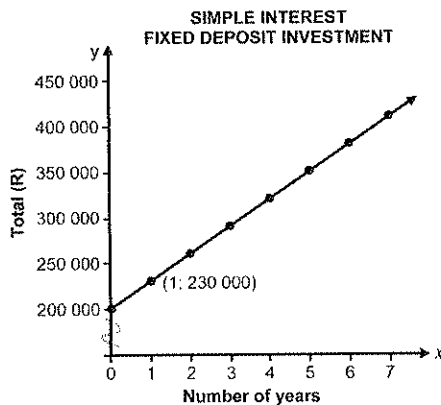
SIMPLE INTEREST

- 1.2 Calculate the total value that Jan needs to repay at the end of the 3 months.

$$\begin{aligned}\text{Total amount} &= \text{principal} + \text{interest} \\ &= R800 + R120 = R920\end{aligned}$$

2. Penny invests R20 000 into a fixed deposit account for 7 years.

The graph below shows the value of her investment, based on **simple interest**:



- 2.1 What is the total value of Penny's investment after 7 years?

$R410\ 000$

- 2.2 After how many years is Penny's investment worth more than R300 000?

$\approx 3,5\ \text{years}$

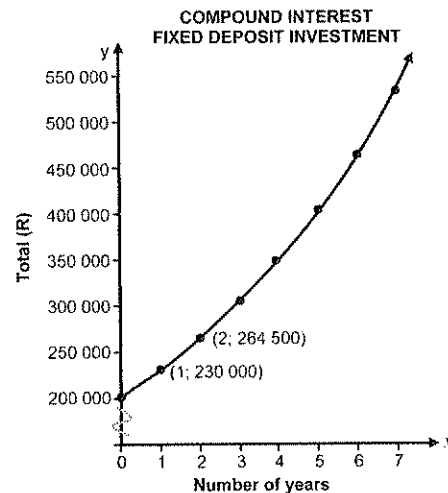
COMPOUND INTEREST

- 1.2 Calculate the total value that Jan needs to repay at the end of the 3 months.

$$\begin{aligned}\text{Total amount} &= \text{principal} + \text{interest} \\ &= R800 + R126,10 = R926,10\end{aligned}$$

2. Penny invests R20 000 into a fixed deposit account for 7 years.

The graph below shows the value of her investment, based on **compound interest**:



- 2.1 What is the total value of Penny's investment after 7 years?

$\approx R530\ 000$

- 2.2 After how many years is Penny's investment worth more than R300 000?

$\approx 3\ \text{years}$

- 2.3 What type of graph is illustrated here?

straight line graph

- 2.4 How much interest has Penny earned after 5 years?

$$\begin{aligned}\text{Interest} &= R350\ 000 - R200\ 000 \\ &= R150\ 000\end{aligned}$$

- 2.5 Calculate the interest rate that Penny received on her investment.

$$\text{Interest rate} = \% \text{ Increase}$$

$$\begin{aligned}\therefore \text{Increase from Yr 0 to Yr 1} &= R230\ 000 - R200\ 000 \\ &= R30\ 000\end{aligned}$$

$$\begin{aligned}\therefore \% \text{ Increase} &= \frac{R30\ 000}{R200\ 000} \times 100 \\ &= 15\%\end{aligned}$$

$$\begin{aligned}\therefore \text{Interest rate} &= 15\% \text{ p.a. simple interest}\end{aligned}$$

- 2.3 What type of graph is illustrated here?

exponential graph

- 2.4 How much interest has Penny earned after 5 years?

$$\begin{aligned}\text{Interest} &\approx R405\ 000 - R200\ 000 \\ &\approx R205\ 000\end{aligned}$$

- 2.5 Calculate the interest rate that Penny received on her investment.



NOTE! Same calculation could be done as for simple interest.

If values were unclear; always calculate the interest between 2 consecutive years' given values.

$$\begin{aligned}\text{Increase from Yr 1 to Yr 2} &= R264\ 500 - R230\ 000 \\ &= R34\ 500\end{aligned}$$

$$\begin{aligned}\therefore \% \text{ Increase} &= \frac{R34\ 500}{R230\ 000} \times 100 \\ &= 15\%\end{aligned}$$

$$\begin{aligned}\therefore \text{Interest rate} &= 15\% \text{ p.a. compounded interest}\end{aligned}$$

Hire Purchase Agreements



- Hire Purchase (HP) agreements are used when customers don't have the full cash amount to purchase a product, usually appliances (e.g. TV's, fridges, washing machines etc.) and cars.
- Customers are able to put down a **deposit** (usually a percentage of the value of the product) and then pay the remaining outstanding balance off by means of **monthly instalment repayments**.
- The **remaining outstanding balance is charged simple interest**.
- Often insurance is added onto the monthly repayments.
- Repayment period is usually 12; 24; 36; 48 or 60 months.



The buyer is in fact 'hiring' the goods until he has 'purchased' it (after the last instalment has been paid).

Worked Example

Amanda buys a second-hand car worth R142 000. She pays a 15% deposit and the balance over 4 years using a hire purchase agreement. Calculate Amanda's monthly instalments if interest is charged at 12,5% per annum.

$$\text{Deposit} = \frac{15}{100} \times R142\,000 = R21\,300$$

$$\begin{aligned}\text{Outstanding balance} &= R142\,000 - R21\,300 \\ &= R120\,700\end{aligned}$$

$$\begin{aligned}\text{Interest charged} &= \left(\frac{12,5}{100} \times R120\,700 \right) \times 4 \text{ years} \\ &= R15\,087,50 \times 4 \\ &= R60\,350\end{aligned}$$

$$\begin{aligned}\therefore \text{Total repayment amount} &= \text{outstanding balance} + \text{interest} \\ &= R120\,700 + R60\,350 \\ &= R181\,050\end{aligned}$$

REMEMBER! Interest is not charged on the deposit; as you've already paid this money to the supplier!

$$\begin{aligned}\therefore \text{Monthly instalment} &= \frac{\text{total repayment amount}}{\text{number of months}} \\ &= \frac{R181\,050}{(4 \text{ yrs} \times 12)} \\ &= \frac{R181\,050}{48} \\ &= R3\,771,88\end{aligned}$$

Test Your Understanding

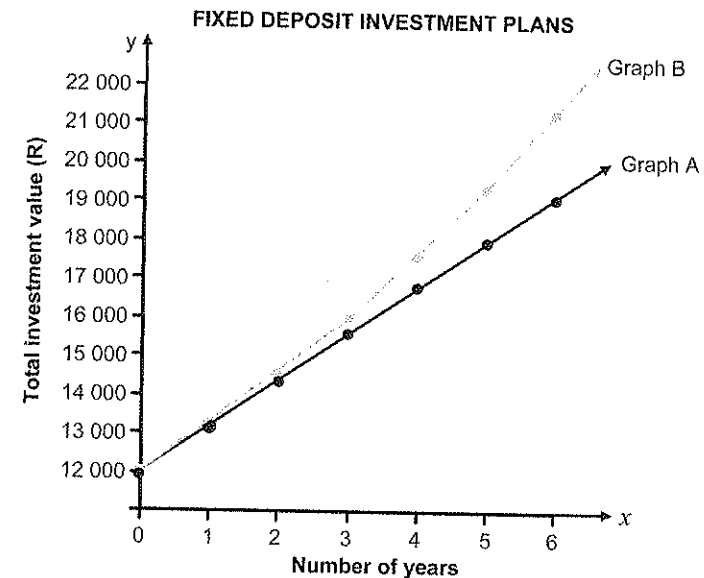
- The owner of the Baggy Pants Golf Shop decides to invest some of the profit from the shop. He investigates some options for investing R250 000,00. Bank A is willing to offer him an interest rate of 6% compounded annually for 2 years, while Bank B is willing to offer him a simple interest rate of 5,8% per annum for 3 years.

Which Bank should the owner invest with? Use calculations to justify your answer.

- You invest R7 400 at 10,5% simple interest for 3 months.
 - Calculate the interest you will receive during this period.
 - Calculate how much money you will have in your account after this time period.

REMEMBER! Express 3 months as a fraction of a year because the interest rate is always per annum, unless otherwise specified.

- Roxanne invests R10 000 in a fashion magazine. How much profit will she have made after 3 years if she earns 11% per year compound interest?
- Clint buys a laptop for R3 999. He signs a hire purchase agreement that stipulates charges of 18% p.a. interest, a 15% deposit and an insurance fee of R30 be paid monthly. Calculate:
 - His monthly instalment over the 3 year period.
 - The total amount he will have paid at the end of the 3 years.
 - How much more does the laptop cost if Clint buys it on HP as opposed to cash?
- If I invested R13 000 for 4 years and I now have R17 160, what was the simple interest rate given to me by the bank?
- Isabel has a choice of two fixed deposit investment plans. Plan A is illustrated by Graph A and Plan B is illustrated by Graph B, shown below:



- What type of graph is illustrated by Graph A and B respectively?
- What type of interest is illustrated by Graph A and B respectively?

Copyright © The Answer: Photocopying of this material is illegal

- 6.3 How much money will Isabel be investing in the fixed deposit?
- 6.4 In which year will Plan A equal Plan B?
- 6.5 Which plan would you recommend Isabel to choose? Give a reason for your answer.
- 6.6 Calculate the interest rate quoted on the investment.

UNIT 7

BANKING, LOANS AND INVESTMENTS

Common Banking Terminology

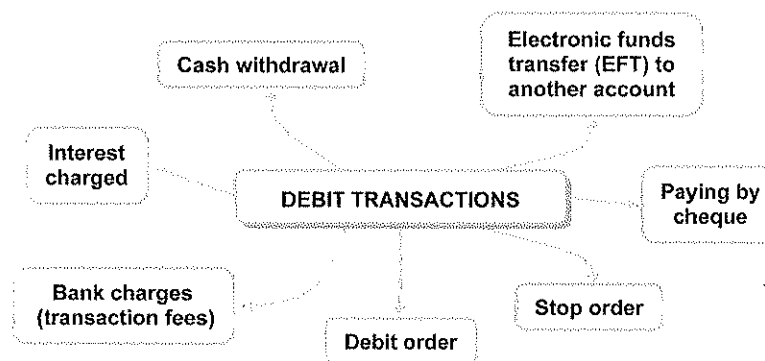
- **Opening balance** - the amount of money in the account at the beginning of the statement period.
- **Closing balance** - the total amount of money in the account at the end of the statement period.
- **Deposit** - money paid into a client's account.
- **Withdrawal** - money that is withdrawn/taken out of a client's bank account.
- **Electronic funds transfer (EFT)** - the electronic transfer of money from a client's account into another account by means of computer-based systems.
- **Cheque** - an instruction to the client's bank to withdraw the amount indicated on the cheque from his/her account and to pay this amount to the payee indicated on the cheque.
- **Stop order** - an arrangement between the client and the bank to pay a fixed amount of money from his/her account into another account on a specific date every month, e.g. rent being paid to a landlord.
- **Debit order** - an arrangement between the client and a company where the client gives the company permission to withdraw an agreed amount of money from his/her bank account, usually on a monthly basis. The company instructs the bank to make the payment. E.g. telephone accounts, loan repayments, insurance premiums and car repayments.
- **Bank charges/ transaction fees** - the fees charged by the bank for various services.



- **Interest** can be earned or charged, depending on the context:
 - ▷ **Interest earned** - money earned for saving money or investing money at the bank. Interest is also paid on a credit balance in a cheque account at some banks, but this interest rate is very low. The interest earned on a credit balance is called **credit interest**.
 - ▷ **Interest charged** - the fee that the bank charges for a debit balance (overdrawn account or negative balance). Interest charged on a debit balance is also called **debit interest**.
- **Overdraft** - is a facility that allows money to continue being withdrawn from a bank account even after a zero balance has been reached, i.e. it allows a negative balance to occur, thus allowing people to borrow money. There is usually an authorised limit set on the amount for borrowing and interest is charged accordingly.
- **PIN** - **Personal Identification Number** is a number you choose and is used as a safety measure when accessing personal information and banking accounts.
- **ATM** - **Automated Teller Machine** is a computerised telecommunications device that provides clients of a financial institution with access to financial transactions in a public space without the need of a cashier or a bank clerk.
- **Bank Statement** - A record (from the bank) indicating the opening and closing balances as well as all the transactions, for a particular period (usually a month).
- **Debit transactions** - when money is taken out of a client's account.

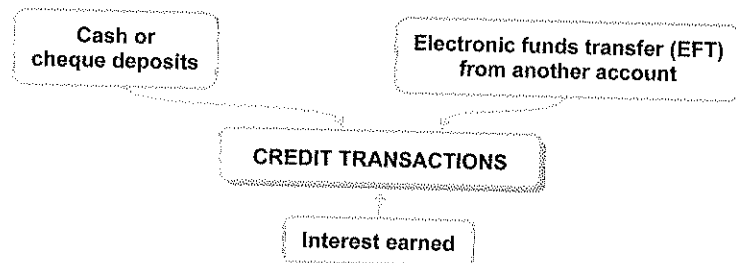


Examples of types of transactions that cause a bank account to decrease are illustrated below:



- **Credit transactions** - when money is paid into a client's account.

Examples of types of transactions that cause a bank account to increase are illustrated below:



Types of Bank Accounts

Banks offer different types of bank accounts; in order to meet the different needs of their clients. The most common types of bank accounts include:

1. SAVINGS ACCOUNT

- A savings account allows the account holder to keep the money that is saved in a safe place, while it earns some interest.
- Interest is calculated daily, but only added to the account total at the end of the month.
- The larger the amount of money invested, the higher the interest rate.

ADVANTAGES

- Money is easily accessible in the savings account.
- Money earns interest while in the savings account.

DISADVANTAGES

- Cheques or credit cards cannot be linked to a savings account.
- Interest rate is usually lower than other accounts (such as a fixed deposit account).
- Banking fees are charged on transactions.

2. CHEQUE/CURRENT ACCOUNT

- This account is used for daily deposit and withdrawal transactions.
- The account holder's salary is usually paid into this account.
- Stop orders or debit orders are usually deducted from a cheque account.
- An overdraft facility can be linked to this account.
- It allows the account holder to write cheques against the funds in the account.
- A debit card can be linked to a current/cheque or savings account.

If you buy goods or services and pay with your debit card, the amount will immediately be deducted from the balance in your account and will be paid directly into the shop or other creditor's bank account.

- Interest is calculated daily, but only added to the account total at the end of the month.

ADVANTAGES

- Money is easily accessible.
- The account is convenient, as it allows the account holder to link a debit card, stop orders, debit orders or write cheques.
- Provides the possibility of an overdraft facility.

DISADVANTAGES

- Low interest rates as money stays in the account for a short period of time.
- High interest rates charged on overdraft facility.
- Banking fees are charged on transactions.

3. CREDIT CARD

- A credit card allows the account holder to buy goods and services now; but only repay the full value (plus interest) over a period of time back to the bank.
- The bank is actually paying for the goods and the cardholder repays the bank through regular monthly payments.
- The amount of money the bank allows the client to spend is called the **credit limit** and usually depends on the client's monthly income.
- Once a month the bank sends a credit card statement, which gives a summary of all the transactions in the specific time period, to the account holder.

ADVANTAGES

- Allows consumers to buy goods and services - even if they don't have the full cash value at the time of purchase.
- It is convenient and safe as consumers don't have to carry large sums of cash on you (e.g. for buying large expensive goods).

DISADVANTAGES

- High interest rates are charged on outstanding balances.
- The bank owns the goods until the goods are paid for in full.
- Consumers can live beyond their means and rack up huge credit problems if they are not able to repay their monthly payments.
- Banking fees are charged on transactions.

4. FIXED DEPOSITS

- A fixed deposit is an investment account; where the account holder either deposits a once-off lump sum or makes regular deposits, for a fixed period of time.
- Withdrawals from these accounts are not allowed (with the exception of proven emergencies, then penalty fees are incurred).

ADVANTAGES

- Offers high interest rates; due to the fact that the account holder's money is secured for a fixed period of time.

- Banking fees are minimal.



DISADVANTAGES

- Money is not easily accessible, as its fixed for a specified period of time.
- Penalties are incurred if money is withdrawn (in a proven emergency).

Bank Charges



- Also known as transaction fees.
- Banks charge different fee structures for the services they offer, e.g. deposits; withdrawals; stop orders; debit orders; cheques, etc.
- Banks do allow clients some free transactions; depending on the type of account they hold.
- Interest is charged on unpaid outstanding credit card balances.



Worked Examples

1. Study the bank fee table on the next column for Golden Bank:

1.1 How much would it cost to deposit R1 500 over the counter?

$$\begin{aligned}
 \text{Cost} &= R7,50 + 2\% \text{ of } R1\,500 \\
 &= R7,50 + \left(\frac{2}{100} \times 1\,500 \right) \\
 &= R7,50 + R30 \\
 &= R37,50
 \end{aligned}$$

GOLDEN BANK FEE TABLE

TRANSACTION

FEE

MONTHLY FEES

Monthly maintenance fee	R25,00
Self-service banking subscription fee	R15,00

DEPOSITS

Cash over the counter	R7,50 + 2% of deposit amount
Cash (at ATM)	R5,00
Cheque (over the counter or at ATM)	R6,00

CASH WITHDRAWALS

Over the counter	R10,00 + R2,50 for every R100 withdrawn (or part thereof)
Golden Bank ATM	R5,00
Another bank's ATM	R7,00
Tillpoint - cash only	R1,00
Tillpoint - cash with purchase	R2,00

ACCOUNT PAYMENT AND PURCHASES

Electronic transfers between accounts	Free
Electronic account payment	Free
Stop order	R25,00
Debit order - internal	R2,50
Debit order - external	R15,00

BALANCE ENQUIRIES

Over the counter	First free per month, then R10,00
Golden Bank ATM	First free per month, then R1,00
Another bank's ATM	R12,00
Self-service banking	Free

1.2 Determine the costs of withdrawing R480 over the counter?

$$\begin{aligned}
 \text{Cost} &= R10,00 + R2,50 \text{ for every } R100 \text{ or part thereof} \\
 &= R10,00 + (R2,50 \times 5) \\
 &= R10,00 + R12,50 \\
 &= R22,50
 \end{aligned}$$

$R480 \div R100 = 4,8$
 Since R2,50 is charged per part of R100, we round up to 5.



- 1.3 Calculate the total monthly banking fees, if a customer made the following transactions in a month:

- 4 cash withdrawals at Golden Bank ATM
- 1 cash deposit of R1 200 at an ATM
- 5 electronic account payments
- 2 stop orders
- 2 external debit orders
- 1 cheque deposit over the counter
- 3 balance enquires at Golden Bank ATM

REMEMBER! First balance enquiry free
 $\therefore$ 2 charged at R1 each.

Total monthly banking fees

$$= (4 \times R5) + (1 \times R5) + (5 \times R0) + (2 \times R25) + (2 \times R15) + (1 \times R6) + 2 \times R1$$

$$= R20 + R5 + R0 + R50 + R30 + R6 + R2$$

$$= R113$$

2. George compares the bank fees for withdrawing money from 3 different banks:

- Bucks Bank fees: R5,00 per transaction + R1,50 for every R100 withdrawn (or part thereof)
- Money Bags Bank fees: R2,50 for every R100 withdrawn (or part thereof).
- Richie Rich Bank fees: R4,50 for the first R500 withdrawn; then R2 per every additional R100 withdrawn (or part thereof)

- 2.1 Which bank would charge the lowest bank fees if George needed to withdraw R1 250?

$$\text{Bucks Bank fees} = R5 + (R1,50 \times 13)$$

$$= R5 + R19,50$$

$$= R24,50$$

$$\text{Money Bags Bank fees} = R2,50 \times 13$$

$$= R32,50$$

$$\text{Richie Rich Bank fees: } R1\,250 - R500 = R750$$

$$\therefore R750 \div R100 = 7,5 \approx 8$$

$$\therefore \text{Fees} = R4,50 + (R2 \times 8)$$

$$= R4,50 + R16$$

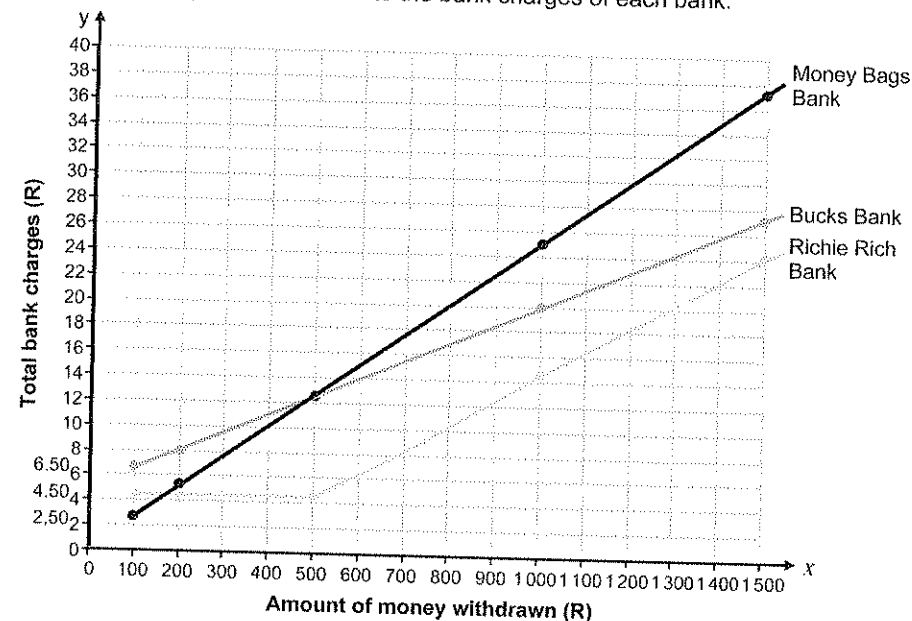
$$= R20,50$$

$\therefore$ Richie Rich Bank would charge the lowest bank fees.

REMEMBER!
 $R1\,250 \div R100 = 12,5$
 $\therefore$ Since R1,50 is charged per part of R100, we round up to 13.



- 2.2 The graphs below illustrate the bank charges of each bank.



- 2.2.1 Which bank is cheapest if George withdraws R300? *Richie Rich Bank*

- 2.2.2 Which bank is the most expensive if George withdraws R600? *Money Bags Bank*

- 2.2.3 How much money must George withdraw to have the same bank charges at either Money Bags Bank or Bucks Bank? *R500*

- 2.2.4 Estimate the difference in bank charges between Money Bags Bank and Richie Rich Bank, if George withdraws R700.
Difference $\approx$ R17 - R8 $\approx$ R9

3. Karien purchased goods on her credit card to the value of R3 720. Her credit card agreement says that she must pay at least 10% of the value of her credit card per month; failing that, she will be charged 23% interest on her outstanding balance.

- 3.1 Determine how much Karien must pay on her credit card.

$$\text{Minimum payment} = \frac{10}{100} \times R3\,720 = R372$$

- 3.2 Calculate the interest Karien will pay if she is unable to make the minimum payment on her credit card.

$$\text{Interest} = \frac{23}{100} \times R3\,720 = R855,60$$

The interest rates charged on outstanding balances of credit cards is very high. Therefore, it is wise to make at least the minimum payment per month, in order to avoid being charged a lot of interest.



Loans



- A loan agreement is a contract between **two parties**; whereby the **lender** gives money to the **borrower**, with the expectation that the money will be repaid, together with **interest** within a fixed time period.
- The interest on the loan is calculated using either a fixed or variable interest rate; based either on simple or compound interest.
- The **monthly repayment** amount comprises of both paying off part of the original borrowed amount; plus a part of the interest.
- The fixed time period of the loan is often referred to as the **term of the loan**; and is usually stated in months.
- People can enter into either informal loan agreements with family members; or formal loan agreements with banks and micro-lenders.
- People can either take out loans to cover the full value of the goods or services; or can enter into hire-purchase agreements.
- Typically, people take out loans to cover personal expenses; student loans; car loans; home appliances; house loans (mortgage); furniture etc.

NOTE! More on simple interest, compound interest and hire purchase agreements in Unit 6 (see p. 78).



The Real Cost of a Loan



- The real cost of the loan is the value of the borrowed amount, plus the interest charged.
- The following formulae can be used to calculate the total amount of money that a person will pay back:

Total amount paid back = monthly repayment × number of months of loan

- One of the best ways to reduce the total amount of money that you end up paying back to the bank for a loan is to increase the monthly repayments.
- Another way to reduce the cost of a loan is to deposit any extra money directly into the home loan. This automatically decrease the amount of money on which interest can be calculated. The loan is paid off faster and with much less interest.



Worked Examples



- The table alongside provides a summary of the conditions of Kate's home loan.

Home loan amount	R725 000,00
Length of the loan	20 years
Current interest rate	14,5% per year
Monthly repayment	R9 280,00

- 1.1 Calculate the total amount that Kate will pay back to the bank for the loan.

Total amount paid back
 $= R9\ 280,00 \text{ per month} \times 12 \text{ months} \times 20 \text{ years} = R2\ 227\ 200,00$

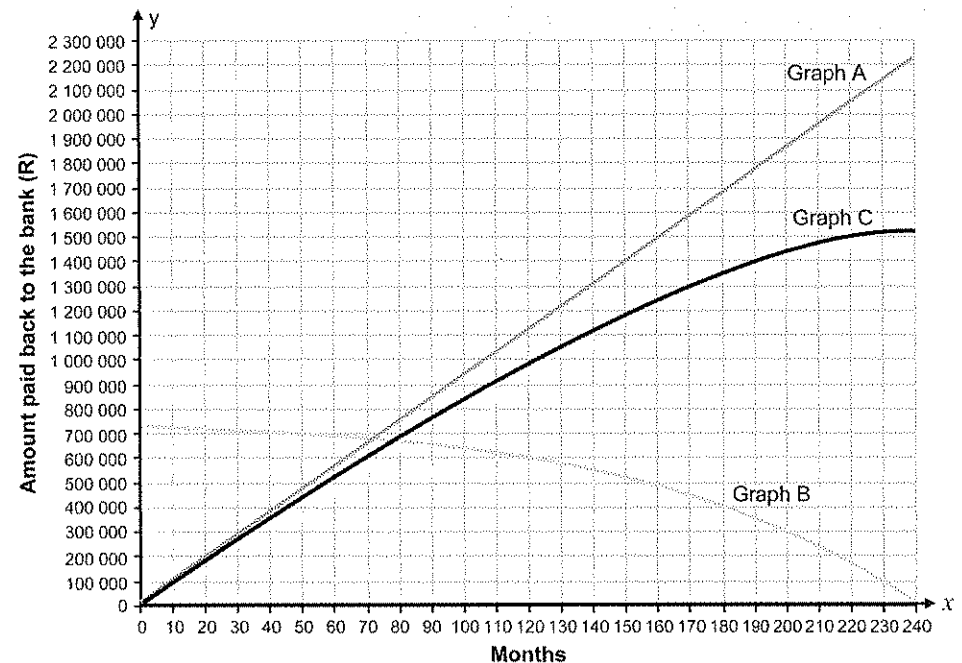
- 1.2 Calculate the total amount of interest that Kate will pay back to the bank for the home loan.

Total interest = total paid back - original loan
 $= R2\ 227\ 200,00 - R725\ 000,00 = R1\ 502\ 200,00$

- 1.3 Compare the total amount that Kate will pay back to the bank to the original value of the loan and describe the compound interest.

Kate will pay back more than double the original value of the loan. This is due to compound interest calculated on the loan. Interest decreases every month, but it is still substantial.

2. Kate's home loan scenario is illustrated in the graphs below.



- 2.1 Graph A shows the real cost of Kate's loan over the 20 year life of the loan. In other words, the graph shows the total amount that Kate will pay back to the bank over time for the loan.

2.1.1 Explain why the graph is a straight line.

Every month Kate will pay the same amount back to the bank i.e. R9 280,00. After 1 month she will have paid back R9 280,00; after 2 months R9 280,00 + R9 280,00 = R18 560,00 etc. The graph will be a linear graph increasing by a constant value of R9 280,00 for every 1 month that passes.

2.1.2 Use the graph to approximate how much Kate will have paid back to the bank after 134 months.

$\approx R1\ 250\ 000,00$

2.2 Graph B shows the total amount of money that Kate will still owe on the loan per month during the 20 year life of the loan.

2.2.1 Explain why the graph slopes downwards and why the graph is curved and not straight.

Every month Kate will pay back a portion of the loan. The graph decreases because the amount of money still owed gets smaller every month. The graph decreases at an increasing rate because as the amount that she owes gets smaller, so the interest calculated on the outstanding balance also decreases. Towards the end of the loan, less and less interest is added every month and the amount owed on the loan decreases.

2.2.2 Approximately how long will it take for Kate to pay back half of the loan?

≈ 190 months (accurate answer is 187 months)

2.3 Graph C shows the cumulative (total) amount of interest that Kate will pay on the loan over the 20 year life of the loan. Explain why this graph curves upwards at a decreasing rate.

During the first 190 months the monthly interest is high as it is calculated on a large outstanding balance. However, as the outstanding balance decreases, so the interest calculated decreases at a faster rate. This is represented on the graph as the graph increases at a slower and slower rate towards the end of the loan.

Monthly Repayments



- The monthly repayments on a home loan are determined by three things:

- size of the bond
- interest rate
- number of years of repayment

- Banks use the following formula to calculate monthly repayments:

$$\text{Monthly repayment} = \frac{\text{loan amount}}{1\ 000} \times \text{factor}$$



- 'Factor' is a value that depends on the number of years and the interest rate of the bond. The higher the interest rate, the higher the factor and, thus, the higher the monthly repayments.

Similarly, the shorter the number of years of repayment, the higher the factor and, thus, the higher the monthly repayments.

The table below gives the various factors as dependent on the length of the bond and the interest rate:

FACTOR TABLE										
Term Year	Interest Rate									
	10,5%	11%	11,5%	12%	12,25%	12,5%	12,75%	13%	13,25%	13,5%
15	11,05	11,37	11,68	12	12,16	12,33	12,49	12,65	12,82	12,98
20	9,98	10,32	10,66	11,01	11,18	11,36	11,54	11,71	11,89	12,07
25	9,44	9,8	10,16	10,53	10,71	10,9	11,1	11,28	11,47	11,66
30	9,15	9,52	9,9	10,29	10,47	10,67	10,87	11,06	11,26	11,45
Term Year	Interest Rate									
	13,75%	14%	14,25%	14,5%	14,75%	15%	15,25%	15,5%	15,75%	16%
15	13,15	13,32	13,49	13,66	13,83	14	14,17	14,34	14,51	14,69
20	12,25	12,44	12,62	12,8	12,92	13,17	13,35	13,54	13,73	13,91
25	11,85	12,04	12,23	12,42	12,61	12,81	13	13,2	13,39	13,59
30	11,65	11,85	12,05	12,25	12,44	12,64	12,84	13,05	13,25	13,45

- By voluntarily increasing the monthly repayment, one can reduce the total cost of a loan.

Worked Examples

1. Calculate the monthly repayments on a bond of R550 000, taken over 20 years and at an interest rate of 14% p.a.

Factor on loan at 14% interest over 20 years = 12,44 (12th column, 2nd row).

$$\begin{aligned}\text{Monthly Repayment} &= (\text{loan amount} \div 1\,000) \times \text{factor} \\ &= [R550\,000 \div 1\,000] \times 12,44 \\ &= 550 \times 12,44 \\ &= R6\,842,00 \text{ per month.}\end{aligned}$$

From the
factor table.

2. Kate takes out a loan of R725 000 from the bank. The bank charges an interest rate of 14,5% per annum. To keep track of how much she still owes on the loan at the end of every month, Kate makes the following summary of her loan for the first 2 years of the loan:

Month	Opening Balance	Interest	Balance Before Payment	Monthly Repayment	Closing Balance After Payment
0					R725 000,00
1	R725 000,00	R8 760,42	R733 760,42	R9 280,00	R724 480,42
2	R724 480,42	R8 754,14	R733 234,56	R9 280,00	R723 954,56
3	R723 954,56	R8 747,78	R732 702,34	R9 280,00	R723 422,34
4	R723 422,34	R8 741,35	R732 163,69	R9 280,00	R722 883,69
5	R722 883,69	R8 734,84	R731 618,54	R9 280,00	R722 338,54
6	R722 338,54	R8 728,26	R731 066,79	R9 280,00	R721 786,79
7	R721 786,79	A	R730 508,38	R9 280,00	R721 228,38
8	R721 228,38	R8 714,84	R729 943,23	R9 280,00	R720 663,23
9	R720 663,23	R8 708,01	R729 371,24	R9 280,00	R720 091,24
10	R720 091,24	R8 701,10	R728 792,34	R9 280,00	R719 512,34
11	R719 512,34	R8 694,11	R728 206,45	B	R718 926,45
12	R718 926,45	R8 687,03	R727 613,48	R9 280,00	R718 333,48
13	R718 333,48	R8 679,86	R727 013,34	R9 280,00	R717 733,34
14	R717 733,34	R8 672,61	R726 405,95	R9 280,00	R717 125,95
15	R717 125,95	R8 665,27	R725 791,23	R9 280,00	R716 511,23
16	C	R8 657,84	R725 169,07	R9 280,00	R715 889,07
17	R715 889,07	R8 650,33	R724 539,40	R9 280,00	R715 259,40
18	R715 259,40	R8 642,72	R723 902,11	R9 280,00	R714 622,11
19	R714 622,11	R8 635,02	R723 257,13	R9 280,00	R713 977,13
20	R713 977,13	R8 627,22	R722 604,35	R9 280,00	R713 324,35
21	R713 324,35	R8 619,34	R721 943,69	R9 280,00	R712 663,69
22	R712 663,69	R8 611,35	D	R9 280,00	R711 995,04
23	R711 995,04	R8 603,27	R720 598,32	R9 280,00	R711 318,32
24	R711 318,32	R8 595,10	R719 913,41	R9 280,00	E

- 2.1 Show how the *Balance before payment* value of R733 760,42 for Month 1 was calculated.

$$\begin{aligned}\text{Balance Before Payment} &= \text{opening balance} + \text{interest} \\ &= R725\,000 + R8\,760,42 \\ &= R733\,760,42\end{aligned}$$

- 2.2 Show how the *Closing Balance after payment* value of R719 512,34 for Month 10 was calculated.

$$\begin{aligned}\text{Closing Balance after payment} &= \text{opening balance} + \text{interest} - \text{monthly repayment} \\ &= R720\,091,24 + R8\,701,10 - R9\,280,00 \\ &= R719\,512,34\end{aligned}$$

- 2.3 Show how the *interest* value of R8 760,42 for Month 1 was calculated.

$$\begin{aligned}\text{Interest per year} &= \frac{14,5}{100} \times R725\,000 \\ &= R105\,125\end{aligned}$$

$$\begin{aligned}\therefore \text{Interest per month} &= R105\,125 \div 12 \\ &= R8\,760,42\end{aligned}$$

NOTE!

The interest rate is
14,5% **per annum**
 $\therefore$ we divide by 12.

- 2.4 Calculate the missing values A - E from the table

$$\begin{aligned}A &= R730\,508,38 - R721\,786,79 \\ &= R8\,721,59\end{aligned}$$

$$B = R9\,280,00$$

$$C = R716\,511,23$$

$$\begin{aligned}D &= R712\,663,69 + R8\,611,35 \\ &= R721\,275,04\end{aligned}$$

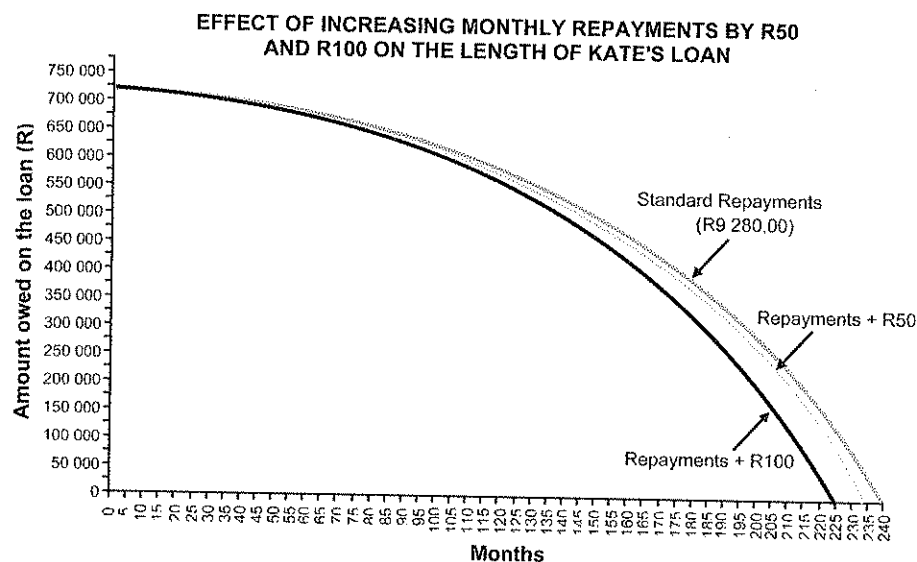
$$\begin{aligned}E &= R711\,318,32 + R8\,595,10 - R9\,280 \\ &= R710\,633,42\end{aligned}$$

- 2.5 Is the interest charged compound or simple interest?
Give a reason for your answer.

The interest is compounded; as it is calculated on the principal amount plus any interest accrued.

An easy way of spotting the difference between simple and compound interest, is that the simple interest value remains the same/constant; while the compound interest value changes.

3. The graph below shows the amount of money that Kate will owe on her loan over time if she pays the standard monthly repayments of R9 280,00; if she pays R50,00 extra; and if she pays R100,00 extra.



- 3.1 Approximately how long will it take Kate to pay off the loan if she increases the monthly repayments by R50,00?
235 months
- 3.2 By how many months earlier will Kate's home loan end, if she pays an additional R100 back per month?
 $240 - 225 = 15 \text{ months}$
- 3.3 How much will Kate owe on the loan at 165 months, if she pays an additional R100 back per month?
 $\approx R400\,000$
- 3.4 After how many months will Kate's home loan equal R175 000, if she pays an additional R50 back per month?
 $\approx 215 \text{ months}$
- 3.5.1 How much will Kate pay back to the bank in total for the loan if she pays R50,00 more than she has to each month?

Total amount paid back with increased repayment
 $= (R9\,280,00 + R50,00) \times 235 \text{ months}$
 $= R9\,330,00 \times 235 = R2\,192\,550,00$

- 3.5.2 How much will Kate save by paying back R50,00 more than her monthly repayment?

Total paid back without increased repayment
 $= R9\,280,00 \times 240 \text{ months} = R2\,227\,200,00$

So, by increasing her repayments by R50,00, Kate will save
 $R2\,227\,200,00 - R2\,192\,550,00 = R34\,650,00$

Interest Rates and Interest

- The monthly repayments are directly related to the current interest rate.
- If the **interest rate increases**, so will the monthly repayments; in order to ensure that the loan is still paid back in the same period of time.
- If the **interest rate decreases**, then the option is available to decrease the monthly repayments. However, if interest rate decreases and the monthly repayments are kept the same, then the loan will be paid off much more quickly, which will in turn save money.

Worked Examples

- 1.1 Use the equation and factor table below to calculate what Kate's monthly repayment will be if interest on her loan of R725 000 is calculated at 15% rather than 14,5%, over 20 years.

Monthly Repayment = (Loan Amount $\div$ 1 000) $\times$ Factor											
Term Year	Interest Rate										
	13.75%	14%	14.25%	14.5%	14.75%	15%	15.25%	15.5%	15.75%	16%	16,5%
15	13.15	13.32	13.49	13.66	13.83	14	14.17	14.34	14.51	14.69	15,04
20	12.25	12.44	12.62	12.8	12.92	13.17	13.35	13.54	13.73	13.91	14,29
25	11.85	12.04	12.23	12.42	12.61	12.81	13	13.2	13.39	13.59	13,98

Monthly repayment = (loan amount $\div$ 1 000) $\times$ factor
 $= (R725\,000,00 \div 1\,000) \times 13,17$
 $= R725 \times 13,17 = R9\,548,25$

- 1.2 Calculate the total amount of money that Kate will pay back to the bank for the loan at an interest rate of 15%.

$$\begin{aligned} \text{Total paid back to the bank} &= R9\,548,25 \text{ per month} \times 12 \text{ month} \times 20 \text{ years} \\ &= R2\,291\,580,00 \end{aligned}$$

- 1.3 What effect will an interest rate of 14,5% have on the total amount that Kate will pay back to the bank for the loan?

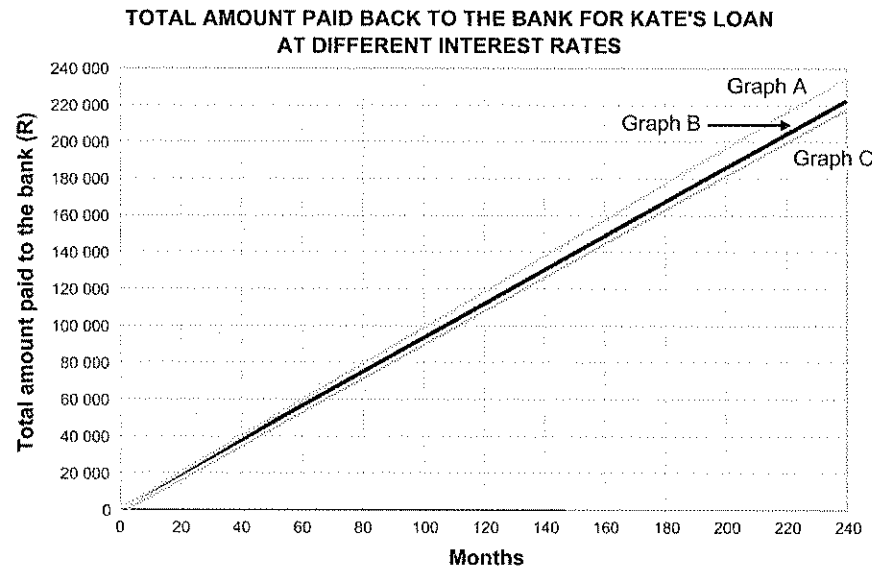
$$\begin{aligned} \text{Monthly repayment at 14,5\%} &= (R725\,000 \div 1\,000) \times 12,8 \\ &= R725 \times 12,8 = R9\,280 \end{aligned}$$

$$\begin{aligned} \therefore \text{Total paid back at 14,5\%} &= (R9\,280,00 \times 12 \text{ months}) \times 20 \text{ years} \\ &= R2\,227\,200,00 \end{aligned}$$

$$\text{Total paid back at 15\%} = R2\,291\,580,00 \text{ [Answer Question 1.2]}$$

$$\begin{aligned} \therefore \text{Extra paid back if the interest rate increases by 0,5\%} \\ &= R2\,291\,580,00 - R2\,227\,200,00 \\ &= R64\,380,00 \end{aligned}$$

2. The graphs below show the total amount of money that Kate will pay back to the bank for the home loan at different interest rates.



- 2.1 What is the length of this loan? Give your answer in years.

$$240 \text{ months} = 240 \div 12 \text{ months} = 20 \text{ years}$$

- 2.2 What is the total amount paid back to the bank after 200 months, on Graph C?

$$R180\,000$$

- 2.3 After how many months will the total paid back to the bank be approximately R130 000 on Graph B?

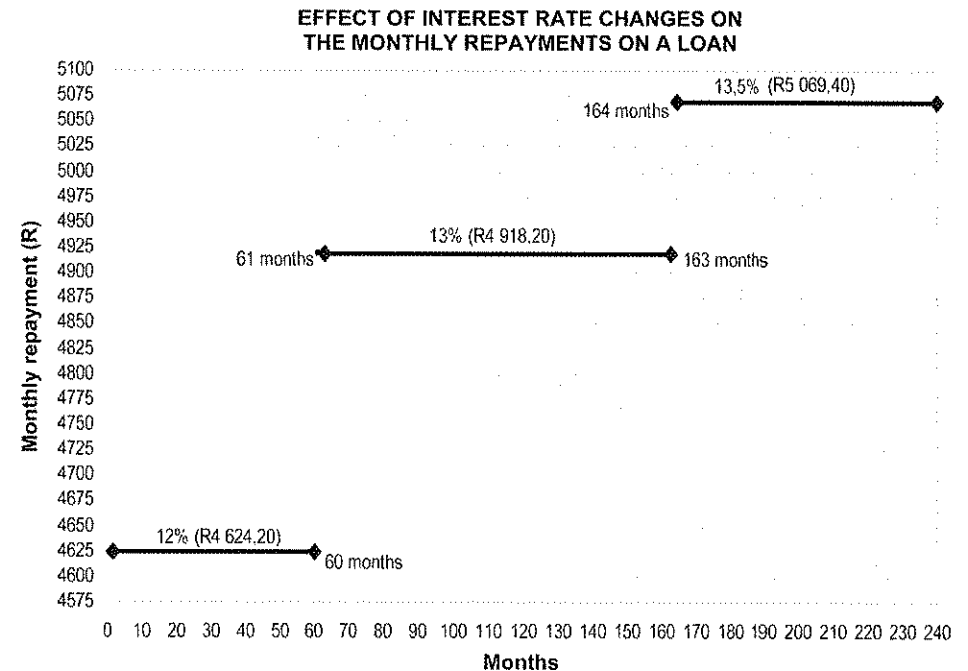
$$140 \text{ months}$$

- 2.4 Graph B represents the loan option that Kate has been offered by ASBA bank with a 14,5% interest rate. Explain, with reasons, which graph represents a loan option at 14% interest rate and which graph a loan option at 15,5% interest rate.

Graph C - 14% interest rate. i.e. The lower interest rate means that a lower amount will be paid back to the bank for the loan.

Graph A - 15,5% interest rate. i.e. The higher interest rate means that a larger amount will be paid back to the bank for the loan.

3. The graph below shows the effect of various interest rate changes on the monthly repayments on a loan.



3.1 Explain why the graph is stepped.

When the interest rate increases, this causes an immediate increase in the monthly repayment. i.e. The monthly repayment is dependent on the length of the loan (which remains fixed) and the interest rate (which can change). So, when the interest rate changes, the 'factor' changes, and the monthly repayment also changes.

3.2 For how many years and months did the interest rate remain at 13%?

*From 61 months to 163 months = 103 months
= 8 years 7 months*

3.3 Using the graph, calculate as accurately as possible the total amount that would have been paid back to the bank on this loan.

12% interest rate

*Length of time = 60 months
Monthly repayment = R4 624,20
Total paid back during this period = R4 624,20 per month $\times$ 60 months
= R277 452,00*

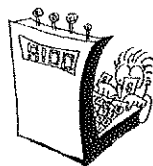
13% interest rate

*Length of time = 103 months
Monthly repayment = R4 918,20
Total paid back during this period = R4 918,20 per month $\times$ 103 months
= R506 574,60*

13,5% interest rate

*Length of time = 77 months
Monthly repayment = R5 069,40
Total paid back during this period = R5 069,40 per month $\times$ 77 months
= R390 343,80*

*$\therefore$ Grand total paid back
= R277 452,00 + R506 574,60 + R390 343,80
= R1 174 370,40 ✓*



Investments



There are four common types of investments, which require monthly deposits:

1. Stokvel

- A stokvel is a typical South African club/society; whereby members of the club contribute a fixed amount of money on a weekly, fortnightly or monthly basis; into a pooled fund.
- The pooled fund is not necessarily invested, but rather used by a different member of the stokvel each month, for their personal use.
- The stokvel therefore acts as a rotating credit union.
- A stokvel allows members to make big and expensive purchases.

Worked Example



Peggy and five of her closest friends form a stokvel. They decide to each contribute R40 of their pocket money per month, in order for each girl to go to the hairdresser to have her hair done.

1. How often will Peggy get to go to the hairdresser?

Every 6 months

2. How much will Peggy be able to spend at the hairdresser?

R40 $\times$ 6 = R240



Since there are 6 members in the stokvel, Peggy will get her turn every six months.

Investments (continued)

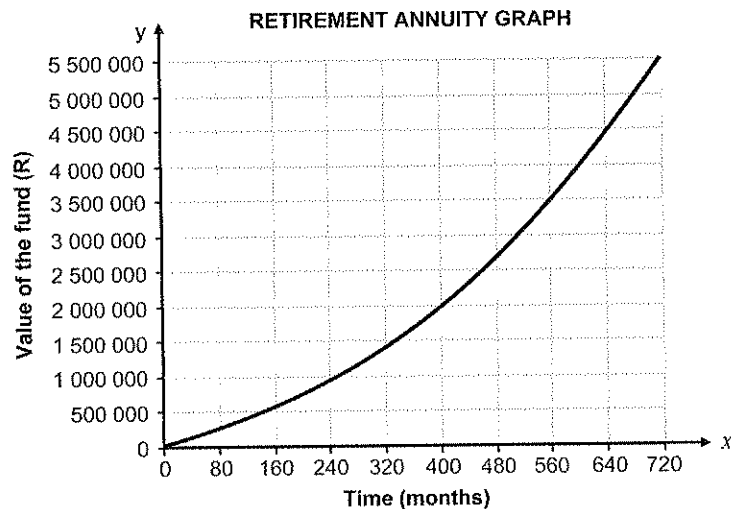


2. Retirement Annuity

- A retirement annuity is an investment fund; whereby a person makes a monthly deposit into a fund that matures at retirement age (usually 65 years).
- The investor can then decide to either be paid the retirement fund out in a lump sum; or to receive regular monthly payments.
- The retirements annuity therefore makes provision for a person when they retire.

Worked Example

Study the retirement annuity investment graph below and answer the following questions:



- After how many months will the fund be valued at approximately R4 500 000?
640 months
- What is the approximate value of the fund after 240 months?
 $\approx$ R1 000 000
- How old will this investor be upon retirement; when the fund matures to the value of R5 500 000?
 $720 \text{ months} \div 12 = 60 \text{ years}$
- If this retirement annuity fund matured to the value of R5 500 000; how long could the investor live off the fund if he wanted to be paid out R25 000 per month?
Give your answer in years and months.

$$\text{Number of months} = R5\,500\,000 \div R25\,000 = 220 \text{ months}$$

$$\therefore \text{Number of years} = 220 \div 12 = 18,333 \text{ years}$$

$$\therefore 18 \text{ years} + 0,3333 \times 12 \text{ months} \\ = 18 \text{ years} + 4 \text{ months}$$



REMEMBER! To convert to months, take the **un-rounded** decimal value and multiply by 12.



Investments (continued)



3. Pension Fund

- A pension fund is also a fund set up to provide income upon retirement.
- Both the employee and employer contribute to the fund; usually based on a percentage of the employee's salary.
- Upon retirement, the pension fund pays out a monthly stipend/amount to the contributor.
- A fund can be paid out before retirement age; but is heavily taxed.

Worked Example



Themba would like to set up a pension fund. He earns R25 000 per month and would like to contribute 5% of his salary towards the pension fund. As part of his employment contract, his employer will match his monthly contribution and add an additional 2%.

Calculate his total monthly contribution towards his pension fund.

$$\begin{aligned} \text{Themba's contribution} &= \frac{5}{100} \times R25\,000 \\ &= R1\,250 \end{aligned}$$

$$\begin{aligned} \text{Employer's contribution} &= \frac{7}{100} \times R25\,000 \\ &= R1\,750 \end{aligned}$$

$$\begin{aligned} \therefore \text{Total monthly contribution} \\ &= R1\,250 + R1\,750 \\ &= R3\,000 \end{aligned}$$



REMEMBER!

His employer will match the contribution of 5% plus an additional 2%.

$$\therefore 5 + 2 = 7\%$$

Investments (continued)



4. Funeral Plan

- A funeral plan is a fund set up to cover funeral costs, in the event of death.
- The policy holder will make fixed monthly contributions; and upon death, the funeral plan will pay out a lump sum to cover costs.
- A funeral plan often includes benefits to help the remaining family members.

Worked Example



Patricia took out a funeral plan for herself and her husband. Her monthly premium is R12,50 per member. The funeral plan pays only a percentage of each members' premium back, based on the number of years she has contributed. This is illustrated in the table below:

Percentage payment of funeral plan premiums, based on the number of years of contribution:

Number of years	1	2	3	4	5	6	7	8	9	10+
% Payment	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

- Calculate Patricia's total monthly premium for her family. $R12,50 \times 2 = R25$
- Determine the value of Patricia's lump sum payment, should her husband die 7 years after starting the funeral plan contributions.

$$\text{Total contributions over 7 years for her husband} = R12,50 \times 12 \text{ mnths} \times 7 \text{ yrs} = R1\ 050$$

$$\therefore \text{Funeral payment} = 70\% \text{ of total contributions} = \frac{70}{100} \times R1\ 050 = R735$$

Test Your Understanding



- Pieter joins stokvel with a total of 20 members. Each member contributes R750 per month towards the fund. Pieter plans to buy a tractor when the stokvel pays him out. The tractor that Pieter is interested in buying is R21 000.
 - Will Pieter be able to afford the tractor? Show all calculations to substantiate your answer.
 - If Pieter is unable to pay cash for the tractor, then what percentage of the value of the tractor will Pieter need to secure a loan?
- Pieter's retirement annuity is maturing, so he thinks of using some of the annuity money to pay for the tractor. His retirement annuity fund is worth R800 000 and stipulates that he will be paid out the lump sum in equal monthly payments over 12 years.
 - Calculate Pieter's monthly retirement annuity payment.
 - If Pieter adds his monthly retirement annuity payment to his stokvel payment, will he be able to afford the tractor in a month? Explain your answer.

2.1 Below is a bank statement for Solly's bank account for the month of February.

BRO* BANK

Bank Statement – Savings Account

Customer Name: Solly van Heerden

Statement date: February

Address: Flat 3C, Ladswood Heights

Account No: ***#2189

Transaction Date	Transaction Details	Amount	Account Balance
1	Opening Balance		R 9 872,50
1	Debit Card Purchase - Groceries	R 1 516,72	R 8 355,78
1	Transaction fee	R 1,50	R 8 354,28
2	Stop order - Rental	R 3 600,00	R 4 754,28
2	Transaction fee	R 6,00	R 4 748,28
2	Debit order - Cell phone account	R 289,32	R 4 458,96
2	Transaction fee	R 6,00	R 4 452,96
3	Debit order - Car insurance		R3 800,96
3	Transaction fee	R 6,00	R 3 794,96
6	ATM Withdrawal	R 1 000,00	R 2 794,96
6	Transaction fee	R 6,00	R 2 788,96
10	ATM Withdrawal	R 880,00	R 1 908,96
10	Transaction fee	R 6,00	R 1 902,96
21	Cash deposit at Branch	R 325,00	R 2 227,96
21	Transaction Fee	R 5,59	
27	Electronic Deposit - Salary	R 7 675,90	R 9 898,27
28	Monthly account fee	R 8,20	R 9 890,07
	Final balance		R 9 890,07

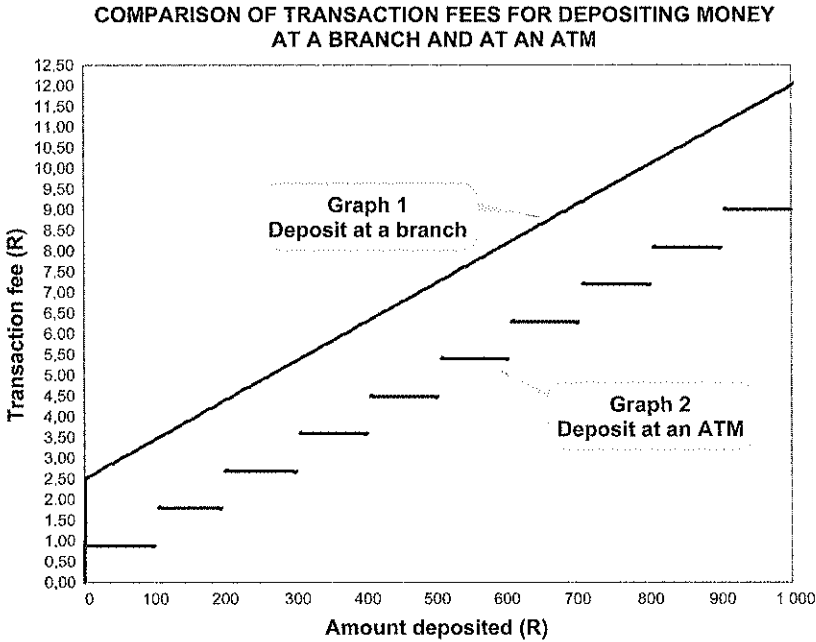
Amount of money in account	Interest Rate (% per annum)
R1 – R4 999,99	1,2%
R5 000 – R9 999,99	1,5%
R10 000 +	1,8%

* BRO - Big Rip-Off

- What was the closing balance in the account on 20 February?
- Calculate the total amount that Solly withdrew from his account through ATM withdrawals.
- Calculate the missing Account Balance value for 21 February.
- Calculate how much Solly pays for Car Insurance (3 February).

- 2.2.1 Solly did not realise that he could deposit money at an ATM machine, and so on 21 February Solly visited a branch of BRO bank and deposited R325,00 into his account.
- BRO bank uses the following formula to calculate the transaction fee on money deposited into an account at a branch:
- Transaction fee = R2,50 + 0,95% of the amount deposited**
- Use the formula to show how the transaction fee value of R5,59 on 21 February was calculated.
- 2.2.2 If Solly had deposited the money at an ATM, then the following method would have been used to calculate the transaction fee:
- Transaction fee = R0,90 for every R100 (or part thereof) deposited**
- Use the formula to calculate the transaction fee.
- 2.2.3 Calculate how much Solly would have saved by depositing the money into his account through the ATM rather than at the branch.

- 2.3 Two graphs are given below:
- One graph shows the transaction fees for depositing money into a BRO account at the branch. This graph represents the equation:
- Transaction fee = R2,50 + 0,95% of the amount deposited**
- The other graph shows the transaction fees for depositing money into a BRO account at an ATM. This graph represents the equation:
- Transaction fee = R0,90 for every R100 (or part thereof) deposited**



- 2.3.1 Approximately how much will it cost to deposit R650,00 into a BRO account at a bank branch?
- 2.3.2 Approximately how much will it cost to deposit R725,00 into a BRO account at an ATM?
- 2.3.3 Explain, by referring to the equation for Graph 1, why this graph is a straight line.
- 2.3.4 Explain why a vertical line has deliberately been included in Graph 1 to show that it starts at R0,00 and not at R2,50.
- 2.3.5 Explain by referring to the equation for Graph 2 why this graph is 'stepped'.
- 2.3.6 According to the graphs above, is it ever better to deposit money into a BRO account at the branch compared to at an ATM? Explain.
- 2.4 A competitor bank – ARO* Bank – is trying to convince Solly to close his account with BRO Bank and open an account with ARO Bank. They provide him with the following summary of bank charges.

ARO BANK – ACCOUNT FEES	
Type of Transaction	Transaction Fee
ATM Withdrawal	R2,50 for the first R100 plus R1,10 for every additional R100 (or part thereof)
Debit Order	R5,00 per debit order
Stop Order	R5,00 per stop order
Debit Card Purchase	R2,30 per purchase
Cash deposit at Branch	R2,80 + 0,90% of the amount deposited
Monthly service fee	R6,70

Based on his transactions for February, do you think Solly should change to ARO Bank or stick with BRO Bank? You must explain your answer and show all working.

* ARO - Another Rip-Off

3. The following poster appeared in a shop window:



HONEST HARRY'S LOANS

Affordable Cash Loans ...

No hassle, no fuss. For easy money, just call on us!

Loan	Amount owed after 1 month	Loan	Amount owed after 1 month
R100,00	R119,50	R600,00	R717,00
R200,00	R239,00	R800,00	R956,00
R300,00	R358,00	R1 000,00	R1 195,00
R400,00	R478,00	R1 500,00	R1 792,50
R500,00	R597,50	R2 000,00	R2 390,00

For more information contact: Honest Harry on 073 452 1891



- 3.1 How much would a customer owe after 1 month, if he loaned R400?
- 3.2 What is the value of a loan, if a customer has to pay R956,00 back after 1 month?
- 3.3 Show that *Honest Harry's Loans* is charging a monthly interest rate of 19,5% on the loan.
4. Callan would like to buy a flat to the value of R975 000. He approaches the following 2 banks and gets their terms and conditions:

BNF
Loan conditions:
• Interest rate - 15,75%
• Length of the loan - 20 years

ASBA
Loan conditions:
• Interest rate - 15,5%
• Length of the loan - 15 years

Use the monthly repayment formula and interest rate table below, to decide which bank will offer Callan the lowest monthly repayments. Show all calculations.

Monthly Repayment = (Loan Amount ÷ 1 000) × Factor										
Term Year	Interest Rate									
	13,75%	14%	14,25%	14,5%	14,75%	15%	15,25%	15,5%	15,75%	16%
15	13,15	13,32	13,49	13,66	13,83	14	14,17	14,34	14,51	14,69
20	12,25	12,44	12,62	12,8	12,92	13,17	13,35	13,54	13,7	13,91
25	11,85	12,04	12,23	12,42	12,61	12,81	13	13,2	13,39	13,59

- 5.1 The table below shows a summary of a small short-term loan scenario. The interest rate on this loan remains the same during the whole life of the loan.

End of Month	Interest	Balance Before Payment	Payment	Closing Balance After Payment
July	R 90,00	R 1 090,00	R 225,00	R 1 000,00
Aug	R 77,85	R 942,85	R 225,00	R 865,00
Sept	R 64,61	R 782,46	R 225,00	R 717,85
Oct	R 50,17	R 607,63	R 225,00	R 557,46
Nov	R 34,44	R 417,06	R 225,00	R 382,63
Dec	R 17,29	R 209,35	R 209,35	R 0,00

- 5.1.1 What was the original loan amount?
- 5.1.2 Show that interest on this loan is being calculated at 9% per month.
- 5.1.3 Show how the Closing Balance After Payment value for October has been calculated.
- 5.1.4 Explain why the Payment value in December is different from every other month.
- 5.1.5 Calculate the real cost of this loan.

- 5.2 Now consider what would happen if the interest rate changed from 9% to 10% in September and remained at 10% until the end of the loan period.

End of Month	Interest	Balance Before Payment	Payment	Closing Balance After Payment
				R 1 000
July	R 90,00	R 1 090,00	R 225,00	R 865,00
Aug	R 77,85	R 942,85	R 225,00	R 717,85
Sept	R 71,79	R 789,64	R 225,00	R 564,64

- 5.2.1 Show that interest in September is calculated at 10%.
- 5.2.2 Complete the table to include the months of October to January.
- 5.2.3 Calculate the real cost of this loan.
- 5.2.4 Explain the effect of a 1% increase in the interest rate on this loan.

UNIT 8



INFLATION

- Inflation is a measure of change in the purchasing power of money over time.
 - Inflation therefore represents the average increase in the price of goods and services over a period of time.
 - It is usually calculated from a particular month in one year to the same month in the following year (although it can also be calculated month-to-month or as an average over a year).
 - The inflation rate is the **percentage change** (normally an increase) in the cost of goods from one year (or month) to the next.
- For example, an inflation rate of 7,4% for January 2014 means that, from January 2013 to January 2014, prices increased on average by 7,4%

- General formula for inflation rate is:

$$\text{Inflation rate} = \frac{\text{price difference}}{\text{original price}} \times 100$$

- The rate of inflation is determined by Statistics South Africa. They determine the rate of change in the price of about 2 500 goods and services on a month-to-month basis.
- Inflation represents **compound interest**.

See Module 1 Unit 7 for more examples on % increase.



RECAP! Percentage Increase



Since the inflation rate works within percentage increase, we'll recap the two methods of calculating percentage increase here:

e.g. Increase R500 by 20%

METHOD 1

Calculate the increase and add it to the original amount

$$\therefore \text{Increase} = \frac{20}{100} \times R500 = R100$$

$$\begin{aligned} \therefore \text{Total} &= \text{original amount} + \text{increase} \\ &= R500 + R100 \\ &= R600 \end{aligned}$$

METHOD 2

Multiply the original amount by (100% + % increase)

$$\begin{aligned} \therefore \text{Total} &= \text{original} \times (100\% + \% \text{ increase}) \\ &= R500 \times (100\% + 20\%) \\ &= R500 \times 120\% \\ &= R500 \times \frac{120}{100} \\ &= R600 \end{aligned}$$

NOTE!

The 100% represents the original amount.



Worked Examples

1. If 1 l of juice cost R12,49 in January 2015 and the inflation rate for January 2016 was 6,2%, calculate the cost of 2 l of juice in January 2016.

METHOD 1

$$\begin{aligned} \text{Increase} &= \frac{6,2}{100} \times R12,49 \\ &= 77c \end{aligned}$$

$$\begin{aligned} \therefore \text{Cost} &= R12,49 + 77c \\ &= R13,26 \end{aligned}$$

METHOD 2

$$\begin{aligned} \text{Cost} &= R12,49 \times (100\% + 6,2\%) \\ &= R12,49 \times \frac{106,2}{100} \\ &= R13,26 \end{aligned}$$

2. Kent evaluates the price of a new car over time. This is represented in the table below:

Year	2014	2015	2016
Price (R)	156 000	166 452	178 103,64

- 2.1 Which year showed the highest inflation rate - from 2014 to 2015 or from 2015 to 2016?

Inflation rate 2014 to 2015:

$$\begin{aligned} \text{Inflation rate} &= \frac{\text{price difference}}{\text{original price}} \times 100 \\ &= \frac{166\,452 - 156\,000}{156\,000} \times 100 \\ &= \frac{10\,452}{156\,000} \times 100 \\ &= 6,7\% \end{aligned}$$

Inflation rate 2015 to 2016:

$$\begin{aligned} \text{Inflation rate} &= \frac{\text{price difference}}{\text{original price}} \times 100 \\ &= \frac{178\,103,64 - 166\,452}{166\,452} \times 100 \\ &= 7\% \end{aligned}$$

$\therefore$ Highest inflation rate from 2015 to 2016.

- 2.2 Calculate what Kent can expect to pay in 2017 for a new car, if the inflation rate is expressed to be at 7,2%

$$\text{Increase} = \frac{7,2}{100} \times 178\,103,64 = R12\,823,46$$

$$\therefore \text{Cost of car} = R178\,103,64 + R12\,823,46 = R190\,927,10$$

3. The company ATMAN Ltd is offering their mechanics a salary increase of 10,2% p.a. The union of mechanics disagrees, and proposes a salary increase of 13,4% p.a. The average annual salary of a mechanic working at ATMAN Ltd is R141 000 p.a.

- 3.1 What is the difference in salary per month, between what the company is offering and the union is proposing?

$$\text{Monthly salary} = R141\,000 \div 12 = R11\,750$$

$$\therefore \text{Company's increase} = \frac{10,2}{100} \times R11\,750 = R1\,198,50$$

$$\begin{aligned} \therefore \text{Company's proposed salary} &= R11\,750 + R1\,198,50 \\ &= R12\,948,50 \end{aligned}$$

$$\text{Union's increase} = \frac{13,4}{100} \times R11\,750 = R1\,574,50$$

$$\therefore \text{Union's proposed salary} = R11\,750 + R1\,574,50 = R13\,324,50$$

$$\therefore \text{Difference in proposed salary} = R13\,324,50 - R12\,948,50 = R376$$

- 3.2 Determine the annual salary of a mechanic, if the company and union negotiations resulted in an increase that met both parties half-way.

$$\therefore \text{Half-way salary \% increase} = \frac{10,2 + 13,4}{2}$$

$$= 11,8\% \text{ p.a.}$$

$$\therefore \text{Annual salary increase} = \frac{11,8}{100} \times R141\,000$$

$$= R16\,638$$

$$\therefore \text{Total annual salary} = R141\,000 + R16\,638$$

$$= R157\,638$$

4. If 200 g of nuts cost R14,20 in January 2014 and the inflation rate for January 2015 was -5,3%, calculate the cost of 200 g of nuts in January 2015.

$$\text{Decrease} = \frac{5,3}{100} \times R14,20$$

$$= R0,75$$

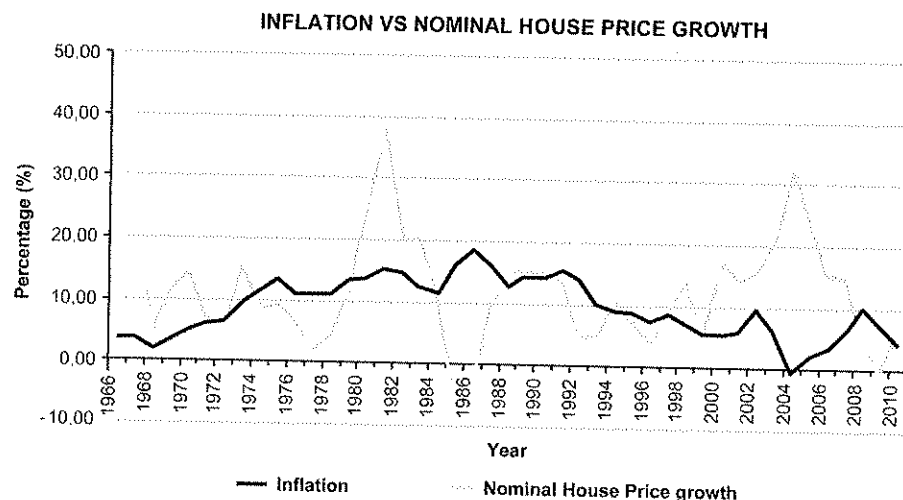
$$\therefore \text{Price} = R14,20 - R0,75$$

$$= R13,45$$

A **negative inflation** rate is known as **depreciation** and results in a decrease in the price of goods.



5. Study the Graph of Inflation vs Nominal House Price Growth below and answer the following questions:



- 5.1 Estimate the nominal house price growth in 1970. 15%
- 5.2 Estimate the inflation rate in 1993. 10%

- 5.3 During which years did the nominal house price growth remain constant and at what percentage?

1982 - 1983 at 20%

- 5.4 Explain what the graph is showing in 1985; with respect to the nominal house price growth.

The nominal house price growth percentage dropped to below 0%; to approximately -7,5%. This implies that there wasn't any growth at all in the house price, but rather that the house prices were losing value.

Test Your Understanding



- A Mercedes VITO could be bought, brand new, for R307 230 in 2013. In 2014 the cost of a new VITO was R321 993 and in 2015 it was R335 730.
 - Which year showed the highest inflation rate?
 - Calculate the cost of a new VITO in 2016, if inflation was predicted to be at 5,4%.
- Although the shop is new, research has shown that in order to keep it looking professional, it will have to be renovated in 3 years' time. Sibusiso got some quotes and discovered that it would cost them approximately R30 000,00 to do the renovations according to this year's prices. Sibusiso has a fixed deposit that will mature in 3 years' time, at a value of R35 000. Will Sibusiso's fixed deposit investment be able to cover the costs of the shop renovations in 3 years' time, if inflation remains constant at 6,8% p.a.?
- The Habib family's water and electricity bill for April 2014 was R1 050. If the inflation rate for April 2015 was -2,3%, what could the Habib's expect to pay for water and electricity in April 2015?
- The average price of PACSA Food Basket provided on the following page, appeared in a newspaper on 27 March 2014.

PACSA
*Pietermaritzburg Agency for
Community Social Action*

 - Answer the following questions.
 - By how much did the price of cabbage change from January 2014 to February 2014?
 - By how much did the price of chicken change from January 2014 to February 2014?
 - By what percentage did the price of maize meal increase from January 2014 to February 2014? If necessary, round off your answer to one decimal place.

4.1.4 If food price inflation represents the increase in the price of food, calculate the food price inflation of the whole basket of groceries represented in the extract alongside from January 2014 to February 2014.

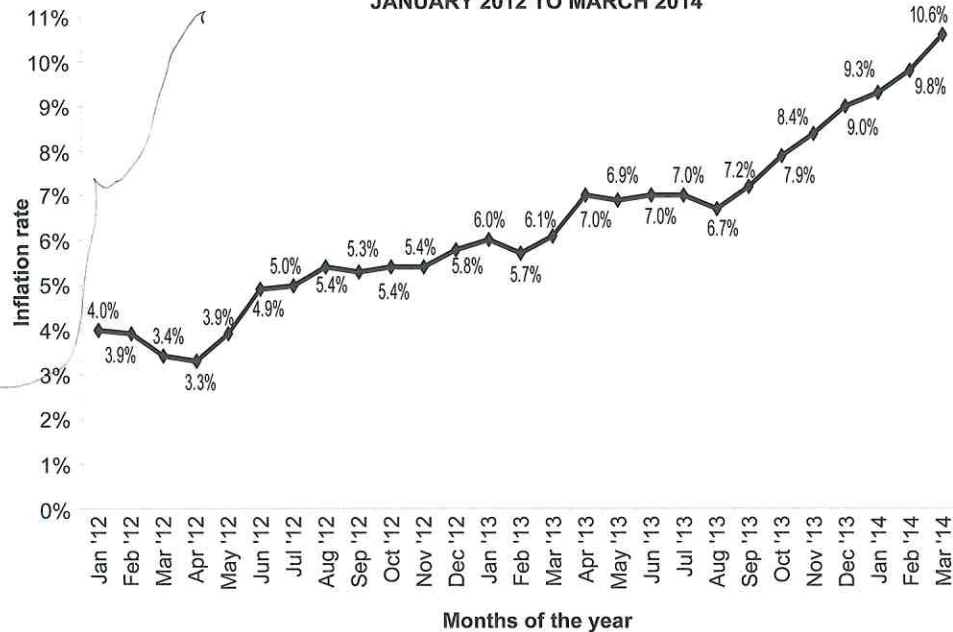
4.1.5 If the inflation rate from February 2014 to March 2014 was 1,6%, determine how much Solly might have paid for a 12,5 kg packet of flour in March 2014.

AVERAGE PRICE OF PACSA FOOD BASKET

Commodities	Jan '14	Feb '14
Maize Meal 50 kg	162,23	212,65
Flour 12,5 kg	64,72	72,24
Rice 10 kg	40,47	44,47
Chicken 2 kg	28,23	24,92
Potatoes 10 kg	27,73	20,23
Cooking oil 5 L	59,48	71,48
Beans 5 kg	54,48	48,22
Tomatoes 1 kg	6,28	7,77
Onions 1 kg	6,72	6,64
Sugar 12,5 kg	68,72	68,84
Tea bags 250 g	8,48	9,11
Coffee 500 g	30,23	31,22
Milk 2 L	18,11	16,84
Bread 600 g (x 20 loaves)	126,50	117,65
Salt 500 g	1,98	3,17
Eggs 30 tray	25,73	23,28
Cabbage 5 pcs	23,68	27,46
Margarine 500 g	8,96	9,86
Soup (NO1) 400 g	9,64	9,30
Yeast p/sachet 10 g	1,72	1,42
Stock 500 g	7,93	6,73
TOTAL	782,02	833,50

4.2 The graph below shows the annual average inflation rate over the period January 2012 to March 2014.

ANNUAL AVERAGE INFLATION RATE -
JANUARY 2012 TO MARCH 2014



4.2.1 What does it mean that the inflation rate in March 2013 was 6,1%?

4.2.2 In which year and month was the inflation rate the lowest and then the highest?

4.2.3 By what percentage did prices increase from October 2012 to October 2013?

UNIT



TAXATION

VAT (Value Added Tax)



- VAT is a charge added to the purchase price of certain goods and services. It is a **form of taxation** and the money is paid to the government.
- In South Africa, **VAT** is currently **charged at a rate of 14%**.
- Some goods and services that are regarded as basic necessities are exempted from VAT, e.g. maize meal, hen's eggs, fresh fruit and vegetables, milk, rice, paraffin, petrol, diesel, brown bread etc.

VAT Calculations



- The **VAT exclusive amount** (VAT excl.) is always equivalent to **100%**.
- VAT** is always equivalent to **14%** of the VAT-exclusive amount.
- The **VAT inclusive amount** (VAT incl.) is always equivalent to **114%** of the VAT exclusive amount (i.e. VAT excl. amount of 100% + VAT of 14%).

Worked Examples



1. **Calculate 14% VAT:** e.g. Determine VAT on R550.

$$\begin{aligned} \text{VAT} &= \frac{14}{100} \times R550 \\ &= R77 \end{aligned}$$

2. Calculate the VAT incl. amount on a VAT excl. item:
e.g. Determine the VAT-incl. price if a R550 item is VAT-excl.

METHOD 1

Calculate the VAT and add it to the VAT excl. price

$$\begin{aligned}\therefore \text{VAT} &= \frac{14}{100} \times R550 \\ &= R77\end{aligned}$$

$$\begin{aligned}\therefore \text{VAT incl. price} &= \text{VAT excl. price} + \text{VAT} \\ &= R550 + R77 \\ &= R627\end{aligned}$$

METHOD 2

Multiply the VAT excl. price by (100% + VAT of 14%)

$$\begin{aligned}R550 \times 114\% \\ &= R550 \times \frac{114}{100} \\ &= R627\end{aligned}$$

3. Calculate the VAT excl. amount from the VAT incl. amount.
e.g. Determine the VAT excl. price of a R627 VAT incl. priced item.

$$\text{Since VAT incl. price} = \text{VAT excl. price} \times 114\%$$

manipulate the formula

$$\begin{aligned}\therefore \text{VAT excl. price} &= \text{VAT incl. price} \div 114\% \\ &= R627 \div \frac{114}{100} \\ &= R550\end{aligned}$$

REMEMBER! You can't just calculate VAT and subtract it from the VAT incl. amount - it won't work!

SUMMARY

Given the selling price of an item 'before VAT', i.e. given the **VAT exclusive price**:

- ▶ To find the **VAT** (of 14%):
multiply the price by 0,14 - i.e. VAT exclusive amount $\times$ 0,14
- ▶ To find the **VAT inclusive price** (i.e. $100\% + 14\% = 114\%$):
multiply the price by 1,14 - i.e. VAT exclusive amount $\times$ 1,14

Given the price including VAT, i.e. given the **VAT inclusive price**:

- ▶ To find the **VAT exclusive price**:
divide by 1,14 - i.e. VAT inclusive price $\div$ 1,14

UIF (Unemployment Insurance Fund)



- The Unemployment Insurance Fund provides short-term relief to employees, when they become unemployed or are unable to work due to maternity or illness.
- All employees have to contribute **1% of their gross income**; while all employers also have to contribute 1% of their employee's gross income (up to a maximum ceiling).
- When a person becomes unemployed, he/she can claim back a portion of his/her UIF contributions, based on a predetermined formula.
- It is the responsibility of the employer to make the deductions and pay the money into the UIF.

Worked Example



Matt's gross monthly income is R42 315. His monthly deductions include medical aid (R4 100), pension (R2 800) and a retirement annuity (R1 200).

1. Calculate Matt's monthly UIF contribution.

$$\begin{aligned}\text{UIF} &= \frac{1}{100} \times R42\,315 \\ &= R423,15\end{aligned}$$

REMEMBER! UIF is calculated at 1% of gross income
($\therefore$ the deductions are not relevant!)

2. Determine the total UIF contribution that Matt and his employer makes per annum.

$$\begin{aligned}\text{UIF of Matt and Employer} &= R423,15 \times 2 \\ &= R846,30\end{aligned}$$

Both employee and employer have to contribute 1%

$$\begin{aligned}\therefore \text{Annual total UIF contribution} &= R846,30 \times 12 \\ &= R10\,155,60\end{aligned}$$

Income Tax



- Every person who earns an income needs to declare it to the South African Revenue Services (SARS) and pay tax on their income.
- The income that is taxed, called the **taxable income**, is calculated by taking your **gross income** and **subtracting** any **non-taxable deductions**, such as retirement annuities.

Income Tax (continued)



- Income tax is always **calculated on your annual taxable income**; using the tax tables issued by SARS

e.g. **TAX TABLE FOR THE YEAR ENDING 28 FEBRUARY 2015**

Tax Bracket	Taxable Income (R)	Rate of Tax
1	0 – 174 550	18% of taxable income
2	174 551 – 272 700	31 419 + 25% of taxable income above 174 550
3	272 701 – 377 450	55 957 + 30% of taxable income above 272 700
4	377 451 – 528 000	87 832 + 35% of taxable income above 377 450
5	528 001 – 673 100	140 074 + 38% of taxable income above 528 000
6	673 101 and above	195 212 + 40% of taxable income above 673 100

- The government provides an element of **tax relief** to citizens, the form of **tax rebates**, which is a set amount of money that is **subtracted after your annual income tax has been calculated**.
- Every person qualifies for the primary rebate; while older people are granted additional rebates (secondary and tertiary rebates); which are subtracted from their annual income tax.

e.g. **TAX REBATES FOR THE YEAR ENDING 28 FEBRUARY 2015**

Rebates	
Primary	R12 726
Secondary (Persons 65 and older)	R7 110
Tertiary (Persons 75 and older)	R2 367

- Net pay is the amount of money a person receives in their bank account, after deductions (e.g. income tax, medical aid, UIF, pension etc.) have been made from their gross income.

$$\therefore \text{Net Pay} = \text{gross income} - \text{deductions}$$

Worked Examples



- Solly is a 47 year-old baker for High Rise Bakery, that provides him with the following payslip:

High Rise Bakery			Pay Date: 25 March 2015
Employee Name: Solly van Heerden	Occupation: Specialist Baker	ID No: 761106912024	
Address: Flat 3C, Ladswood Heights	Bank Account No: 1745193208	Tax No: ZA123718	
Incomes	Amount	Deductions	Amount
Basic Salary	R18 090,00	Income Tax	A
		Medical Aid	R772,00
		Pension	R415,00
		UIF	B
Gross Income	R18 090,00	Total Deductions	C
		Net Pay	D

- 1.1 Use the income tax table and tax rebate shown alongside to calculate Solly's monthly income tax (A).

$$\begin{aligned} \text{Annual salary} &= R18\,090 \times 12 \\ &= R217\,080 \end{aligned}$$

$$\therefore \text{Taxable income bracket 2: } R174\,551 - R272\,700$$

$$\begin{aligned} \therefore \text{Rate of tax} &= R31\,419 + 25\% \text{ of taxable income above } R174\,550 \\ &= R31\,419 + 25\% \text{ of } (R217\,080 - R174\,550) \\ &= R31\,419 + 25\% \text{ of } R42\,530 \\ &= R31\,419 + \frac{25}{100} \times R42\,530 \\ &= R31\,419 + R10\,632,50 \\ &= R42\,051,50 \text{ per annum} \end{aligned}$$

$$\begin{aligned} \therefore \text{Annual income tax} &= \text{rate of tax} - \text{tax rebate (primary)} \\ &= R42\,051,50 - R12\,726 \\ &= R29\,325,50 \text{ per annum} \end{aligned}$$

$$\begin{aligned} \therefore \text{Monthly income tax} &= R29\,325,50 \div 12 \\ A &= R2\,443,79 \end{aligned}$$

1.2 Calculate Solly's UIF contribution (B)

UIF = 1% of gross income

$$= \frac{1}{100} \times R18\,090$$

$$B = R180,90$$

1.3 Determine Solly's Total Deductions (C) on his payslip.

$$\begin{aligned} \text{Total Deductions} &= \text{income tax} + \text{medical aid} + \text{pension} + \text{UIF} \\ &= R2\,443,79 + R772,00 + R415,00 + R180,90 \end{aligned}$$

$$C = R3\,811,69$$

1.4 Calculate Solly's Net Pay (D).

$$\begin{aligned} \text{Net Pay} &= \text{gross Income} - \text{total Deductions} \\ &= R18\,090 - R3\,811,69 \end{aligned}$$

$$D = R14\,278,31$$

2. Solly's sister is a cake decorator at High Rise Bakery. She is 65 years old earns a gross salary of R22 734 p.m. Calculate her monthly income tax.

$$\text{Annual salary} = R22\,734 \times 12 = R272\,808$$

$$\therefore \text{Taxable income bracket 3: } R272\,701 - R377\,450$$

$$\therefore \text{Rate of tax} = R55\,957 + 30\% \text{ of taxable income above } R272\,700$$

$$= R55\,957 + 30\% \text{ of } (R272\,808 - R272\,700)$$

$$= R55\,957 + 30\% \text{ of } R108$$

$$= R55\,957 + \frac{30}{100} \text{ of } R108$$

$$= R55\,957 + R32,40$$

$$= R55\,989,40$$

$$\therefore \text{Annual income tax} = \text{rate of tax} - \text{primary tax rebate} - \text{secondary tax rebate}$$

$$= R55\,989,40 - R12\,726 - R7\,110$$

$$= R36\,153,40 \text{ p.a.}$$

$$\therefore \text{Monthly income tax}$$

$$= R36\,153,40 \div 12$$

$$= R3\,012,78$$

**REMEMBER!**

Solly's sister is 65 years old; so she is also granted the secondary tax rebate, in addition to the primary rebate.

3. Study the IRP5 income tax certificate provided (see p. 103, 104) and answer the following questions:

3.1 What is the name of the employee?

Allen Tshabile

3.2 Who does the employee work for?

P × B South African (Pty) Ltd

3.3 Calculate the total income received.

$$\text{Total income} = R25\,000 + R4\,000 = R29\,000$$

3.4 How much did the employee Pay As You Earn? R3 590,38

3.5 Show how the total UIF contribution amount was calculated.

UIF = 1% of gross income

$$= \frac{1}{100} \times R29\,000$$

$$= R290$$

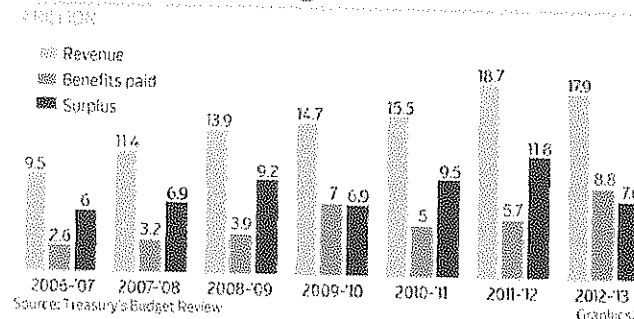
Since the employee and employer both have to contribute 1%;
then $R290 \times 2 = R580$.

Test Your Understanding



1. An I-pod shuffle costs R820,99. Calculate how much VAT you are paying on this item.
2. Determine the VAT-inclusive price if a builder quotes R12 700 excl. VAT to build a boundary wall.
3. Calculate the VAT-exclusive price on a Plasma TV that costs R9 899 VAT-inclusive.
4. Peter buys candles at R6,59 per candle, exclusive of VAT. He would like to buy a case of candles, which houses 120 per case. Additionally, Peter receives a 5% discount if he purchases a case of candles. Calculate the VAT-inclusive amount that Peter will pay for a case of candles.
5. Study the graph below indicating the UIF fund statistics; and answer the questions that follow.

Unemployment Insurance Fund –
what comes in, what goes out



- 5.1 What was the UIF revenue in 2007 - 2008?
- 5.2 Determine the surplus in the UIF in 2009 - 2010?
- 5.3 How much was paid out in benefits in 2012 - 2013?
- 5.4 What percentage of the revenue was paid in benefits in 2006 - 2007?
- 5.5 In which year(s) was the benefits paid out greater than the surplus?

6. Mr Riet is a secretary at a school and currently earns a gross monthly salary of R7 986,50.

The following amounts are deducted from his gross monthly salary:

Union Membership Fee: R35

Pension Fund: 7,5% of gross salary

PAYE (Tax): (Gross salary - R4 750) $\times$ 18%

UIF: 1% of gross salary

Medical Aid: $\frac{1}{3}$ of the total medical aid subscription due, as shown in TABLE 1 below.

TABLE 1: MEDICAL AID MEMBERSHIP SUBSCRIPTION COSTS

Gross Monthly Salary	SUBSCRIPTION COSTS		
	Main Member	Wife / Partner / Parent	Each Child
R0 – R 8 000	R1 152	R816	R424
R8 001 – R10 500	R1 256	R900	R468
More than R10 500	R1 396	R992	R516

- 6.1 Mr Riet, his wife and two children belong to a medical aid fund. Use TABLE 1 to calculate his monthly contribution to the medical aid fund.
- 6.2 Calculate the total deductions from Mr Riet's monthly salary. Use the table below.

MONTHLY DEDUCTIONS	
Union membership fee	
Pension fund	
PAYE	
UIF	
Medical aid contribution	
TOTAL	

- 6.3 Hence, calculate Mr Riet's net annual salary.

- 7.1 The table below shows the income tax brackets for the 2016/2017 financial year. (Source: South African Revenue Service, 2016 Budget Tax Guide, p.2)

Income Tax Brackets 2008/2009

Income Tax for Individuals

Tax Bracket	Taxable Income (R)	Rates of Tax (R)
1	0 - 188 000	18% of each R1
2	188 001 - 293 600	33 840 + 26% of the amount above 188 000
3	293 601 - 406 400	61 296 + 31% of the amount above 293 600
4	406 401 - 550 100	96 264 + 36% of the amount above 406 400
5	550 101 - 701 300	147 996 + 39% of the amount above 550 100
6	701 301 and above	206 964 + 41% of the amount above 701 300

Tax Rebates

Rebates	R
Primary	13 500
Secondary (Persons 65 and older)	7 407

Tax Threshold

Age	Tax Threshold (R)
Below age 65	75 000
Age 65 and over	116 150

- 7.1.1 Into which tax bracket does a person who earns R232 520,00 per year fall?
- 7.1.2 Into which tax bracket does a person who earns R17 100,00 per month fall?
- 7.1.3 What tax rebate will a 52 year old person receive?
- 7.1.4 How old do you have to be to qualify for the R116 150,00 tax threshold?



Tax threshold refers to the maximum amount of money that a person can earn per year without having to pay tax.

- 7.2 Solly is a 28 year-old who earns R15 090 per month.
- 7.2.1 Use the tax table for 2016/2017 above to calculate Solly's monthly income tax contribution.
- 7.2.2 If Solly is given a 13th cheque at the end of every year, how will this affect the total amount of tax that he has to pay on his salary?
- 7.2.3 If Solly is given a 8% increase on his gross salary, how much of the increase will he actually receive after tax every month? (For this question, ignore the 13th cheque and work with 12 salaries for the year.)



Transaction Year (CCYY)

2013

Year of Assessment (CCYY)

2013

Period of Reconciliation (CCYYMM)

201302

Employee Income Tax Certificate

IRP5/IT3(a)

Certificate No.

7710947223207302VIPPO007300107

Type of Certificate

IRP5

Surname / Trading Name

TSHABILE

IRPINF01

First Two Names

ALLEN

Home Tel No.

Initials

A

Nature of Person

A

Date of Birth (CCYYMMDD)

19860516

ID No.

8605165897088

Bus. Tel No.

031582754

Passport No.

Passport Country of Issue (e.g. South Africa = ZAF)

Income Tax Ref No.

8294567141

Fax No.

Contact Email

ALLEN@PXB.CO.ZA

Cell No.

Unit No.

Complex (if applicable) Street/Name of Farm

MALUTI AVENUE

PAYE Ref No.

7710947223

Street No.

10

SDL Ref No.

L710947223

Suburb / District

QUEENSPARK

UIF Ref No.

U710947223

City / Town

QUEENSPARK

Postal Code

9909

Mark here with an X if same as above or complete your Postal Address

X

Postal Code

Account Holder Relationship:

Own

X

Joint

3rd Party

Account Type:

Cheque / Current Acc.

Bond Acc.

Savings Acc.

X

Credit Card Acc.

Transmission Acc.

Subscription Share Acc.

Mark here with an X if not paid electronically or if foreign bank account

Account No.

65348999750

Branch No.

250655

Bank Name

FNB

Branch Name

UNIVERSAL

Account Holder Name

A TSHABILE

IRP5T3a

L 0

FV 2013.01.00 SV 1202

CT 03

NO 7710747223



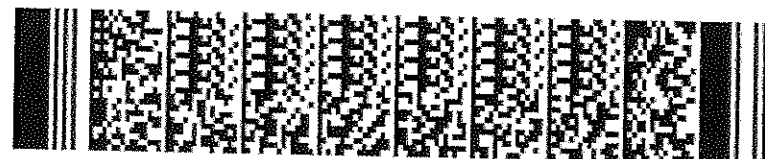
P

Y

2014

b91c9121-0a17-4b26-a09d-d5980eb532db

001/002



ITCIRA01

Trading or Other Name

PXB SOUTH AFRICA (PTY) LTD

Unit No.

4

Complex (if applicable)

WATTLE OFFICE PARK

Street No.

9

Street / Name of Farm

HENRYFORD DRIVE

Suburb / District

FOURWAYS

City / Town

JOHANNESBURG

Postal Code

2055

Amount

Source Code

R	25000	3601
R	4000	3701
R		
R		
R		
R		
R		
R		
R		
R		
R		
R		
R		
R		

Non-Taxable Income

R 3696

Gross Retirement Funding Income

R 3697

Gross Non-Retirement Funding Income

R 30000 3698

Amount

Source Code

R		
R		
R		
R		
R		
R		
R		
R		
R		
R		
R		

Total Deductions / Contributions

R 4497

SITE

R 4101

PAYE

R 3590 38 4102

PAYE on Lump Sum Benefit

R 4115

Employee and Employer UIF Contribution

R 580 00 4141

Employer SDL Contribution

R 290 00 4142

Total Tax, SDL and UIF

R 4460 38 4149

Medical Scheme Fees Tax Credit

R 4116

Reason for Non-Deduction of Employees' Tax

4150

Periods in Year of Assessment

365 0000

No. of Periods Worked

059 0000

Period Employed From (CCYYMMDD)

20130101

Period Employed To (CCYYMMDD)

20130228

Directive No.

Directive No.

Directive No.

IRP5T3a L 0 FV 2013.01.00 SV 1202 CT 03 NO 7710747223



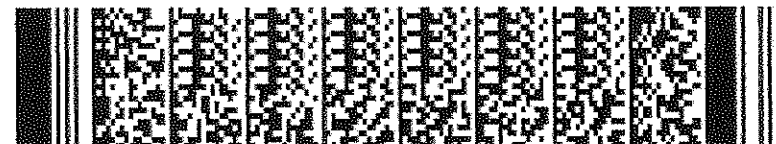
P

Y

2014

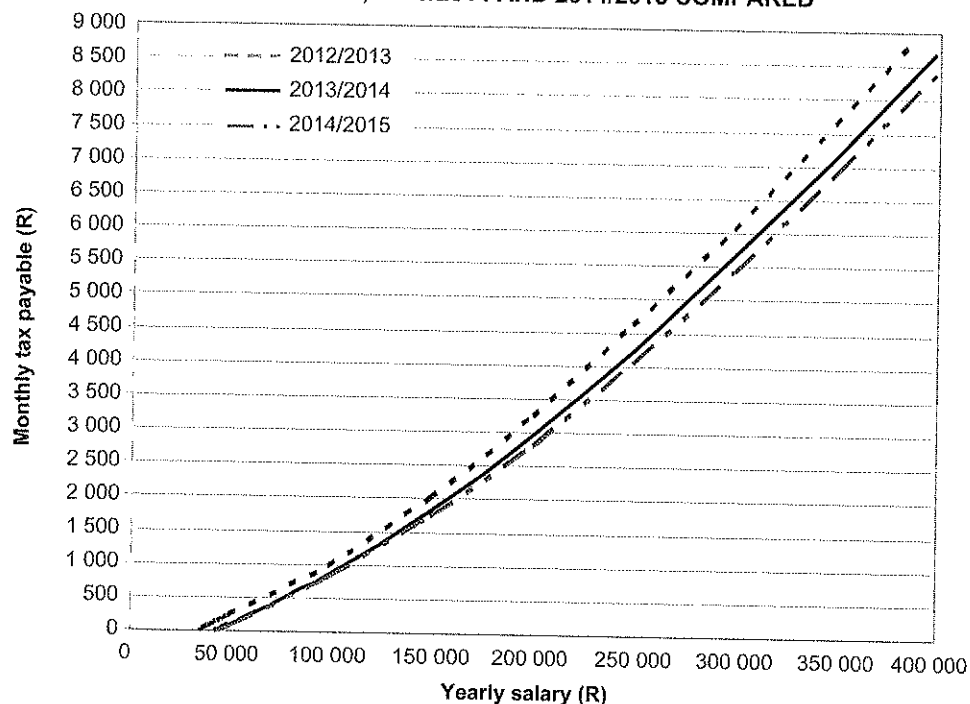
b91c9121-0a17-4b26-a09d-d5980eb532db

002/002



8. The graph below shows a comparison between the income tax payable on various salaries (for people under 65 years old) in the 2012/2013, 2013/2014 and 2014/2015 financial years.

MONTHLY INCOME TAX PAYABLE ON VARIOUS YEARLY SALARIES -
2012/2013, 2013/2014 AND 2014/2015 COMPARED



- 8.1 Explain why the graphs do not start at R0,00 on the horizontal axis.
- 8.2 Approximately how much less tax will a person who earns R270 000,00 per year be paying per month in 2014/2015 than in 2012/2013 (assuming their income remained the same).
- 8.3.1 Describe the trend/changes in the amount of tax that taxpayers have been taxed on their income since 2012/2013.
- 8.3.2 Why do you think there is this trend in the amount of tax that taxpayers have had to pay on their income since 2012/2013?

UNIT

10

EXCHANGE RATES

- Each country buys and sells goods and services within their own country, using a particular **currency**, e.g. **rands** in South Africa; **dollars** in USA; **pounds** in the United Kingdom (UK); **euros** in the European Union (EU); etc.
- A **strong currency** means that the currency has a **higher value** or **greater worth** than another currency:
 - it is an indication of a stable economic market
 - ideally, one would like to export goods when you have a strong currency; as you would get a higher price for the goods.
- A **weak currency** means that the currency has a **lower value** or a **lesser worth** than another currency:
 - it is an indication of an unstable or volatile economic market
 - ideally, one would like to import goods from a country with a weak currency; as you would get more product for your unit currency.
- An **exchange rate** is used when countries trade; as it **specifies** how much one currency is **worth** in terms of the other:
 - e.g. \$1 = R10 ... i.e. for every dollar, one would need ten rand
 - R1 = \$0,10 ... i.e. for every rand, one would need a tenth of a dollar.
 In both cases, the dollar is the stronger currency and the rand is weaker.
- Exchange rates change on a daily basis; due to market trends.
- When travelling, it is difficult to know how much of your own currency you need to exchange for the foreign currency; as you don't know if 100 of your own currency can buy 1 foreign cold drink or 1 foreign car! Therefore, be sure to investigate the **buying power** of the foreign currency, i.e. **the value of the currency in relation to the cost of living** in the foreign country.

Worked Examples

Given the following exchange rates, answer the questions below:

£1 : \$1,61

\$1 : €0,79

1. When comparing the pound to the dollar, which currency is the strongest?

pound

For every pound, you get more than one dollar ∴ the pound is stronger



2. What happens to the cost of British goods, if the exchange rate strengthens against the pound?

The British goods will become cheaper if the dollar strengthens.

3. When comparing the dollar to the euro, which currency is the weakest?
dollar



For every €0,79 (79 cents of a euro), you need to pay \$1
∴ the dollar is weaker

4. What happens to the cost of European goods, if the dollar weakens against the euro?

The European goods become more expensive.

Exchange Rate Calculations



- Exchange rates are written in **ratio form**

i.e. **unit currency : multiple currency**

e.g. **\$1 : R11**

Exchange Rate = 11

Unit currency
= 1 × currency

Multiple currency = any
number except 1 of currency.

- When doing exchange rate calculations, determine whether they are asking for how many of the unit currency or how many of the multiple currency; given a specific value.

- Use the following general formulae to calculate:

i) **Unit currency = multiple currency ÷ rate**

e.g. How many **dollars** do I get if I have **R50**?

Unit currency?

Specific value of multiple currency.

∴ Unit currency = multiple currency ÷ rate

∴ Dollars = rands ÷ rate

= R50 ÷ 11 = \$4,55

ii) **Multiple currency = unit currency × rate**

e.g. How many **rands** do I get for **\$50**?

Multiple currency?

Specific value of unit currency.

∴ Multiple currency = unit currency × rate

∴ Rands = dollars × rate

= \$50 × 11 = R550

Worked Examples



1. Edmond cannot decide on his holiday location, so he looks at the exchange rate table below:



- R1 : Thai baht 2,93
- Chinese yen 1 : R1,81
- Australian dollar 1 : R9,69
- R1 : Mexican peso 1,21

Show by means of calculations, which location Edmond should choose, to get the greatest number of currency to that specific location for R25 000.

- R1 : Thai baht 2,93

∴ Multiple currency = unit currency × rate

∴ Thai baht = rands × rate

= R25 000 × 2,93

= Thai baht 73 250

unit currency : multiple currency
Rand : Thai Baht
with exchange rate = 2,93

- Chinese yuan 1 : R1,81

∴ Unit currency = multiple currency ÷ rate

∴ Chinese yuan = R25 000 ÷ 1,81

= Chinese yuan 13 812,15

unit currency : multiple currency
Chinese yuan : Rand
with exchange rate = 1,81

- Australian dollar 1 : R9,69

∴ Unit currency = multiple currency ÷ rate

∴ Australian dollar = rands ÷ rate

= R25 000 ÷ 9,69

= Australian dollar 2 579,98

unit currency : multiple currency
Australian dollar : Rand
with exchange rate = 9,69

- R1 : Mexican peso 1,21

∴ Multiple currency = unit currency × rate

∴ Mexican peso = rands × rate

= R25 000 × 1,21 = Mexican peso 30 250

unit currency : multiple currency
Rand : Mexican peso
with exchange rate = 1,21

∴ Edmond should go to Thailand, as the Thai baht has the greatest number of currency to his rands.



2. Tammy is travelling from the USA to Brazil. Calculate how many Brazilian real Tammy will get for \$100; using the exchange rates below:

1 Argentine peso = 0,12 US dollars

1 Argentine peso = 0,28 Brazilian real

unit currency : multiple currency
Argentine peso : \$
with exchange rate = 0,12

Unit currency = multiple currency ÷ rate

Argentine peso = dollar ÷ rate

= \$100 ÷ 0,12

= 833,33 Argentine peso

Multiple currency = unit currency × rate

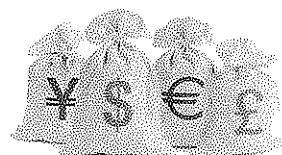
Brazilian real = Argentine peso × rate

= 833,33 × 0,28

= 233,33 Brazilian real

∴ \$100 = 233,33 Brazilian real

unit currency : multiple currency
Argentine peso : Brazilian real
with exchange rate = 0,28



Exchange Rate Estimations

- It is often far easier and more practical, to estimate the exchange rate and thereafter, convert between the currencies.
- Estimation allows one to do basic exchange rate calculations without the need for a calculator.



More examples on Estimation in
Module 1: Unit 2 (see p. 2).



Worked Example

Neil has just finished Matric and would like to take a short holiday in London, before starting Varsity. He plans to stay for 9 days and 8 nights at a local backpackers.

Neil plans to have breakfast at the Backpackers and then buy lunch and dinner while he is out exploring London.

He will use the London Tube for transport and would like to see a few local attractions. Neil finds the following prices on the internet:

3-Day Tube Travel Card	£21,20
London Eye Back Packers	£28,30 per night, incl. breakfast
Average Meal Cost	£8,50
Madame Tussauds	£21,50
London Eye	£18,10
Thames River Cruise	£19,00
West End Theatre Show	£57,00

1. Use the information above, together with the exchange rate of £1 : R17,56, to estimate the total cost in Rand of Neil's London holiday.

- Tube Travel Costs ≈ (9 days ÷ 3 day-pass ticket) × £20
≈ 3 three-day tube travel cards × £20
≈ £60
- Back Packers Costs ≈ 8 nights × £30 ≈ £240
- Meal Costs ≈ (2 meals per day × 9 days) × £10
≈ 18 meals × £10
≈ £180

REMEMBER! Breakfast is included in the costs of the accommodation at the backpackers, so Neil only has to buy lunch and dinner per day.

- Madame Tussauds ≈ £20
- London Eye ≈ £20
- Thames River Cruise ≈ £20
- West End Theatre Show ≈ £60

∴ Total estimated cost (£) ≈ £60 + £240 + £180 + £20 + £20 + £20 + £60
≈ £600

∴ Total estimated cost (R):

Multiple currency = unit currency × rate

$$\text{Rand} \approx \text{£}600 \times 20$$

$$\approx \text{R}12\,000$$

unit currency : multiple currency

£ : R

with exchange rate ≈ 20

2. Now calculate the difference in the estimated total cost of Neil's holiday in London, to the actual costs.

- Tube Travel Costs = 3 three-day tube travel cards × £21,20 = £63,60
- Back Packers Costs = 8 nights × £28,30 = £226,40
- Meal Costs = 18 meals × £8,50 = £153
- Madame Tussauds = £21,50
- London Eye = £18,10
- Thames River Cruise = £19,00
- West End Theatre Show = £57,00



$$\begin{aligned} \therefore \text{Total actual cost (£)} &= \text{£}63,60 + \text{£}226,40 + \text{£}153 + \text{£}21,50 + \text{£}18,10 \\ &\quad + \text{£}19 + \text{£}57,00 \\ &= \text{£}558,60 \end{aligned}$$

∴ Total actual cost (R):

Multiple currency = unit currency × rate

$$\text{Rands} = \text{£}558,60 \times 17,56$$

$$= \text{R}9\,809,02$$

∴ Difference in Estimated and Actual Cost

$$= \text{R}12\,000 - \text{R}9\,809,02$$

$$= \text{R}2\,190,98$$

Test Your Understanding



1. Convert DZD 110,35 (Algerian dinar) to rand, if R1 : DZD 9,48
2. The table alongside shows au pairs' salaries in 3 countries.

1 month = 4 weeks

AU PAIRS' SALARIES IN THREE COUNTRIES

Country	Salary	Exchange Rates	
United Kingdom	£55 per week	Rand/British Pound (R/£)	14,66
USA	US\$140 per week	Rand/US Dollar (R/US\$)	7,42
Netherlands	€340 per month	Rand/Euro (R/€)	9,86

- 2.1 You want to work as an au pair. Which country will pay you the best salary per week? Show weekly income in rand to justify your answer.
- 2.2 Which is the strongest currency to the rand? Give a reason for your answer.
- 2.3 If you were living in South Africa; which country would it be best to import goods from? Give a reason for your answer.
3. A South African couple visited Botswana and Zambia. The table below shows the exchange rate between the currencies of the two countries and the South African Rand (ZAR):

SOUTH AFRICAN RAND TO FOREIGN CURRENCY

R1,00 (ZAR) = 0,95 Botswana pula (BWP)

R1,00 (ZAR) = 681,07 Zambian kwacha (ZMK)

- 3.1 The couple budgeted to pay R20,00 per person for lunch. How much is this amount in Botswana pula?
- 3.2 The accommodation in Zambia cost 360 286 ZMK per couple per day. They paid a deposit of 1 021 605 ZMK to secure their accommodation. Dinner costs 85 134 ZMK per person and they both had dinner at the hotel restaurant four times.
Calculate the total amount they will pay in rand at the end of their ten-day (9 nights) stay at the hotel in Zambia.
- 3.3 Is the Botswana Pula or the Zambian kwacha stronger against the rand?
4. Amy would like to import a sewing machine and accessories from Britain.
She is quoted the following:
Over-locker Sewing Machine: £510,99
Needles: £9,50 for a pack of 5
Thread: £4,85 per bobbin
- 4.1 Estimate the total cost in rand; if Amy buys the over-locker sewing machine; 10 packs of needles and 50 bobbins of thread; given the exchange rate of £1 : R18,42
- 4.2 If Amy budgeted R15 000 for the sewing machine and accessories, would she be able to afford to import it? Show all accurate calculations.

4 MEASUREMENT

UNIT

1

MEASUREMENT SYSTEMS

The Metric System

- The metric system which was first developed during the French Revolution (end of the 18th century) is a **decimal based system of measurement**.
- The metric system is referred to as the **International System of Units**, abbreviated to **SI**, and has been officially adopted as the system of Measurement around the world (with the exception of the USA, Burma and Liberia).
- A **general set of prefixes** is used when really large or really small measurements need to be indicated. The role of the prefix is to indicate the **factor of 10 by which the unit needs to be multiplied or divided**, e.g. **kilo** indicates 'multiply by 1 000'; **milli** indicates 'divide by 1 000', etc.
- Examples of the metric system units of measurement include: **kilogram, centimetre, millimetre, seconds**, etc.

NOTE: You will be working with the **metric system** and are expected to learn the conversions and work with them **from memory**.

The Imperial System

- The imperial system was first defined in the British Empire in 1824 and is still used in the USA (whereas most other countries use the metric system).
- Examples of the imperial system units of measurement include: **gallons, pints, yards, pounds, inches**, etc.

NOTE: You will **not** be expected to learn these units **from memory**, but rather to be able to work with them from a conversion table.

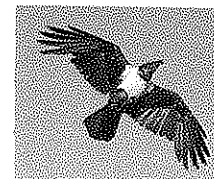
UNIT

2

MEASURING LENGTH AND DISTANCE

Definition and Units

- **Length:** The measurement of an object from one end to the other in one dimension (usually the longest dimension)
e.g. the length of a soccer field is between 100 - 110 m.
- **Distance:** The physical length between two points,
e.g. the distance between Cape Town and New York is approximately 12 557 km (as the crow flies).
- The **basic unit** of length is the **metre (m)**.
Other units, such as kilometre (km), centimetre (cm) and millimetre (mm), are multiples or fractions of a metre.



Measuring Length and Distance

- Estimating length is known as the '**rule of thumb**' method. Typical estimations include:
 - 1 large step $\approx$ 1 m
 - from the tip of your nose to the tip of your outstretched arm and fingers $\approx$ 1 m
- Accurate **measuring instruments** can also be used, such as the ruler, tape measure, odometer (in a car), etc.



Conversions and Calculations involving Metric Units of Length and Distance



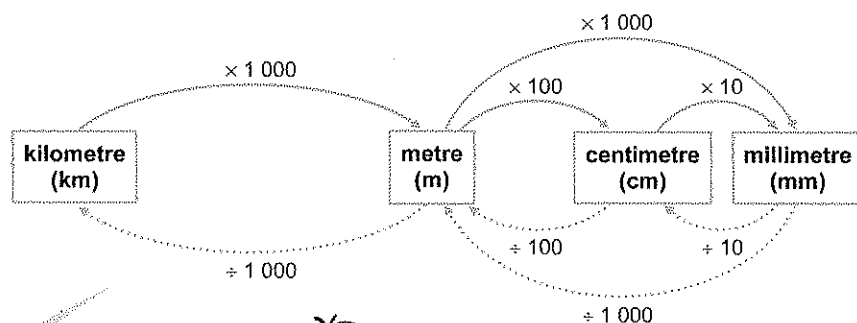
General method:

- ▶ **BIG** unit down to a **SMALLER** unit → **MULTIPLY** by the conversion factor
- ▶ **SMALL** unit up to a **BIGGER** unit → **DIVIDE** by the conversion factor

Conversion diagram:

km → m → cm → mm

big to small ∴ × by
conversion factor



This diagram should be
committed to memory!



mm → cm → m → km

small to big ∴ ÷ by
conversion factor



Conversion tables:

km to m:	× by 1 000	mm to cm:	÷ by 10
m to cm:	× by 100	cm to m:	÷ by 100
cm to mm:	× by 10	m to km:	÷ by 1 000

Calculations

- ▶ Always make sure that you are working in the same units, e.g. if mm and km are given, convert all measurements into either mm or into km.
- ▶ When you are given a cost per length, ensure that you work in the same unit of length, e.g. if you are given a cost of R2,50/m ensure that you are also working in metres.

Worked Examples



1. Convert the following:

1.1 2,39 cm to m

$$2,39 \text{ cm} \div 100 = 0,0239 \text{ m}$$



m cm
÷ 100
small to big ∴ divide

1.2 645 km to m

$$645 \text{ km} \times 1\,000 = 645\,000 \text{ m}$$

× 1 000
km m
big to small ∴ multiply



1.3 12 999 mm to m

$$12\,999 \text{ mm} \div 10 = 1\,299,9 \text{ cm}$$

$$\therefore 1\,299,9 \text{ cm} \div 100 = 12,999 \text{ m}$$



m cm mm
÷ 100 ÷ 10
small to big ∴ divide

2. Three friends measure their height in different units:

Jabulani is 152 cm; Pelu is 1,49 m; and Samuel is 1 886 mm.
Which friend is the tallest?

- First determine the most appropriate unit of measurement → metres
∴ Pelu = 1,49 m
- To convert cm to m: divide by 100:
∴ Jabulani = $152 \text{ cm} \div 100 = 1,52 \text{ m}$
- To convert mm to m: divide by 1 000:
∴ Samuel = $1\,886 \text{ mm} \div 1\,000 = 1,886 \text{ m}$
- Therefore, Samuel is the tallest of the three friends.



3. A florist works out that she needs 125 mm of ribbon per bunch of flowers. Calculate the total cost of her ribbon, if the ribbon costs R9,80 per metre; and she needs to wrap 120 bunches of flowers.

- To convert mm to cm: divide by 10:
∴ $125 \text{ mm} = 125 \div 10 = 12,5 \text{ cm}$
- To convert cm to m: divide by 100:
∴ $12,5 \text{ cm} = 12,5 \div 100 = 0,125 \text{ m}$
- Total ribbon needed = $0,125 \text{ m} \times 120 \text{ bunches} = 15 \text{ m}$
∴ Total cost of ribbon = $15 \text{ m} \times \text{R}9,80/\text{m} = \text{R}147$



EXERCISE 1

1. Convert the following:

- | | | | |
|-----|-----------------|------|-------------------|
| 1.1 | 2 m = ... mm | 1.6 | 0,89 km = ... cm |
| 1.2 | 3,18 km = ... m | 1.7 | 63 km = ... mm |
| 1.3 | 80 m = ... cm | 1.8 | 5,29 cm = ... m |
| 1.4 | 6,7 cm = ... mm | 1.9 | 8 mm = ... km |
| 1.5 | 400 m = ... km | 1.10 | 9 301 cm = ... km |

2. Liam would like to run a half-marathon, which is a distance of 21 km. How many times would he have to run around his school's athletics track (which measures 400 metres) in order to at least equal the distance of a half-marathon?
3. Sally would like to decorate her window-sill in her kitchen with mosaic tiles. The mosaic tiles are 20 cm x 20 cm and cost R6,20 per centimetre. Calculate the cost of tiling her window-sill with mosaic tiles, if her window-sill measures 720 mm.



Conversions and Calculations involving Imperial Units of Length and Distance



- Imperial units of length and distance include:
inches (in); feet (ft); yards (yd) and miles (mi).



NOTE! You will be given the conversion tables in order to convert between imperial and metric units (and vice versa); so you **don't** need to memorise the tables; but you do need to know how to convert between the different units.

General method:

- If you are given: **1 imperial unit** = multiple metric units
e.g. 1 inch = 2,54 cm
and asked for: **How many imperial** units, given a metric number?
→ **DIVIDE** by rate
e.g. How many inches in 15 cm?
∴ 15 cm = $15 \div 2,54 = 5,91$ in
- or asked for: **How many metric** units, given an imperial number?
→ **MULTIPLY** by rate
e.g. How many centimetres in 15 inches?
∴ 15 in = $15 \times 2,54 = 38,1$ cm

- If you are given: **1 metric unit** = multiple imperial units

e.g. 1 cm = 0,3937 inches

- and asked for: **How many metric** units, given an imperial number?
→ **DIVIDE** by rate

e.g. How many centimetres in 15 inches?
∴ 15 in = $15 \div 0,3937 = 38,1$ cm



- or asked for: **How many imperial** units, given a metric number?
→ **MULTIPLY** by rate

e.g. How many inches in 15 cm?
∴ 15 cm = $15 \times 0,3937 = 5,91$ in



Therefore, if you are given **1 imperial** and asked for **imperial** → then divide by rate (same goes for metric).
Alternatively, if you are given **1 imperial** and asked for **metric** → then multiply by rate (same goes for metric).

Conversion tables:

From METRIC to IMPERIAL

1 cm = 0,3937 inches (in)

1 m = 1,0936 yards (yd)

1 km = 0,6214 miles (mi)

1 m = 3,2808 feet (ft)

From IMPERIAL to METRIC

1 inch (in) = 2,54 cm

1 foot (ft) = 0,3048 m

1 yard (yd) = 0,9144 m

1 mile (mi) = 1,6093 km

These conversion tables **don't** need to be memorised!
Just ensure that you are able to use them in order to convert between metric and imperial conversions (and vice versa).



Calculations

Always ensure you work in the same imperial/metric units.

Worked Examples



Use the conversion tables on p. 111 to answer the following questions:

1. Convert the following:

1.1 29 m to yards

- If you use the conversion: $1 \text{ m} = 1,0936 \text{ yd}$
 $\therefore 29 \text{ m} = 29 \times 1,0936 = 31,71 \text{ yd}$



REMEMBER! Given 1 metric = multiple imperial and asked for imperial $\rightarrow$ multiply by rate.

- If you use the conversion: $1 \text{ yd} = 0,9144 \text{ m}$
 $\therefore 29 \text{ m} = 29 \div 0,9144 = 31,71 \text{ yd}$

REMEMBER! Given 1 imperial = multiple metric and asked for imperial $\rightarrow$ divide by rate.



1.2 63 miles to km

- If you use the conversion: $1 \text{ km} = 0,6214 \text{ mile}$
 $\therefore 63 \text{ mile} = 63 \div 0,6214 = 101,38 \text{ km}$
- If you use the conversion: $1 \text{ mile} = 1,6093 \text{ km}$
 $\therefore 63 \text{ mile} = 63 \times 1,6093 = 101,39 \text{ km}$



Note the rounding-off discrepancy (i.e. 101,38 km vs 101,39 km), because the conversion factor/rate is a rounded-off number!

2. Two friends compare their records for their long-jump trials at school. Nelly jumped 3,4 ft, while Pam jumped 1,87 yds. Who jumped the furthest?

$$\text{Nelly: } 3,4 \text{ ft} = 3,4 \times 0,3048 = 1,04 \text{ m}$$

$$\text{Using the conversion } 1 \text{ ft} = 0,3048 \text{ m}$$

$$\text{Pam: } 1,87 \text{ yds} = 1,87 \times 0,9144 = 1,71 \text{ m}$$

$$\text{Using the conversion } 1 \text{ yd} = 0,9144 \text{ m}$$

$\therefore$ Pam jumped the furthest.

3. Eunice would like to put up an electric fence around her farm. She measures the perimeter to be 4,3 miles. Determine the total cost of the electric fence, if they charge R29,50 per metre.

$$4,3 \text{ mile} = 4,3 \times 1,6093 = 6,92 \text{ km}$$

$$\therefore 6,92 \text{ km} = 6,92 \times 1\,000 = 6\,920 \text{ m}$$

$$\therefore \text{Total cost} = 6\,920 \text{ m} \times \text{R}29,50/\text{m} = \text{R}204\,140$$

$$\text{Using the conversion } 1 \text{ mile} = 1,6093 \text{ km}$$



EXERCISE 2

Use the conversion tables on p. 111 in order to answer the following questions:

1. Convert the following:

1.1 40 miles to km

1.5 3,7 m to yards

1.2 650 yards to m

1.6 900,4 km to miles

1.3 50 inches to cm

1.7 180 cm to feet

1.4 0,24 feet to mm

1.8 22 169 mm to inches

2. In a high-jump competition, Lindy jumps 2,4 feet and Lester jumps 0,85 m. Who jumps the highest?

3. Andy travels 3 250 metres. Determine the cost of his gas, if gas costs \$2,50 per mile.

Test Your Understanding



1. Jacques will win a club cycling cup if he is able to log at least 600 km of cycling distance in a six month period. He cycles the Vineyard Race in January (65 miles); the Ocean to Ocean Race in March (120 000 yards); the Karoo Fun Race in April (888 500 feet); and the Charity Fun Sprint in June (5 632 144 inches). Will Jacques win the club cycling cup? (Use the conversions provided.)

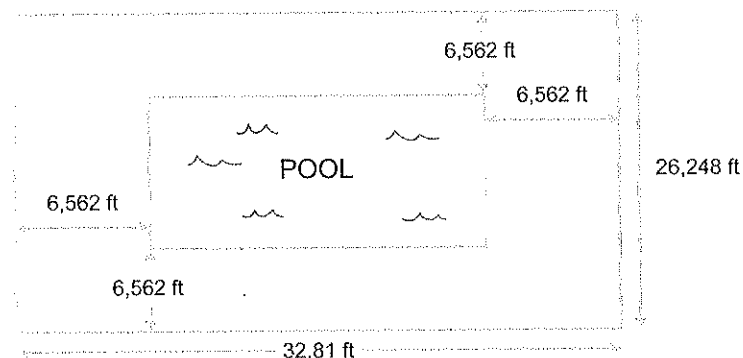
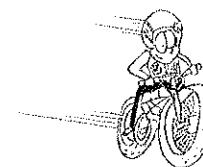
$$1 \text{ km} = 0,6214 \text{ miles}$$

$$1 \text{ m} = 3,281 \text{ feet}$$

$$1 \text{ yard} = 0,9144 \text{ metres}$$

$$1 \text{ inch} = 2,54 \text{ cm}$$

2. Austin wants to pave around his pool, as shown below:



He is able to import pavers that measure 2,1872 yards by 2,1872 yards.

Determine the total cost (in rands) to Austin, if the pavers cost \$42,99 each; plus import duties of 15% of the total cost of the product.

Use the conversions: 1 m = 3,281 feet; 1 yard = 0,9144 m; \$1 = R9,68.

3. Amy needs to travel from Cape Town to New York. She finds this distance table below (distance given in miles):

London		New York	
3 460			
570	3 960	Berlin	
6 010	7 800	5 980	Cape Town

- 3.1 Which route is the shortest distance:

Cape Town → Berlin → New York or

Cape Town → London → New York?

Show all calculations to substantiate your answer.

- 3.2 If 1 mile = 1,6093 km, determine the total distance (in kilometres) that Amy will travel if she picks the shortest route. Round off your answer to the nearest km.

- 3.3 Use the information given above to calculate the average speed (in km/h) at which Amy will fly, if it takes 11 hours and 45 minutes to fly from Cape Town to London. Use the formula: **Average Speed = Distance ÷ Time**



UNIT

3

MEASURING MASS (WEIGHT)

Definition and Units

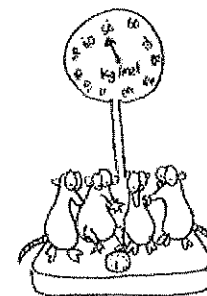


- **Mass (weight):** The amount of matter in an object.
- The **basic unit** of mass (weight) is the **kilogram**. Other units, such as grams (g), milligrams (mg) and tons (t) are multiples or fractions of a kilogram.

Measuring Mass (weight)



- **Estimation** is very useful in everyday activities, such as cooking (e.g. a pinch of salt) and travelling (e.g. ensuring your luggage falls below the maximum weight limit) etc.
- Typical **measuring instruments** for weight include different types of scales e.g. bathroom, kitchen, fruit/veg, electronic luggage scale etc.



Conversions and Calculations involving Metric Units of Mass (Weight)



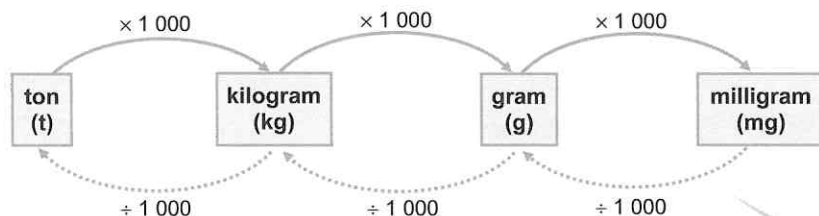
General Method:

- ▶ **BIG** unit down to a **SMALLER** unit
→ **MULTIPLY** by the conversion factor
- ▶ **SMALL** unit up to a **BIGGER** unit
→ **DIVIDE** by the conversion factor

Conversion Diagram:

t → kg → g → mg

big to small ∴ × by
conversion factor



This diagram should be
committed to memory!

mg → g → kg → t

small to big ∴ ÷ by
conversion factor



Conversion tables:

t to kg:	× by 1 000	mg to g:	÷ by 1 000
kg to g:	× by 1 000	g to kg:	÷ by 1 000
g to mg:	× by 1 000	kg to t:	÷ by 1 000

Calculations:

- ▶ Always make sure that you are working in the same units, e.g. if given g and mg, convert into either g or mg.
- ▶ When you are given a cost per mass/weight, ensure that you work in the same unit of mass/weight, e.g. R2 000/kg.
- ▶ A very useful calculation involving mass is the **Body Mass Index (BMI)**, which is a measure of body weight, based on a person's weight and height:

$$\text{BMI} = \frac{\text{mass (kg)}}{[\text{height (m)}]^2}$$

Did you know?

BMI does not actually measure the percentage of body fat, it is used to estimate a healthy body weight based on a person's height, assuming an average body composition. Due to its ease of measurement and calculation, it is the most widely used diagnostic tool to identify weight problems within a population.



- ▶ The BMI can be used to classify a person's weight status (as underweight, normal, overweight or obese) based on the BMI Weight Classification Table:

BMI	Classification
<18,5	Underweight
≥ 18,5 and < 25	Normal
≥ 25 and < 30	Overweight
≥ 30	Obese

- ▶ Other useful mass (weight) calculations can be done, based on values read from typical **growth charts**.
 - These growth charts provide an indication of the typical weight, age and height growth patterns of children and babies.
 - Based on a child's growth pattern, they will fall along a curved line, which represents a specific percentile of the population, i.e. 95th percentile means that 95% of children are at or under that reading (weight or height).

Stature-for-age and Weight-for-age percentiles

RECORD #

[illegible]

SCIENCE Development by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000). <http://www.cdc.nchp.growthcharts>

- The top of the chart shows ages, 2 to 20 years. The left and right sides of the chart show heights (above) and weights (below).
- Find a child's height (stature) on the left side, and read horizontally across until you reach the vertical line for the child's age. Then determine that point in relation to the percentile lines.
- For example, a boy 61 inches (155 cm) tall and 12 years old is just above the 75% percentile.

$$\therefore \text{Total cost of compost} = 10 \text{ kg} \times R19,99/\text{kg} = R199,90$$


4



$\times 1\,000$
 kg g
 big to small $\therefore$ multiply



g mg
 $\div 1\,000$
 small to big $\therefore$ divide



t kg g
 $\div 1\,000$ $\div 1\,000$
 small to big $\therefore$ divide



4. Tommy is an adult male who weighs 63 kg and measures 1,62 m in height. Determine whether Tommy is overweight, based on the Body Mass Index (BMI) table (see p. 114).

$$\begin{aligned} \text{BMI} &= \frac{\text{mass (kg)}}{[\text{height (m)}]^2} \\ &= \frac{63 \text{ kg}}{(1,62 \text{ m})^2} = 24,01 \end{aligned}$$

$\therefore$ Tommy is classified as 'Normal' weight and is $\therefore$ not overweight

5. Use the growth chart for 2 to 20 year old boys (see p. 115) to determine:

- 5.1 In what percentile does an 8-year old boy who weighs 35 kg, fall?
95th percentile
- 5.2 How tall are 20-year old boys if they fall in the 5th percentile of the population?
165 cm
- 5.3 What is the weight of a 15-year old boy, if he falls in the 25th percentile?
50 kg

EXERCISE 1

1. Convert the following:

- | | |
|---------------------|--------------------------|
| 1.1 3 mg = ... g | 1.6 84 kg = ... t |
| 1.2 75 g = ... kg | 1.7 0,0002 t = ... g |
| 1.3 64,7 t = ... kg | 1.8 1,492 mg = ... kg |
| 1.4 28 g = ... mg | 1.9 89,3 kg = ... mg |
| 1.5 0,03 kg = ... g | 1.10 49110763 mg = ... t |

2. Determine which bag of sweets is the better buy:

Sweet Chews: 280 g for R13,99 Happy Tastes: 320 g for R16,50

3. Sandra is a 54 year old female who weighs 58 kg and is 1,24 m in height. Use the BMI table provided (see p. 114) to classify Sandra's weight status.
4. Use the growth chart for 2 - 20 year old boys (see p. 115), to determine the difference in weight between a 10 year old boy in the 50th percentile and a 13 year old boy in the 90th percentile.

Conversions and Calculations Involving Imperial Units of Mass (Weight)



- Imperial units of mass (weight) include: pounds (lb) and ounces (oz).



NOTE! You will be given the conversion tables in order to convert between imperial and metric units (and vice versa); so you **don't** need to memorise the tables; but you do need to know how to convert between the different units.

- General Method:**

- If you are given: **1 imperial unit** = multiple metric units
e.g. 1 pound (lb) = 0,453 kg

and asked for: **How many imperial units**, given a metric number?
→ **DIVIDE** by rate

e.g. How many pounds in 20 kg? $\therefore 20 \text{ kg} = 20 \div 0,453 = 44,15 \text{ lb}$

or asked for: **How many metric units**, given an imperial number?
→ **MULTIPLY** by rate

e.g. How many kilograms in 20 lb? $\therefore 20 \text{ lb} = 20 \times 0,453 = 9,06 \text{ kg}$

- If you are given: **1 metric unit** = multiple imperial units
e.g. 1 kg = 2,204 pounds (lb)

and asked for: **How many metric units**, given an imperial number?
→ **DIVIDE** by rate

e.g. How many kilograms in 20 lb? $\therefore 20 \text{ lb} = 20 \div 2,204 = 9,07 \text{ kg}$

or asked for: **How many imperial units**, given a metric number?
→ **MULTIPLY** by rate

e.g. How many pounds in 20 kg? $\therefore 20 \text{ kg} = 20 \times 2,204 = 44,08 \text{ lb}$



Therefore, if you are given **1 imperial** and asked for **imperial** → then **divide by rate** (same goes for metric).
Alternatively, if you are given **1 imperial** and asked for **metric** → then **multiply by rate** (same goes for metric).

- Conversion tables:**

From METRIC to IMPERIAL

1 g = 0,0352 ounces (oz)

1 kg = 2,2046 pounds (lb)

From IMPERIAL to METRIC

1 ounce (oz) = 28,3495 g

1 pound (lb) = 0,4535 kg

- Calculations**

Always ensure that you work in the same imperial/metric units.

Worked Examples



1. Convert the following, using the conversion tables on p. 116:

1.1 32 g to ounces

- If you use the conversion: $1 \text{ g} = 0,0352 \text{ oz}$
 $\therefore 32 \text{ g} = 32 \times 0,0352 = 1,13 \text{ oz}$

REMEMBER! Given 1 metric = multiple imperial and asked for imperial $\rightarrow$ multiply by rate



- If you use the conversion:

$$1 \text{ oz} = 28,3495 \text{ g}$$

$$\therefore 32 \text{ g} = 32 \div 28,3495 = 1,13 \text{ oz}$$

1.2 7,4 ounces to g

- If you use the conversion: $1 \text{ g} = 0,0352 \text{ oz}$
 $\therefore 7,4 \text{ oz} = 7,4 \div 0,0352 = 210,23 \text{ g}$

- If you use the conversion: $1 \text{ oz} = 28,3495 \text{ g}$
 $\therefore 7,4 \text{ oz} = 7,4 \times 28,3495 = 209,79 \text{ g}$

REMEMBER! Given 1 imperial = multiple metric and asked for imperial $\rightarrow$ divide by rate

NOTE! The discrepancy between the answers is due to the fact that the conversion rate/factor is rounded off!

2. A Weigh-Sure diet claims to cause a weight loss of 9 lb in 3 weeks; while the Super-Less diet claims to help you lose 4 kg a week. Which diet claims the most weight loss?

- Weigh-Sure diet: $9 \text{ lb} = 9 \times 0,4535 = 4,08 \text{ kg}$
 $\therefore$ Total loss in 3 weeks = 4,08 kg

- Super-Less diet: 4 kg per week
 $\therefore$ Total loss in 3 weeks = $4 \times 3 = 12 \text{ kg}$
 $\therefore$ Super-Less diet claims the most weight loss

Using the conversion:
 $1 \text{ lb} = 0,4535 \text{ kg}$



3. Saffron is the most expensive spice in the world, and sells for approximately \$72,95 per ounce. Determine the cost of Saffron per portion (in rand), if a recipe that serves 8 people, calls for 5 g of Saffron.

$$\text{Mass of Saffron per portion} = 5 \text{ g} \div 8 \text{ people} = 0,625 \text{ g}$$

$$\text{Mass of Saffron in ounces} = 0,625 \text{ g} \div 28,3495 = 0,022 \text{ oz}$$

$$\therefore \text{Cost of Saffron per portion in dollars} = 0,022 \text{ oz} \times \$72,95 = \$1,60$$

$$\text{Cost of Saffron per portion in rands} = \$1,60 \times 8,97 = \text{R}14,35$$

Using the conversion:
 $1 \text{ ounce} = 28,3495 \text{ g}$
 $\$1 = \text{R}8,97$

EXERCISE 2

Use the conversion tables on p. 116 to answer the following questions:

1. Convert the following:

1.1 4,9 kg to pounds

1.5 8 ounces to kg

1.2 630 g to ounces

1.6 2 pounds to g

1.3 21 ounces to g

1.7 99,32 kg to ounces

1.4 0,03 pounds to kg

1.8 818,7 g to pounds

2. A crane can lift a maximum load of 3,5 tons. If a container weighs 6 820 pounds, will the crane be able to lift the container?

3. Calculate the cost of 0,25 pounds of gold, if gold costs R13 070,58 per ounce.

Test Your Understanding



1. Which baby powder is most economical?

125 000 mg

500 g

0,75 kg



R8,95

R32,20

R38,99

2. A muffin recipe calls for 3,6 lb of dates and 5,4 ounces of coconut. Determine the total cost (rands) of the dates and the coconut per muffin, if dates cost 4 cents per gram and coconut costs 25 cents per gram. This muffin recipe makes a dozen and a half muffins. (Assume that all ingredients are equally spread per muffin.)

Use the conversions: $1 \text{ kg} = 2,2046 \text{ lb}$; $1 \text{ oz} = 28,3495 \text{ g}$

3. Paul and Pauline are 20-year old twins. Use the information given below to determine which twin needs to choose a healthier lifestyle:



Use the formula: $\text{BMI} = \frac{\text{mass (kg)}}{[\text{height (m)}]^2}$

Twin	Mass	Height
Paul	76 kg	165 cm
Pauline	99 lb	60 inches

BMI	Classification
< 18,5	Underweight
$\geq 18,5$ and < 25	Normal
≥ 25 and < 30	Overweight
≥ 30	Obese

$1 \text{ lb} = 0,4536 \text{ kg}$
 $1 \text{ inch} = 0,0254 \text{ m}$



4. Study the 2-20 year old girls' growth chart alongside and answer the questions that follow:

- 4.1 Explain what it means that a 7 year old girl has a weight-for-age ratio that places her on the 50th percentile graph?
- 4.2 Write down the average weight of a 12 year old girl.
- 4.3 Write down the average height (stature) of a 17 year old girl.
- 4.4 A 6 year old girl weighs 25 kg. Does this girl have an above average, average, or below average weight-for-age ratio? Explain.
- 4.5 A 14 year old girl has a height (stature) of 152 cm. Would this girl be considered to have an above average, an average, or a below average height-for-age ratio? Explain your answer.
- 4.6 A 13 year old girl weighs 42 kg. Between which two percentile graphs does the girl's weight-for-age ratio lie?
- 4.7 A 15 year old girl is 1,67 m tall. Between which two percentile graphs does the girl's height-for-age ratio lie?
- 4.8 What do you think the weight of an overweight 16 year old girl might be? You need to justify your answer by referring to the percentile curves on the growth chart.
- 4.9 What do you think the weight of an underweight 19 year old girl might be? You need to justify your answer by referring to the percentile curves on the growth chart.
- 4.10 A 14 year old girl has a height of 1,77 m. Do you think this girl is tall? Explain your answer.
- 4.11 According to the weight-for-age curves, between what ages does the weight of girls increase the most? Explain your answer by referring to the weight-for-age curves.



GROWTH CHART FOR GIRLS AGED 2 TO 20 YEARS

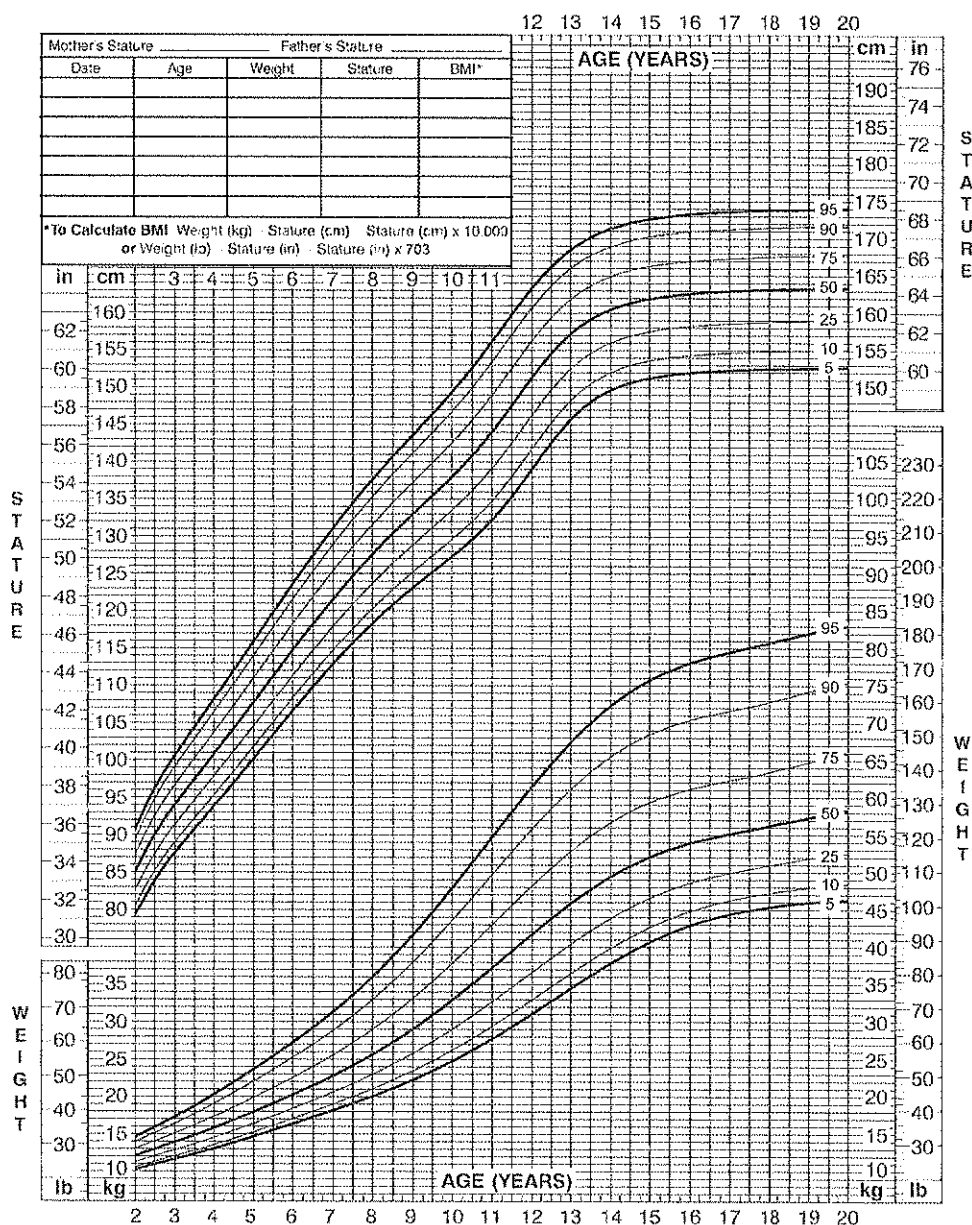
'Stature' = height

2 to 20 years: Girls

Stature-for-age and Weight-for-age percentiles

NAME _____

RECORD # _____



MEASURING VOLUME

Definition and Units

- **Volume:** the three-dimensional space occupied by a gas, a liquid, or a solid substance.

NOTE: The word 'capacity' also refers to volume.

- **Liquid volumes:**

- ▶ The basic unit of volume for liquids is the **litre (ℓ)**.
- ▶ Other units, such as **millilitres (mℓ)** and **kilolitres (kℓ)** are multiples or fractions of a litre.

- **Solid volumes:**

- ▶ The basic unit of volume for solids is the **cubic centimetre (cm³)**.
- ▶ Other units, such as the **cubic millimetre (mm³)** and **cubic metre (m³)** are multiples or fractions of a cubic centimetre.

Measuring Volume

- **Estimation** of volume can be useful for:

- ▶ pouring enough water into a kettle for four cups of coffee.
- ▶ selecting a container to store excess cooking oil.

- **Measuring instruments** include: measuring spoons, cups, jugs, bottles, buckets and wheelbarrows, etc.

Conversions and Calculations including Metric Units of Volume

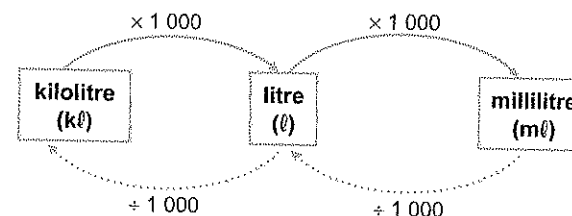
- **General Method:**

- ▶ **BIG** unit down to a **SMALLER** unit → **MULTIPLY** by the conversion factor
- ▶ **SMALL** unit up to a **BIGGER** unit → **DIVIDE** by the conversion factor

- **Conversion diagram:**

kℓ → ℓ → mℓ

This diagram should be committed to memory!



mℓ → ℓ → kℓ

- **Conversion tables:**

kℓ to ℓ	× by 1 000	mℓ to ℓ	÷ by 1 000
ℓ to mℓ	× by 1 000	ℓ to kℓ	÷ by 1 000

- **Baking conversion table:**

1 cup	250 mℓ
1 tablespoon (tbs)	15 mℓ
1 teaspoon (tsp)	5 mℓ

*You do **not** have to memorise these conversions.*

- **Calculations:**

- ▶ Always make sure that you are working in the same units, e.g. if they give kℓ and ℓ, convert all measurements into either kℓ or all into ℓ.
- ▶ When you are given a cost per volume, ensure that you work in the same unit of volume, e.g. if they give you a cost of R2,99/mℓ, ensure that you are also working in millilitres.

Worked Examples

1. Convert the following:

1.1 12 l to ml

$$12 \text{ l} \times 1\,000 = 12\,000 \text{ ml}$$

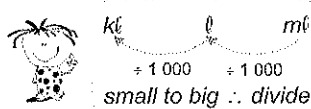
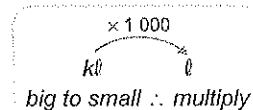
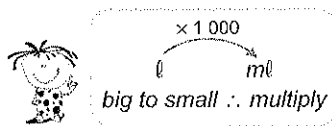
1.2 6,3 kl to l

$$6,3 \text{ kl} \times 1\,000 = 6\,300 \text{ l}$$

1.3 24 780 000 ml to kl

$$24\,780\,000 \div 1\,000 = 24\,780 \text{ l}$$

$$\therefore 24\,780 \text{ l} \div 1\,000 = 24,78 \text{ kl}$$



2. Calculate the total volume of liquid (in litres), if a recipe calls for:

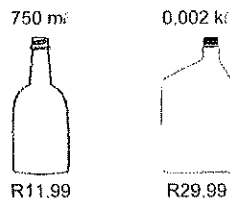
- 2 cups milk
- 4 tbs food colouring
- 3 tsp vanilla essence
- 320 ml coffee

Use the baking conversion table given on p. 119:

- 2 cups milk = $2 \times 250 \text{ ml} = 500 \text{ ml}$
- 3 tsp vanilla essence = $3 \times 5 \text{ ml} = 15 \text{ ml}$
- 4 tbs food colouring = $4 \times 15 \text{ ml} = 60 \text{ ml}$
- coffee = 320 ml
- therefore, total liquid volume = $500 \text{ ml} + 15 \text{ ml} + 60 \text{ ml} + 320 \text{ ml} = 895 \text{ ml}$
- $\therefore 895 \text{ ml} = 895 \div 1\,000 = 0,895 \text{ l}$

3. Which volume of paraffin is more expensive per litre to buy?

- $750 \text{ ml} = 750 \div 1\,000 = 0,75 \text{ l}$
 $\therefore \text{Cost per litre} = R11,99 \div 0,75 = R15,99/\text{l}$
- $0,002 \text{ kl} = 0,002 \times 1\,000 = 2 \text{ l}$
 $\therefore \text{Cost per litre} = R29,99 \div 2 = R14,99/\text{l}$
- Therefore the 750 ml is more expensive



4. A candle-maker adds 7 drops of vanilla scent to each candle that she makes. If she needs to make 150 candles, calculate the total cost of the vanilla scent if 1 drop is 2,5 ml and the vanilla scent sells for R127,40 per 1 l bottle.

- Total no. of drops = $7 \times 150 \text{ candles} = 1\,050 \text{ drops}$
- Volume of drops = $1\,050 \times 2,5 \text{ ml} = 2\,625 \text{ ml}$
 $\therefore 2\,625 \text{ ml} = 2\,625 \div 1\,000 = 2,625 \text{ l}$
- Total cost = $3 \text{ bottles} \times R127,40 = R382,20$

Vanilla scent is sold in 1 litre bottles, so she needs 3 bottles, if she needs 2,625 l.



EXERCISE 1

1. Convert the following:

1.1 60 ml = ... l

1.2 4 l = ... kl

1.3 2 kl = ... l

1.4 0,32 l = ... ml

1.5 2,4 kl = ... ml

1.6 63 000 ml = ... kl

2. Use the baking conversion table on p. 119, to determine the following:

2.1 How many cups are there in 1,75 l?

2.2 How many teaspoons are there in 45 ml?

2.3 How many tablespoons are there in 0,06 l?

3. Calculate the total cost of fuel, if a military tank travels at 70 km/h for one hour, 15 minutes; and the tank's fuel consumption is 1 l per 0,8 km. The cost of fuel is R11,59/l.

Use the formula: **Distance = Speed $\times$ Time**

Conversions and Calculations Involving Imperial Units of Volume



Imperial units of volume:

gallons (gal); pints (pt); fluid ounces (fl. oz) and quarts (qt).



NOTE! You will be given the conversion tables in order to convert between imperial and metric units (and vice versa); so you **don't** need to memorise the tables; but you do need to know how to convert between the different units.

General Method:

► If you are given:

1 imperial unit = multiple metric units

e.g. 1 UK pint = 0,5682 l

and asked for:

How many imperial units, given a metric number?

→ **DIVIDE** by rate

e.g. How many UK pints in 1,2 l?

$$\therefore 1,2 \text{ l} = 1,2 \div 0,5682 = 2,11 \text{ pt}$$

or asked for:

How many metric units, given an imperial number?

→ **MULTIPLY** by rate

e.g. How many litres in 1,2 pints?

$$\therefore 1,2 \text{ pt} = 1,2 \times 0,5682 = 0,68 \text{ l}$$



- If you are given: **1 metric unit = multiple imperial units**
e.g. $1 \ell = 1,7598 \text{ pints}$
- and asked for: **How many metric units, given an imperial number?**
→ **DIVIDE** by rate
e.g. *How many litres in 1,2 pints?*
 $\therefore 1,2 \text{ pt} = 1,2 \div 1,7598 = 0,68 \ell$
- or asked for: **How many imperial units, given a metric number?**
→ **MULTIPLY** by rate
e.g. *How many UK pints in 1,2 l?*
 $\therefore 1,2 \ell = 1,2 \times 1,7598 = 2,11 \text{ pt}$

Therefore, if you are given **1 imperial** and asked for **imperial** → then **divide by rate** (same goes for metric).
Alternatively, if you are given **1 imperial** and asked for **metric** → then **multiply by rate** (same goes for metric).



Conversion tables:

From METRIC to IMPERIAL

- $1 \ell = 1,7598 \text{ UK pint (pt)}$
 $1 \ell = 0,2199 \text{ UK gallons (gal)}$
 $1 \ell = 1,0566 \text{ US quarts (qt)}$
 $1 \text{ ml} = 0,0338 \text{ US fluid ounces (fl. oz)}$

From IMPERIAL to METRIC

- $1 \text{ UK pint (pt)} = 0,5682 \ell$
 $1 \text{ UK gallon (gal)} = 4,5461 \ell$
 $1 \text{ US quart (qt)} = 0,9464 \ell$
 $1 \text{ US fluid ounce (fl. oz)} = 29,5735 \text{ ml}$

NOTE! Both the UK and US use imperial units for volume, e.g. pint, gallon, quart and fluid ounce. So, it is important to specify the country (UK/US) as the conversion rates/factors differ.



REMEMBER!

Don't memorise these tables - just be sure you can use them to do conversions!



Worked Examples

1. Convert the following, using the conversion tables alongside:

1.1 84ℓ to UK gallons

- If you use the conversion: $1 \ell = 0,2199 \text{ UK gal}$
 $\therefore 84 \ell = 84 \times 0,2199 = 18,47 \text{ UK gal}$



REMEMBER! Given 1 metric = multiple imperial and asked for imperial → multiply by rate.

- If you use the conversion: $1 \text{ UK gal} = 4,5461 \ell$
 $\therefore 84 \ell = 84 \div 4,5461 = 18,48 \text{ UK gal}$

REMEMBER! Given 1 imperial = multiple metric and asked for imperial → divide by rate.



Note the rounding-off discrepancy (i.e. $18,47 \text{ UK gal}$ vs $18,48 \text{ UK gal}$), because the conversion factor/rate is a rounded-off number!



1.2 10 US quarts to litres

- If you use the conversion: $1 \ell = 1,0566 \text{ US qt}$
 $\therefore 10 \text{ US qt} = 10 \div 1,0566 = 9,46 \ell$
- If you use the conversion: $1 \text{ US qt} = 0,9464 \ell$
 $\therefore 10 \text{ US qt} = 10 \times 0,9464 = 9,46 \ell$

1.3 250 ml to US fluid ounces

- If you use the conversion: $1 \text{ ml} = 0,0338 \text{ US fl. oz}$
 $\therefore 250 \text{ ml} = 250 \times 0,0338 = 8,45 \text{ US fl. oz}$
- If you use the conversion: $1 \text{ US fl. oz} = 29,5735 \text{ ml}$
 $\therefore 250 \text{ ml} = 250 \div 29,5735 = 8,45 \text{ US fl. oz}$

2. Which is a better buy?

* 1 US quart milk for R15,99 OR * 1 l milk for R16,99

- $1 \text{ US qt} = 0,9464 \ell$
 $\therefore \text{Cost per litre} = R15,99 \div 0,9464 = R16,90/\ell$
- $1 \ell \text{ milk cost} = R16,99/\ell$
Therefore, the 1 US quart is a better buy

3. A vanilla sponge cake calls for 2,5 US fluid ounces of vanilla essence. Determine the cost of the vanilla essence in the sponge cake, if a 1 litre bottle of vanilla essence costs R38,50.

- $2,5 \text{ US fl. oz} = 2,5 \times 29,5735 \text{ ml}$
 $= 73,93 \text{ ml}$
- $1 \text{ l vanilla essence} = 1 \times 1\,000 = 1\,000 \text{ ml}$
- $\text{Cost of vanilla essence/ml} = R38,50 \div 1\,000 = R0,0385/\text{ml}$
- $\text{Cost of vanilla essence in sponge cake}$
 $= 73,93 \text{ ml} \times R0,0385/\text{ml}$
 $= R2,85$



Using the conversion:
 $1 \text{ US fl. oz} = 29,5735 \text{ ml}$

EXERCISE 2

Use the conversion tables from imperial to metric (and vice versa) on p. 121, in order to answer the following questions:

1. Convert the following:

1.1 60 ml to US fluid ounces	1.4 0,8 US quarts to ml
1.2 4 UK pints to l	1.5 479 l to UK gallons
1.3 27 US fluid ounces to l	1.6 1011,21 ml to UK pints
2. Which water reservoir has the biggest capacity?
 - * Silver Stream – which holds 2 750 UK gallons
 - * Placid Water – which holds 13 207,5 US quarts
3. Mrs Hawthorne needs to buy Italian gelato ice-cream for her dinner party of 8 guests. Would Mrs Hawthorne get more gelato if she bought two 1,5849 US quart tubs of gelato or 8 individual tubs of 0,43995 UK pint each?

Converting between Liquids and Solids



- Converting between liquid and solid measurements is useful in:
 - Baking - measuring liquid and dry ingredients
 - Physics - measuring the solid and liquid state of a substance
 - Painting - determining the volume of a liquid needed to cover/paint a solid surface

- Standard Conversion Factors:
 $1 \text{ ml} = 1 \text{ cm}^3$ and $1 \text{ m}^3 = 1\,000 \text{ l}$



To estimate the **weight of a liquid**
 - we use the conversion factor for
pure water: $1 \text{ ml} = 1 \text{ cm}^3 = 1 \text{ g}$



Worked Examples

1. A recipe calls for 750 g of flour, but Maggie doesn't have a scale. How many cups of flour could she use instead, if 1 cup = 250 ml and 1 g flour = 2 ml flour?

$$\begin{aligned}
 1 \text{ g flour} &= 2 \text{ ml flour} \\
 \therefore 750 \text{ g} &= 1\,500 \text{ ml} \\
 \text{If } 250 \text{ ml} &= 1 \text{ cup} \\
 \therefore 1\,500 \text{ ml} &= 6 \text{ cups of flour}
 \end{aligned}$$

2. An Olympic swimming pool holds 2 500 kl of water. Calculate how many cubic metres of water that is, if $1 \text{ m}^3 = 1\,000 \text{ l}$.

$$\begin{aligned}
 2\,500 \text{ kl} &= 2\,500 \times 1\,000 = 2\,500\,000 \text{ l} \\
 \therefore 2\,500\,000 \text{ l} &= 2\,500\,000 \div 1\,000 = 2\,500 \text{ m}^3
 \end{aligned}$$

EXERCISE 3

Use the conversion factors alongside to answer the following questions:

1. If 150 g of ice melts, how many ice-cube blocks can be filled, if one ice-cube block holds 25 ml of water?
2. Grandpa Tommy measures the run-off rate of water from his roof as $0,002 \text{ m}^3$ per minute. Calculate how many litres of water will run-off his roof, if it rains for an hour and a half.
3. A bag of potting soil contains an average $8,5 \text{ cm}^3$ of *E.coli* per 100 g of soil. Determine how many grams of *E.coli* could be found in a 1,5 kg bag of potting soil.

Test Your Understanding



1. Mirriam uses 135 000 ml of water to wash a car; as well as 0,065 l of car shampoo. Calculate the total cost of washing a car if water costs R12,90 per kilolitre and car shampoo costs 40 cents per millilitre.
2. Calculate the total volume of medicine cough syrup that can be consumed in a 24-hour period if the dosage says that a person can take 2 teaspoons cough syrup every 4 hours. Give your answer in US fluid ounces.
 Use the conversions: $1 \text{ tsp} = 5 \text{ ml}$; $1 \text{ US fluid ounce} = 29,5735 \text{ ml}$

3. A shipping container has a capacity for 33 m^3 . However, if a flammable substance is shipped in a container, then the container can only be filled to 85% of its capacity. Calculate how many gallons of flammable paraffin can be shipped in the container. Use the conversions: $1 \text{ m}^3 = 1\,000 \ell$; $1 \text{ UK gallon} = 4,5461 \ell$

4. Calculate the total weight of the pancake batter (in kg) in this recipe:

- | | |
|-------------------------|--------------------------|
| * 2 cups flour | * 1 cup sugar |
| * 2 tsp vanilla essence | * 4 tbs oil |
| * 150 g sugar | * 180 cm^3 milk |

Use the conversion tables on p. 119 and p. 121.



UNIT 5

MEASURING TEMPERATURE

Definition and Units

- **Temperature:** The measurement of how hot or cold a substance or object is.
- The **basic unit** of temperature used in SA is **degrees Celsius ($^{\circ}\text{C}$)**.
- The alternate unit of temperature is **degrees Fahrenheit ($^{\circ}\text{F}$)** which is commonly used in the USA.



Measuring Temperature

- **Estimation** of the temperature can be useful for determining suitable clothes to wear or finding the correct bath temperature for a child etc.
- Typical **measuring instruments** include: digital, mercury and infrared thermometer.



Setting, Monitoring and Interpreting Temperature

- Temperature dials need to be set on household appliances, such as: oven, heater, freezer, fridge, air conditioner etc.
- Appliance temperatures need to be monitored to prevent under- or over-heating.
- The temperature measurement/reading needs to be interpreted in context e.g. 15°C in South Africa is considered a cold day, while 15°C in Russia is considered a warm, balmy day!



Conversions and Calculations involving Units of Temperature



- Temperatures can be converted from Celsius ($^{\circ}\text{C}$) to Fahrenheit ($^{\circ}\text{F}$) or vice versa, using the **Conversion Formulae**:

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32^{\circ}) \div 1,8 \quad ^{\circ}\text{F} = (1,8 \times ^{\circ}\text{C}) + 32^{\circ}$$

- **Calculations**

Always ensure that you are working in the **same units**, when comparing or interpreting temperature readings.

Worked Examples



1. Convert the following:

1.1 25°C to $^{\circ}\text{F}$

$$\begin{aligned} ^{\circ}\text{F} &= (1,8 \times ^{\circ}\text{C}) + 32^{\circ}\text{C} \\ &= (1,8 \times 25^{\circ}\text{C}) + 32^{\circ}\text{C} \\ &= 77^{\circ}\text{F} \end{aligned}$$

1.2 10°F to $^{\circ}\text{C}$

$$\begin{aligned} ^{\circ}\text{C} &= (^{\circ}\text{F} - 32^{\circ}) \div 1,8 \\ &= (10^{\circ}\text{F} - 32^{\circ}) \div 1,8 \\ &= -12,22^{\circ}\text{C} \end{aligned}$$

2. Study the temperature conversion graph given below and answer the questions that follow:

- 2.1 The melting point of ice is 0°C . What is it in degrees Fahrenheit?

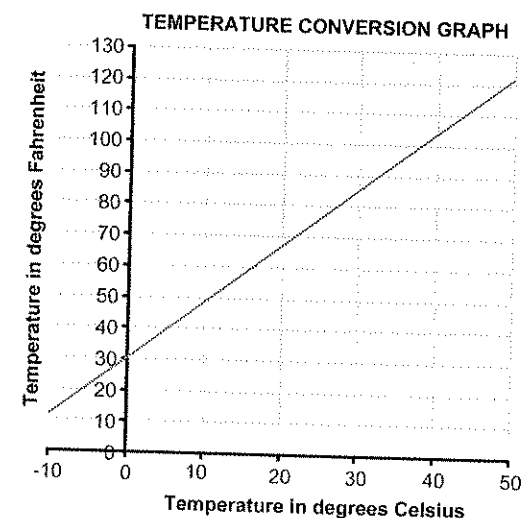
$$32^{\circ}\text{F}$$

- 2.2 Determine the temperature in $^{\circ}\text{C}$, if it is 95°F

$$\approx 35^{\circ}\text{C}$$

- 2.3 Determine the temperature in $^{\circ}\text{F}$, if it is 15°C

$$\approx 60^{\circ}\text{F}$$



Test Your Understanding



Use the conversions on p. 123 to answer the questions that follow:

1. Use the temperatures below to answer these questions:

- | | |
|-------------------|-----------------------|
| * Amsterdam: 10°C | * Dar es Salaam: 91°F |
| * Beijing: 46°F | * Halifax: 2°C |
| * Cairo: 16°C | * Minsk: 21°F |

1.1 In which city would it be the best to hang your washing on the line?

1.2 In which city would you most likely be sitting by the fire?

2. A sick child, Emily, has a temperature of 100.76°F. A temperature of above 40°C is deemed serious and the child needs to go to hospital. After how many hours will Emily's temperature be deemed serious enough to go to hospital, if her temperature increases by 5% per hour?
3. Carrots grow best at average temperature of between 60°F and 65°F. If my garden vegetable patch has an average temperature of 16°C, should I plant carrots?

UNIT

6

MEASURING TIME

Definition and Units



- **Time:** A measure of the passage of time.
- There are two **basic units** of time:
 - ▶ the **day**, whereby other units such as **hours (h)**, **minutes (min)** and **seconds (s)** are fractions of a day.
 - ▶ the **year**, whereby other units such as months, weeks and days are fractions of a year.

DID YOU KNOW?

- A **decade** is made up of 10 years.
- A **century** is made up of 100 years.
- A **millennium** is made up of 1 000 years.



Measuring Time



- **Estimation** of time is done every day, such as how long it will take to walk from point A to point B; or how long it will take to get your food at a restaurant etc.
- Typical **measuring instruments** include: a stopwatch, watch, cellphone computer, etc.

Displaying Time



- **Analogue** time: time is displayed on a clock or watch. The clock is divided into 12 equal parts (each representing 5 minutes) and the hour and minute hands indicate the time.
- **Digital** time: the time is displayed by numbers only.



10:35

Time Formats



- **12-hour format:** 24 hours in the day are broken up into 12 hours before noon (am) and 12 hours after noon (pm)

e.g. 4 o'clock in the:	morning - 4.00 am	afternoon - 4.00 pm
12 o'clock:	midnight - 0.00	midday - 12.00 pm
- **24-hour format:**

24 hours in the day whereby:

the 12 hours before 12 o'clock noon run from 00:00 - 12:00;
the 12 hours after 12 o'clock noon run from 12:00 - 24:00



- Numbers on the clock in the white circle indicate the time before noon (am).
- Numbers on the clock in the black circle indicate the time after noon (pm).

- **time recording format:** the time taken to complete a sports event or task is displayed in a linear, descending format (i.e. the biggest time units down to the smallest time units)

e.g. 1 h 12 min 20 s
1 : 12 : 20

Conversions and Calculations involving Units of Time



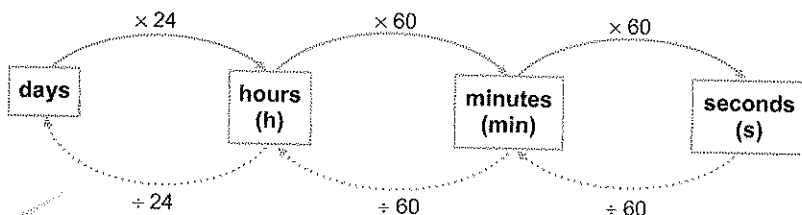
General Method:

- ▶ **BIG** unit down to a **SMALLER** unit → **MULTIPLY** by the conversion factor
- ▶ **SMALL** unit up to a **BIGGER** unit → **DIVIDE** by the conversion factor

Conversion diagram:

days → hours → minutes → seconds

big to small ∴ × by conversion factor



This diagram should be committed to memory!

seconds → minutes → hours → days

small to big ∴ ÷ by conversion factor



Conversion tables:

days to h	× by 24	s to min	÷ by 60
h to min	× by 60	min to h	÷ by 60
min to s	× by 60	h to days	÷ by 24

NB: You are also expected to know the following:

- ▶ 1 year = 365 days
- ▶ 1 year = ± 52 weeks
- ▶ 1 month = ± 4 weeks
- ▶ 1 week = 7 days



Calculations:

- ▶ Always make sure that you are working in the same units of time e.g. if minutes and seconds are given, convert all measurements into the same unit - usually the smallest unit of time e.g. seconds.

Working with fractions of time

- Time is not a decimal system ∴ 2,4 h ≠ 2 h, 40 min
- To convert the fraction, **multiply it by the maximum number of smaller units that make up the whole**

$$\begin{aligned} \text{e.g. } 2,4 \text{ h} &= 2 \text{ h} + 0,4 \text{ h} \\ &= 2 \text{ h} + 0,4 \times 60 \text{ min} \\ &= 2 \text{ hours} + 24 \text{ minutes} \end{aligned}$$

- ▶ Working with **elapsed time**: the difference between the first and second time recording.

- When subtracting, you may need to 'borrow' units from a bigger time unit ∴ remember time conversions.

$$\begin{array}{r} 3 \quad 60 + 45 = 105 \\ \text{e.g. } 4 \text{ min } 45 \text{ s} \\ - 1 \text{ min } 55 \text{ s} \\ \hline 2 \text{ min } 50 \text{ s} \end{array}$$

i.e. $60 \times \text{minutes} = 1 \text{ hour}$
∴ maximum number of whole smaller units



- It is often easier to break time down into the smallest unit and then calculate elapsed time.

$$\begin{aligned} 45 - 55 &\rightarrow \text{need to borrow} \\ 1 \text{ minute} &= 60 \text{ seconds} \\ \therefore 60 + 45 &= 105 \\ \therefore 105 - 55 &= 50 \end{aligned}$$

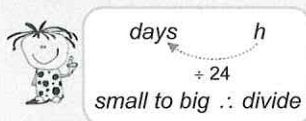




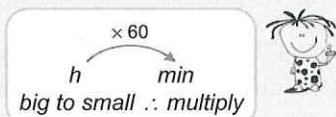
Worked Examples

1. Convert the following:

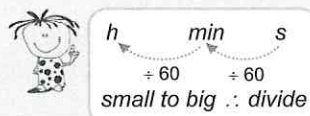
1.1 120 hours to days
 $120 \text{ h} \div 24 = 5 \text{ days}$



1.2 9 h to min
 $9 \text{ h} \times 60 = 540 \text{ min}$



1.3 43 200 s to hours
 $43\,200 \text{ s} \div 60 = 720 \text{ min}$
 $\therefore 720 \text{ min} \div 60 = 12 \text{ h}$



1.4 355 minutes to hours and minutes
 $355 \text{ min} \div 60 = 5,9166 \text{ h}$
 $\therefore 5,9166... \text{ h} = 5 \text{ h} + 0,9166 \text{ h}$
 $= 5 \text{ h} + 0,9166 \text{ h} \times 60 \text{ min}$
 $= 5 \text{ h} + 55 \text{ min}$

To convert a fraction of time, multiply by max number of smaller units in the whole $\therefore 60 \text{ min} = 1 \text{ h}$

1.5 630 hours to days and hours
 $630 \text{ h} \div 24 = 26,25 \text{ days}$
 $\therefore 26,25 \text{ days} = 26 \text{ days} + 0,25 \text{ days}$
 $= 26 \text{ days} + 0,25 \text{ days} \times 24 \text{ h}$
 $= 26 \text{ days} + 6 \text{ h}$

Multiply by max number of smaller units in the whole $\therefore 24 \text{ h in } 1 \text{ day}$

2. How much time has elapsed:

2.1 From 6.15 am to 14:38?
 • 6.15 am to 2.38 pm
 • 6 am $\rightarrow$ 2 pm = 8 h
 • 0:15 $\rightarrow$ 0:38 = 23 min
 $\therefore 8 \text{ hours}; 23 \text{ minutes}$

Convert to the same time format!

2.2 Between 4 days: 6 h: 20 min and 2 days: 19 h: 50 min?

$\begin{array}{r} 3 \\ 4 \text{ days} \end{array} ; \begin{array}{r} 5 + 24 = 29 \text{ h} \\ 6 \text{ h} \end{array} ; \begin{array}{r} 60 + 20 = 80 \text{ min} \\ 20 \text{ min} \end{array}$
 $\begin{array}{r} 2 \\ 2 \text{ days} \end{array} ; \begin{array}{r} 19 \text{ h} \end{array} ; \begin{array}{r} 50 \text{ min} \end{array}$
 $\begin{array}{r} 1 \\ 1 \text{ day} \end{array} ; \begin{array}{r} 10 \text{ h} \end{array} ; \begin{array}{r} 30 \text{ min} \end{array}$

EXERCISE 1

1. Convert the following:

- 1.1 4 days to min
 1.2 666 s to hours
 1.3 121 days to s
 1.4 432 min to h and min
 1.5 1 000 s to min and s
 1.6 557 799 s to days, h, min and s

2. Order the following times of the day from earliest to latest:

10.30 am ; 04:25 ; 10.45 pm ; 22:15 ; 4.15 pm ; 16:05 ; 4.30 am

3. On 18 October 1968, Bob Beamon set the world long jump record when he jumped a distance of 8,90 m. This record was only broken on 30 August 1991. After how many years (rounded off to the nearest year) was Bob Beamon's record broken?

4. The table below shows the running times of two different athletes during the 2015 Comrades Marathon at various points along the route. One of the athletes won the marathon and the other came second.

Athlete A			Athlete B		
Place on the route	Distance run (km)	Total running time (h, min, s)	Place on the route	Distance run (km)	Total running time (h, min, s)
Lion Park	15,9	01:08:07	Lion Park	15,9	01:05:26
Camperdown	26,9	01:55:19	Camperdown	26,9	01:50:39
Halfway	45	03:13:20	Halfway	45	03:05:14
Pinetown	68,9	05:01:32	Pinetown	68,9	04:54:45
Mayville	82,3	06:03:07	Mayville	82,3	06:02:45
Finish	89,3	06:36:03	Finish	89,3	06:37:30

- 4.1 Who out of the two athletes came in first?
 4.2 How far is it from Pinetown to the Finish?
 4.3 How long (in hours, minutes and seconds) did it take Athlete A to run from Lion Park to Halfway?
 4.4 How long (in hours, minutes and seconds) did it take Athlete B to run from Camperdown to Mayville?
 4.5 Approximately where in the race did Athlete A overtake Athlete B? Explain your answer.

Speed

▪ **Speed:** The rate at which a person or object covers a distance in a specified period of time.

▪ **Formula:** Average speed = $\frac{\text{distance}}{\text{time}}$

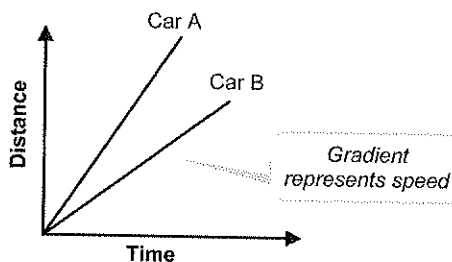
▪ **Calculations:**

Always remember to take note of the given units of the distance and time; in order to calculate the speed in particular units.

▪ **Graph:**

When distance is plotted against time, the resulting straight line represents the speed.

‣ The steeper the straight line, the faster the person or object is travelling.



NOTE!
Car A is travelling faster (i.e. has a greater speed) than Car B, as the straight line A is steeper than line B.

Worked Examples

1. Calculate the speed of a cyclist who cycles 129 km in 3 hours.

$$\begin{aligned}\text{Speed} &= \frac{129 \text{ km}}{3 \text{ h}} \\ &= 43 \text{ km/h}\end{aligned}$$

2. Trudy runs 0,2 km in 4 minutes.

Calculate her average speed in metres per second.

$$\begin{aligned}\text{Speed} &= \frac{0,2 \text{ km}}{4 \text{ min}} \\ &= \frac{200 \text{ metres}}{240 \text{ s}} \\ &= 0,83 \text{ m/s}\end{aligned}$$

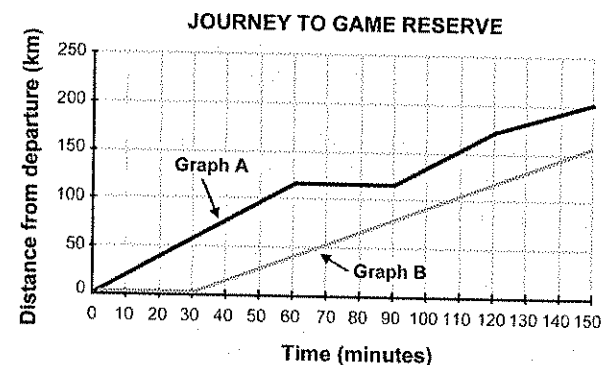
$$\begin{aligned}0,2 \text{ km} &= 0,2 \times 1\,000 = 200 \text{ m} \\ 4 \text{ min} &= 4 \times 60 = 240 \text{ s}\end{aligned}$$

3. Lihle travels the 110 km from Cape Town to Caledon in 1 hour 42 minutes. Calculate Lihle's average speed for the trip in km/h.

$$\begin{aligned}\text{Speed} &= \frac{110 \text{ km}}{1 \text{ h } 42 \text{ min}} \\ &= \frac{110 \text{ km}}{1,7 \text{ h}} \\ &= 64,71 \text{ km/h}\end{aligned}$$

$$\begin{aligned}1 \text{ h } 42 \text{ min} &= 1 \text{ h} + \frac{42}{60} \text{ min} \\ &= 1 \text{ h} + 0,7 \text{ h} \\ &= 1,7 \text{ h}\end{aligned}$$

4. Two families are driving from their home towards the game reserve for the weekend. The graphs below show the distance travelled by the two vehicles over time while travelling towards the game reserve. The game reserve is 200 km from where the families left for their journey.



Four statements are given below. You need to match each of the two graphs above to only one correct statement. Answer as follows: Graph A = Statement ? ; Graph B = Statement ?

- Statement 1: The vehicle travels at a constant speed before stopping for a while. It then continues at a constant speed, but later reduces to a slower constant speed just before arriving at the game reserve.
- Statement 2: The vehicle does not leave at the expected time and is delayed for 30 minutes. Once it begins, it travels at an inconsistent speed until it arrives at the game reserve.
- Statement 3: The vehicle travels at a constant speed before stopping. It then continues its journey at a constant speed, and increases to a faster constant speed just before arriving at the game reserve.
- Statement 4: The vehicle travelled at a constant speed and does not reach its destination within 2,5 hours.

Graph A = Statement 1 ; Graph B = Statement 4

For explanation
(see p. 128).

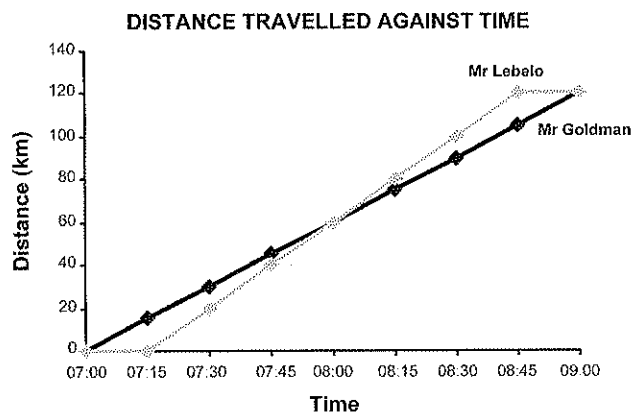
Statement 1 and Statement 3 are quite similar. They both seem to be describing Graph A - except in Statement 3 it says that the vehicle 'increases to a faster constant speed just before arriving at the game reserve'. You need to look at the slope of the graph. It is clear that the slope of the graph is less steep from 120 minutes. This means that less distance was covered per time unit $\therefore$ the vehicle was travelling at a slower constant speed. $\therefore$ Statement 1 matches graph A.

Statement 2 seems to describe Graph B. However, if you look at the last point on the graph - it has a distance value of ≈ 170 km at a time of 150 minutes. The question says that the game reserve is 200 km away from the start of the journey. $\therefore$ They had not yet reached their destination at 150 minutes (2,5 hours). $\therefore$ Statement 4 matches Graph B.



EXERCISE 2

1. A pilot flies a distance of 12 311 km from Cape Town to Miami. Calculate the average speed of the plane if the flight takes 15 hours 48 minutes?
2. Who was travelling the fastest?
* Neil who travelled 280 000 m in 2 hours 10 min
* Larry who travelled 26 000 000 cm in 120 min
3. The graph below shows the distance that Mr Goldman and Mr Lebelo each travelled. Study the graph and answer the questions that follow:



- 3.1 After how many hours does Mr Lebelo catch up with Mr Goldman?
- 3.2 What distance has Mr Goldman travelled when Mr Lebelo catches up with him?
- 3.3 What is the distance between Mr Lebelo and Mr Goldman after Mr Goldman has travelled exactly 90 minutes?
- 3.4 Calculate Mr Lebelo's average speed between 08:00 and 08:45.

Calendars and Timetables



- Remember the following conversions when working with calendars:
 - 1 year = 365 days
 - 1 year = ± 52 days
 - 1 month = ± 4 weeks
 - 1 week = 7 days
 - leap year = 366 days and occurs every 4 years
- Many different timetables are used in our daily life, e.g. school study timetable; transport timetables; construction timetables when building; tide timetables etc.
- Be sure to read the **keys** and study the timetables carefully when answering questions.

Worked Examples



1. Study the 2015 Calendar on p. 129 to answer the following questions:

- 1.1 What percentage of the days in the year are public holidays?

$$\frac{12}{365} \times 100 = 3,29\%$$

- 1.2 How many working days are there in the month of April?

Working days are
Mondays $\rightarrow$ Fridays



19 days

- 1.3 List the months that have five different dates that fall on a Monday.

March; June; August and November

- 1.4 Determine how many weeks and days there are from 28 February to 19 May.

11 weeks and 3 days



Holiday Calendar South Africa - 2015

January	February	March
S M T W T F S	S M T W T F S	S M T W T F S
1 2 3	1 2 3 4 5 6 7	1 2 3 4 5 6 7
4 5 6 7 8 9 10	8 9 10 11 12 13 14	8 9 10 11 12 13 14
11 12 13 14 15 16 17	15 16 17 18 19 20 21	15 16 17 18 19 20 21
18 19 20 21 22 23 24	22 23 24 25 26 27 28	22 23 24 25 26 27 28
25 26 27 28 29 30 31		29 30 31
April	May	June
S M T W T F S	S M T W T F S	S M T W T F S
1 2 3 4	1 2	1 2 3 4 5 6
5 6 7 8 9 10 11	3 4 5 6 7 8 9	7 8 9 10 11 12 13
12 13 14 15 16 17 18	10 11 12 13 14 15 16	14 15 16 17 18 19 20
19 20 21 22 23 24 25	17 18 19 20 21 22 23	21 22 23 24 25 26 27
26 27 28 29 30	24 25 26 27 28 29 30	28 29 30
July	August	September
S M T W T F S	S M T W T F S	S M T W T F S
1 2 3 4	1	1 2 3 4 5
5 6 7 8 9 10 11	2 3 4 5 6 7 8	6 7 8 9 10 11 12
12 13 14 15 16 17 18	9 10 11 12 13 14 15	13 14 15 16 17 18 19
19 20 21 22 23 24 25	16 17 18 19 20 21 22	20 21 22 23 24 25 26
26 27 28 29 30 31	23 24 25 26 27 28 29	27 28 29 30
October	November	December
S M T W T F S	S M T W T F S	S M T W T F S
1 2 3	1 2 3 4 5 6 7	1 2 3 4 5
4 5 6 7 8 9 10	8 9 10 11 12 13 14	6 7 8 9 10 11 12
11 12 13 14 15 16 17	15 16 17 18 19 20 21	13 14 15 16 17 18 19
18 19 20 21 22 23 24	22 23 24 25 26 27 28	20 21 22 23 24 25 26
25 26 27 28 29 30 31	29 30	27 28 29 30 31

■ South African Public Holidays

2. The Aqualine Bus travels to and from Durban North and Umhlanga Rocks in Durban. Study the bus transport timetable below and answer the following questions:

- 2.1 What Route No will the 07:30 bus be, that travels from the market on a Thursday to Durban-North?

Route No 716

- 2.2 How many departure times are there for a bus that travels from Durban-North to the City on a Monday?

2 departure times

- 2.3 What time do the buses run on a Saturday, travelling from Durban-North?

07:05 - 19:10

- 2.4 A worker usually finishes work at 5 pm and then walks for 10 min to catch the 716 bus from the market to Durban-North. If the worker leaves work 10 minutes late, how long must the worker wait for the next 716 bus home?

- Worker usually catches the 17:10 bus.
- Worker will arrive at bus stop at 17:20 (5 pm + 10 min late + 10 min walk).
- Next bus is at 18:10.
- ∴ Worker must wait 50 minutes to catch the next 716 bus.

MONDAY - FRIDAY FROM DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	TO	ROUTE NO.	TIME	TO
716	07:15	City	716	15:00	Market
716	08:00	Market	716	16:10	Market
716	08:20	City	701	16:30	Market
716	08:30	Market	716	17:10	Market
716	09:00	Market	716	18:10	Market
716	10:00	Market	716	19:10	Market
716	13:00	Market			

MONDAY - FRIDAY TO DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	FROM	ROUTE NO.	TIME	FROM
716	06:15	Market	716	08:00	Market
716	07:00	Market	716	09:00	Market
706	07:10	City	716	12:00	Market
701	07:20	City	706	16:10	City
716	07:30	Market	716	18:10	Market

SATURDAY FROM DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	TO	ROUTE NO.	TIME	TO
716	07:05	City-Market	716	17:15	City-Market
716	09:00	City-Market	716	19:10	City-Market
716	13:30	City-Market			

SATURDAY TO DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	FROM	ROUTE NO.	TIME	FROM
716	06:10	Market-City	716	16:15	Market-City
716	08:00	Market-City	716	18:10	Market-City
716	12:30	Market-City			

SUNDAY FROM DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	TO	ROUTE NO.	TIME	TO
716	07:15	City-Market	716	17:15	City-Market
716	09:00	City-Market	716	19:10	City-Market
716	16:15	City-Market			

SUNDAY TO DURBAN NORTH - UMHLANGA ROCKS					
ROUTE NO.	TIME	FROM	ROUTE NO.	TIME	FROM
716	06:10	Market-City	716	16:15	Market-City
716	08:00	Market-City	716	18:10	Market-City
716	08:30	Market-City			

3. Fishermen, surfers and scuba divers all keep a close eye on the tides of the ocean.

Study the tide timetable below and answer the following questions:

Monday 8 December	4.32 am	High Tide	1,89 metres
	4.47 am	Sunrise	
	6.28 am	Moonset	
	10.35 am	Low Tide	0,35 metres
	4.42 pm	High Tide	1,93 metres
	6.47 pm	Sunset	
	8.23 pm	Moonrise	
	10.55 pm	Low Tide	0,27 metres

- 3.1 How many high tides and low tides are there in a day?

2 high tides and 2 low tides

- 3.2 Write down the time the sun sets in a 24-hour format.

Sunset at 6.47 pm = 18:47

- 3.3 Is the time difference between the 2 high tides, the same time difference as between the 2 low tides? Show all calculations to substantiate your answer.

- High tides at 4.32 am and 4.42 pm

$\therefore 16:42$

- 04:32

12:10 $\therefore$ 12 hours; 10 minutes

- Low tides at 10.35 am and 10.55 pm

$\therefore 22:55$

10:35

12:20 $\therefore$ 12 hours; 20 minutes

- Therefore, the time difference is not the same.



Test Your Understanding



1. The Van Zyl family are on holiday in the Kruger Park and are staying at Letaba Camp.

- 1.1 It takes 3 hours and 45 minutes to travel from Letaba Camp to N'wanetsi Get Out Point. Write this time down in decimal format [i.e. 1 h 30 min = 1,5 h]

- 1.2 Using Appendix C and D, determine the closing time of the shop at Letaba Camp during the month of July.

APPENDIX C

Gate Times for all Gates			
	March	April	May - July
Gates open	05:30	06:00	06:00
Gates close	18:00	18:00	17:30

APPENDIX D

Restaurants	
Breakfast	07:00 - 09:00
Lunch	12:00 - 14:00
Dinner	18:00 - 21:00
Shops	
08:00 to half an hour after gate closing time	

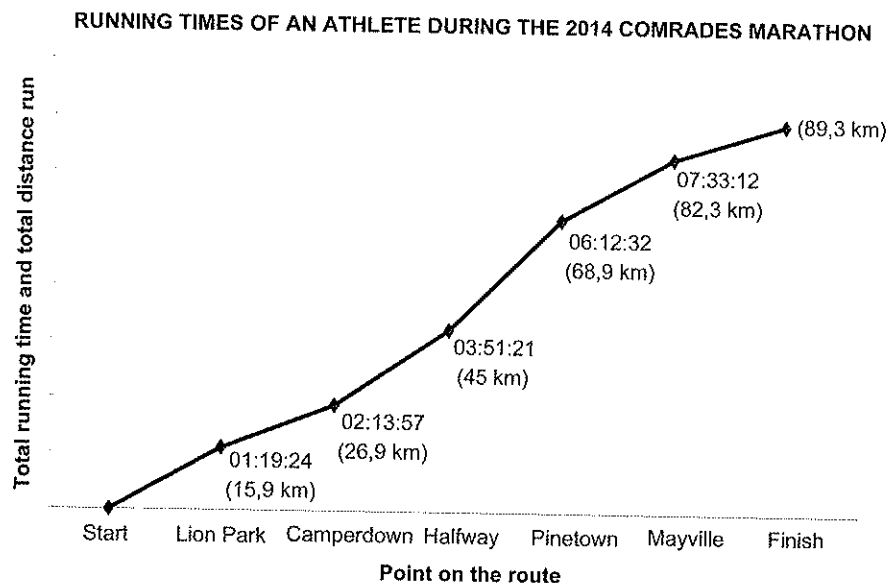
[Source: Visitors Guide to the Kruger National Park]

- 1.3 How long are the Kruger Park gates open for in March?
- 1.4 Whilst having tea one sunny July afternoon at Olifants Camp (which is 30 km from Letaba Camp), the Van Zyl family realise they need to return to Letaba Camp where they will spend the night. They leave Olifants Camp at a quarter past five.

Use the formula: $\text{average speed} = \frac{\text{distance}}{\text{time}}$ to determine:

- 1.4.1 How long will it take them to get to Letaba camp if they stick to the speed limit of 50 kilometres per hour? Give your answer to the nearest minute.
- 1.4.2 By how many minutes will they miss the closing time of Letaba Camp?
- 1.4.3 At what speed would Mr Van Zyl have to travel in order to make the closing time of the gate?

2. The graph below shows the running times of an athlete at various points along the route of the 2014 Comrades Marathon.



- 2.1 Calculate the average speed (in km/min) at which this athlete ran from the Start to Mayville and then use this average speed to approximate what the athlete's finishing time was. Round off the average speed and time to 3 decimal places.
- 2.2 Do you think the athlete ran faster or slower from Mayville to the finish than from Pinetown to Mayville? Explain your answer.
3. Study the UCT 2015 Semester 1 Timetable shown alongside together with the 2015 South African Calendar on p. 129 to answer the following questions.
- 3.1 On what day of the week does Sax Appeal Day fall?
- 3.2 On what date do students graduate?
- 3.3 List the public holidays which fall within the first semester academic term?
[Note: Academic term refers to the time during which students are studying academics at university, i.e. not during vacations nor weekends].
- 3.4 How many academic days are there from the day that lectures resume on 7 April until lectures end on 20 May?

UCT 2015 Semester 1

Week	Dates	Description
1	01 Jan - 03 Jan	New Years day 01 January * University Libraries open 02 January
2	04 Jan - 10 Jan	
3	11 Jan - 17 Jan	
4	18 Jan - 24 Jan	Summer School 19-23 January Summer School commences 19 January Supplementary and Deferred Exams 19-23 January
5	25 Jan - 31 Jan	Summer School 226-30 January Supplementary and Deferred Exam results due to records on 30 January Early Orientation – 31 January
6	01 Feb - 07 Feb	Residences open for new students 2 February Parent Orientation 2 February Early Orientation 2 and 3 February Registration and Orientation 4-6 February
7	08 Feb - 14 Feb	Registration 9-11 February Sax Appeal Day 12 February First meeting of UG classes 13 February Lectures begin 16 February
8	15 Feb - 21 Feb	
9	22 Feb - 28 Feb	
10	01 Mar - 07 Mar	
11	08 Mar - 14 Mar	
12	15 Mar - 21 Mar	Human Rights day 21 March*
13	22 Mar - 28 Mar	GSB 2014 eMBA and MBA results to records by 12h00 on 24 March
14	29 Mar - 04 Apr	Short Vacation 30 March to 6 April Commerce (GSB) FEB 31 March Good Friday 3 April*
15	05 Apr - 11 Apr	Easter 5 April Family Day 6 April* Lectures resume 7 April
16	12 Apr - 18 Apr	
17	19 Apr - 25 Apr	
18	26 Apr - 02 May	Freedom Day 27 April* Workers Day 1 May*
19	03 May - 09 May	
20	10 May - 16 May	
21	17 May - 23 May	Lectures end 20 May Consolidation 21-22 May
22	24 May - 30 May	Consolidation 25-26 May Examinations 27-29 May 2 Examination sessions 29 May
23	31 May - 06 Jun	Examinations
24	07 Jun - 13 Jun	Examinations 8-12 June Graduations 11-12 June 2 Graduations sessions 12 June Vacation commences 13 June
25	15 Jun - 20 Jun	Mid-year Vacation Winter Term commences 15 June Youth Day 16 June*
26	21 Jun - 27 Jun	Mid-year vacation
27	28 Jun - 04 Jul	Mid-year vacation Mid-year results on system by 3 July
28	05 Jul - 11 Jul	Mid-year vacation
29	12 Jul - 18 Jul	Winter term ends 10 July Mid-year vacation Winter Term examinations 14-15 July

4. Sally stays in Rocklands and travels 3,7 km to work, at the Cape Town International Convention Centre. Sally's working hours are Monday to Friday, 8 am to 5 pm. Study the MyCiti Transport Table and Transport Fares below and alongside to answer the following questions.

- 4.1 How long is the bus trip from Rocklands to the Convention Centre?
- 4.2 What is the latest bus that Sally can catch, in order to arrive at work on time?
- 4.3 What time will Sally arrive at home, if she catches the first available bus straight after work finishes?
- 4.4 Sally and her team had a major deadline to meet, so they worked until 6:30 pm; after which they all decided to go out for a celebratory drink at the Waterfront. They plan to meet at 7 pm, for 2 hours. Detail Sally's best transport bus trip route and calculate her total travelling time on the bus for that day.
- 4.5 Calculate how much Sally could save per month, if she travelled on MyCiti Mover Fare package, as opposed to a MyCiti Standard Fare package. Assume Sally always travels to work on the 07:10 bus; and back home on the 17:13 bus; for 4 weeks per month.

104: SEA POINT - WATERFRONT - CIVIC CENTRE

MONDAY TO FRIDAY

☐ Peak fare period

☐ Saver fare period

Direction: to Civic Centre

Queens Beach	Dep	06:42	07:03	07:26	07:41	08:03	08:25	08:48	09:09	0934	09:59
Cassel		06:43	07:04	07:27	07:42	08:04	08:26	08:49	09:14	09:39	10:04
Sea Point Pool		06:44	07:05	07:28	07:43	08:05	08:27	08:50	09:15	09:40	10:05
Boat Bay		06:45	07:06	07:29	07:44	08:06	08:28	08:51	09:16	09:41	10:06
Graaffs Pool		06:46	07:07	07:30	07:45	08:07	08:29	08:52	09:17	09:42	10:07
London		06:47	07:08	07:31	07:46	08:08	08:30	08:53	09:18	09:43	10:08
Promenade		06:48	07:09	07:32	07:47	08:09	08:31	08:54	09:19	09:44	10:09
Rocklands		06:49	07:10	07:33	07:48	08:10	08:32	08:55	09:20	09:45	10:10
Three Anchor Bay		06:51	07:12	07:35	07:50	08:12	08:34	08:57	09:22	09:47	10:12
Lighthouse		06:52	07:13	07:36	07:51	08:13	08:35	08:58	09:23	09:48	10:13
Surrey		06:53	07:14	07:37	07:52	08:14	08:36	08:59	09:24	09:49	10:14
Mouille Point		06:54	07:15	07:38	07:53	08:15	08:37	09:00	09:25	09:50	10:15
Granger		06:55	07:16	07:39	07:54	08:16	08:38	09:01	09:26	09:51	10:16
Somerses Hospital		06:56	07:17	07:40	07:55	08:17	08:39	09:02	09:27	09:52	10:17
Breakwater		06:58	07:19	07:42	07:57	08:19	08:41	09:04	09:29	09:54	10:19
Waterfront		06:58	07:19	07:42	07:57	08:19	08:41	09:04	09:29	09:54	10:19
Waterfront		07:01	07:22	07:45	08:00	08:22	08:44	09:07	09:33	09:58	10:23
Breakwater		07:02	07:23	07:46	08:01	08:23	08:45	09:08	09:34	09:59	10:24
Nobel Square		07:04	07:25	07:48	08:03	08:25	08:47	09:10	09:36	10:01	10:26
Aquarium		07:05	07:26	07:49	08:04	08:26	08:48	09:11	09:37	10:02	10:27
Marina		07:07	07:28	07:51	08:06	08:28	08:50	09:13	09:39	10:04	10:29
Waterfront Silo		07:09	07:30	07:53	08:08	08:30	08:52	09:15	09:41	10:06	10:31
Amsterdam		07:11	07:32	07:55	08:10	08:32	08:54	09:17	09:41	10:06	10:31
Convention Centre		07:13	07:34	07:57	08:12	08:34	08:56	09:19	09:42	10:07	10:32
Foreshore		07:16	07:37	08:00	08:15	08:37	08:59	09:22	09:44	10:09	10:34
Civic Centre	Arr	07:20	07:41	08:04	08:19	08:41	09:03	09:26	09:47	10:12	10:37

104: SEA POINT - WATERFRONT - CIVIC CENTRE

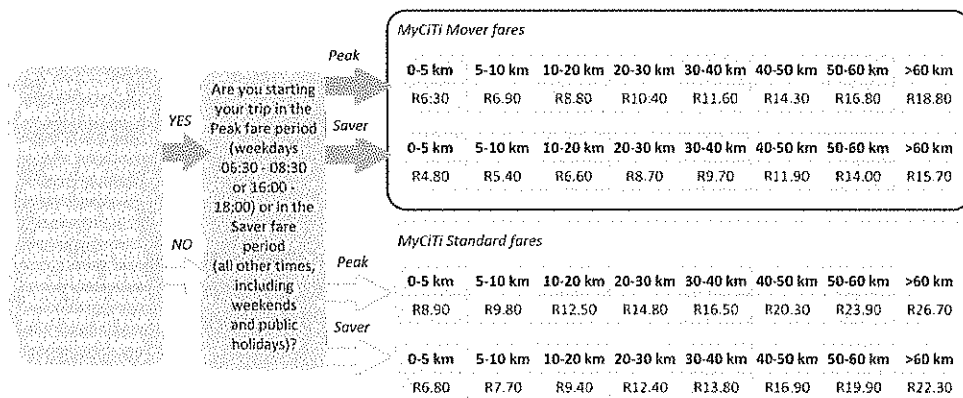
MONDAY TO FRIDAY

☐ Peak fare period

☐ Saver fare period

Direction: to Sea Point

Civic Centre	Dep	16:48	17:06	17:30	17:51	18:12	18:30	18:54	19:34	20:13	20:52
Foreshore		16:52	17:10	17:34	17:55	18:16	18:34	18:58	19:36	20:15	20:54
Convention Centre		16:55	17:13	17:37	17:58	18:19	18:37	19:01	19:39	20:18	20:57
Amsterdam		16:58	17:16	17:40	18:01	18:22	18:40	19:04	19:41	20:20	20:59
Waterfront Silo		17:00	17:18	17:42	18:03	18:24	18:42	19:06	19:43	20:22	21:01
Marina		17:02	17:20	17:44	18:05	18:26	18:44	19:08	19:45	20:24	21:03
Aquarium		17:04	17:22	17:46	18:07	18:28	18:46	19:10	19:46	20:25	21:04
Nobel Square		17:05	17:23	17:47	18:08	18:29	18:47	19:11	19:47	20:26	21:05
Breakwater		17:07	17:25	17:49	18:10	18:31	18:49	19:13	19:49	20:28	21:07
Waterfront		17:08	17:26	17:50	18:11	18:32	18:50	19:14	19:51	20:30	21:09
Waterfront		17:08	17:26	17:50	18:11	18:32	18:50	19:14	19:51	20:30	21:09
Breakwater		17:09	17:27	17:51	18:12	18:33	18:51	19:15	19:52	20:31	21:10
Somerses Hospital		17:11	17:29	17:53	18:14	18:35	18:53	19:17	19:53	20:32	21:11
Granger		17:12	17:30	17:54	18:15	18:36	18:54	19:18	19:54	20:33	21:12
Mouille Point		17:13	17:31	17:55	18:16	18:37	18:55	19:19	19:55	20:34	21:13
Surrey		17:14	17:32	17:56	18:17	18:38	18:56	19:20	19:56	20:35	21:14
Lighthouse		17:15	17:33	17:57	18:18	18:39	18:57	19:21	19:56	20:35	21:14
Three Anchor Bay		17:16	17:34	17:58	18:19	18:40	18:58	19:22	19:57	20:36	21:15
Rocklands		17:18	17:36	18:00	18:21	18:42	19:00	19:24	19:58	20:37	21:16
Promenade		17:19	17:37	18:01	18:22	18:43	19:01	19:25	19:59	20:38	21:17
London		17:20	17:38	18:02	18:23	18:44	19:02	19:26	20:00	20:39	21:18
Graaffs Pool		17:21	17:39	18:03	18:24	18:45	19:03	19:27	20:01	20:40	21:19
Boat Bay		17:22	17:40	18:04	18:25	18:46	19:04	19:28	20:02	20:41	21:20
Sea Point Pool		17:23	17:41	18:05	18:26	18:47	19:05	19:29	20:03	20:42	21:21
Cassel		17:24	17:42	18:06	18:27	18:48	19:06	19:30	20:04	20:43	21:22
Queens Beach	Arr	17:25	17:43	18:07	18:28	18:49	19:07	19:31	20:04	20:43	21:22



5. Pete is an avid leisure fisherman and keeps an eye on the tide timetable. Study the Simon's Town Tide Timetable for a part of November 2014 below in order to answer the following questions:
- 5.1 Write down the solar times for 3 November in 12-hour format.
 - 5.2 Calculate the time difference between the Spring Tide and Lowest Tide on 6 November.
 - 5.3 On which days should Pete go out fishing? Give a reason for your answer.
 - 5.4 How much later is the first low tide on 4 November, as opposed to the first low tide on 5 November?
 - 5.5 Pete is only able to go out fishing on Saturday, 8 November, from 2:15 pm - 5:00 pm. What percentage of his fishing time out will be considered as the 'best fishing time'?

**SIMON'S TOWN TIDE TABLE
NOVEMBER 2014**

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
FISH-O-METER Good fishing days Slow fishing days Solar Theory Please note that the best fishing periods are for the leisure fisherman and peak fishing times during night time have not been listed. Spring Tide: 06 Nov 2014 @ 14:43 (1,92 m) Lowest Tide: 06 Nov 2014 @ 21:02 (0,23 m)						 SOLAR Sunrise 05:45 Sunset 19:14 LUNAR Moonset 01:58 Underfoot 07:42 Moonrise 13:31 Overhead 20:08 TIDE TIMES Low tide 04:00 High tide 10:19 Low tide 17:01 High tide 23:04 BEST FISHING 
 SOLAR Sunrise 05:44 Sunset 19:15 LUNAR Moonset 02:39 Underfoot 08:35 Moonrise 14:37 Overhead 21:01 TIDE TIMES High tide 05:20 High tide 11:33 Low tide 18:07 High tide ---- BEST FISHING 	 SOLAR Sunrise 05:43 Sunset 19:16 LUNAR Moonset 03:19 Underfoot 09:27 Moonrise 15:42 Overhead 21:53 TIDE TIMES High tide 06:12 Low tide 06:21 High tide 12:31 Low tide 18:59 BEST FISHING 	 SOLAR Sunrise 05:42 Sunset 19:17 LUNAR Moonset 03:58 Underfoot 10:20 Moonrise 16:48 Overhead 22:46 TIDE TIMES High tide 01:04 Low tide 07:11 High tide 13:19 Low tide 19:44 BEST FISHING 	 SOLAR Sunrise 05:41 Sunset 19:18 LUNAR Moonset 04:38 Underfoot 11:13 Moonrise 17:54 Overhead 23:39 TIDE TIMES High tide 01:49 Low tide 07:55 High tide 14:02 Low tide 20:24 BEST FISHING 	 SOLAR Sunrise 05:40 Sunset 19:19 LUNAR Moonset 05:19 Underfoot 12:06 Moonrise 18:59 Overhead -- TIDE TIMES High tide 02:30 Low tide 08:36 High tide 14:43 Low tide 21:02 BEST FISHING 	 SOLAR Sunrise 05:40 Sunset 19:20 LUNAR Overhead 00:33 Moonset 06:03 Underfoot 13:00 Moonrise 20:01 TIDE TIMES High tide 02:30 Low tide 08:36 High tide 14:43 Low tide 21:02 BEST FISHING 	 SOLAR Sunrise 05:39 Sunset 19:21 LUNAR Overhead 01:27 Moonset 06:50 Underfoot 13:54 Moonrise 21:01 TIDE TIMES High tide 03:47 Low tide 09:51 High tide 15:58 Low tide 22:12 BEST FISHING 

UNIT

7

CALCULATING PERIMETER, AREA, TOTAL SURFACE AREA AND VOLUME

4

Perimeter



Definition:

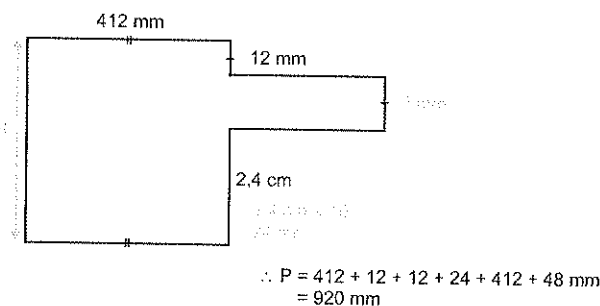
- ▶ The border or outer boundary lengths of a two-dimensional figure
- ▶ The perimeter of a circle is known as the circumference

Units of Perimeter:

- ▶ Since perimeter has to do with lengths, the units of perimeter are mm; cm; m; or km.

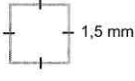
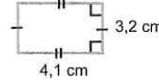
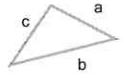
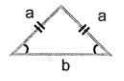
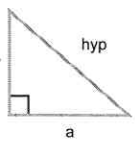
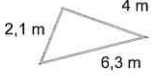
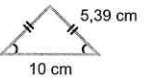
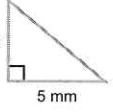
Calculations:

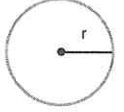
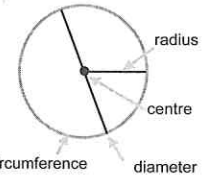
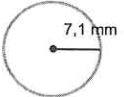
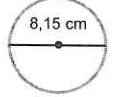
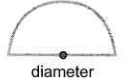
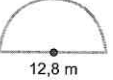
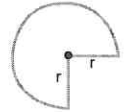
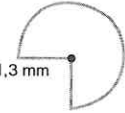
- ▶ Break the complex shape down into its basic shapes, such as a square, rectangle, triangle and/or circle.
- ▶ Be sure to remember that we will only add the outermost lengths of the shape, therefore the formulas for perimeter for basic shapes may not always be applicable.
- ▶ Check that all measurements are in the same units, otherwise convert all measurements into the most appropriate, but same unit e.g. Calculate the perimeter of the following shape:



- ▶ Round off your *final* answer to 2 decimal places; unless otherwise stated.
- ▶ Remember that **products** are often **sold per whole m/m²/m³** etc., so then round off to the nearest whole number when calculating costs.

THE PERIMETER OF A SQUARE, RECTANGLE, TRIANGLE AND CIRCLE

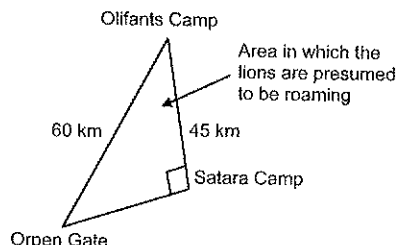
SHAPE	CHARACTERISTICS	PERIMETER (P)	WORKED EXAMPLES
SQUARE  side = s	- All four sides equal in length - Angles of sides meet at 90°	$P = 4 \times \text{side}$ $= 4 \times s$	 $P = 4 \times 1,5 \text{ mm}$ $= 6 \text{ mm}$
RECTANGLE  breadth = b length = l	- Opposite sides are equal in length - Angles of sides meet at 90°	$P = (2 \times \text{length}) + (2 \times \text{breadth})$ $= 2l + 2b$ $= 2(l + b)$	 $P = 2(4,1 + 3,2)$ $= 14,6 \text{ cm}$
TRIANGLE a) scalene  b) isosceles  c) right-angle 	- All sides are different in length - Two sides are equal in length - Base angles of the triangle are equal 90° angle in triangle - Side opposite 90° called the hypotenuse - Theorem of Pythagoras: $\text{hyp}^2 = \text{side}_1^2 + \text{side}_2^2$	$P = \text{side}_1 + \text{side}_2 + \text{side}_3$ $= a + b + c$ $P = \text{side}_1 + \text{side}_2 + \text{side}_3$ $= a + a + b$ $= 2a + b$ $P = \text{side}_1 + \text{side}_2 + \text{side}_3$ $= a + b + \text{hyp}$	 $P = 2,1 + 4 + 6,3$ $= 12,4 \text{ m}$  $P = 2(5,39) + 10$ $= 20,78 \text{ cm}$  $\text{hyp}^2 = \text{side}_1^2 + \text{side}_2^2$ $= (5)^2 + (12)^2$ $= 169$ $\therefore \text{hyp} = \sqrt{169}$ $= 13 \text{ mm}$ [Pythagoras] $\therefore P = 5 + 12 + 13$ $= 30 \text{ mm}$

SHAPE	CHARACTERISTICS	PERIMETER/ CIRCUMFERENCE (C)	WORKED EXAMPLES
CIRCLE a) full/ complete  r = radius	Circle terminology:  r = radius	$C = 2 \times \pi \times \text{radius}$ where $\pi = 3,142$ REMEMBER! C = circumference which is the perimeter. OR $C = 2 \times \pi \times \text{radius}$ $= \pi \times \text{diameter}$ REMEMBER! $2 \times \text{radius} = \text{diameter}$	 $C = 2 \times \pi \times \text{radius}$ $= 2 \times 3,142 \times 7,1$ $= 44,62 \text{ mm}$ Round off to 2 DP. OR  $C = \pi \times \text{diameter}$ $= 3,142 \times 8,15$ $= 25,61 \text{ cm}$
b) semicircle (½)  diameter	- Circumference is the perimeter of the circle - Radius extends from the centre of the circle to the circumference - Diameter extends from one side of the circle, through the centre, to the other side of the circle - diameter = 2 × radius - OR - radius = diameter ÷ 2	$C = \frac{1}{2} \text{ circumference of full circle} + \text{diameter}$ $= [\frac{1}{2} \times (2 \times \pi \times r)] + \text{diameter}$	 $r = \text{diameter} \div 2$ $= 12,8 \div 2 = 6,4 \text{ m}$ $\therefore C = [\frac{1}{2} \times (2 \times 3,142 \times 6,4)] + 12,8$ $= 32,91 \text{ m}$
c) quarter (¼)  r	Quarter of a full circle	$C = \frac{1}{4} \text{ of circumference of full circle}$ $= [\frac{1}{4} \times (2 \times \pi \times r)] + (2 \times r)$	 $C = [\frac{1}{4} \times (2 \times 3,142 \times 6,6)] + (2 \times 6,6)$ $= 23,57 \text{ cm}$
d) three quarter (¾)  r	Three-quarters of a full circle	$C = \frac{3}{4} \text{ of circumference of full circle} + 2 \text{ radii}$ $= [\frac{3}{4} \times (2 \times \pi \times r)] + (2 \times r)$	 $C = [\frac{3}{4} \times (2 \times 3,142 \times 1,3)] + (2 \times 1,3)$ $= 8,73 \text{ mm}$

Worked Examples



1. The angle formed at Satara Camp can be presumed to be 90° . The direct distance between Olifants Camp and Satara Camp is 45 km and the direct distance between Olifants Camp and the Orpen Gate is 60 km. Calculate the direct distance between Satara Camp and the Orpen Gate.



$$\begin{aligned} \text{Hypotenuse}^2 &= \text{side}_1^2 + \text{side}_2^2 \text{ (Pythagoras)} \\ (60)^2 &= (45)^2 + (\text{distance})^2 \\ 60^2 - 45^2 &= \text{distance}^2 \\ 1\,575 &= \text{distance}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Distance} &= \sqrt{1\,575} \\ \text{Distance} &= 39,69 \text{ km} \end{aligned}$$

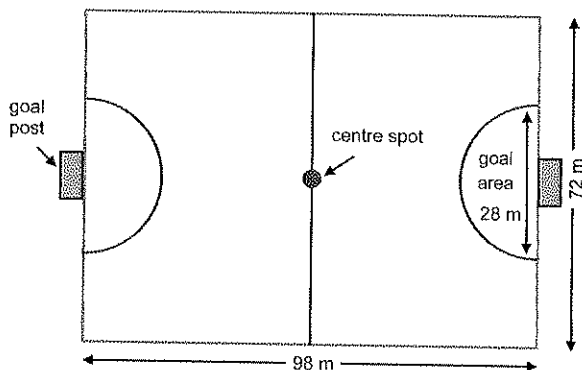
2. Two weeks before the final hockey match, a concert was held on the hockey field. The stadium manager inspected the field after the concert and found that some of the lines on the field were unclear and part of the grass on the field was damaged.

The dimensions of the hockey field are:

length = 98 m

breadth = 72 m

diameter of the goal area = 28 m



- 2.1 If all the boundary lines, plus the hockey field internal lines, have to be re-marked; determine the total length of all the lines that have to be re-marked.

$$\begin{aligned} \text{Boundary lines: } P &= 2(\ell + b) \\ &= 2(98 + 72) = 340 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Internal lines: } P &= 2 \times \frac{1}{2} \text{ circle} + \text{length} \\ &= (2 \times \frac{1}{2} \times 2 \times \pi \times r) + b \\ &= (2 \times \frac{1}{2} \times 2 \times 3,142 \times 14) + 72 \\ &= 159,98 \text{ m} \end{aligned}$$

$$\therefore \text{Total length} = 340 + 159,98 = 499,98 \text{ m}$$

$$\begin{aligned} \text{Radius} &= \text{diameter} \div 2 \\ &= 28 \div 2 = 14 \text{ m} \end{aligned}$$

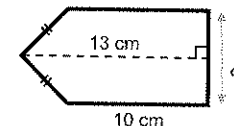


- 2.2 Calculate the total cost of remarking all the lines, if the line tape costs R0,45 per metre.

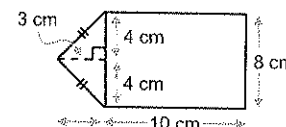
Since line tape is sold per metre, $499,98 \text{ m} \approx 500 \text{ m}$

$$\therefore \text{Total cost} = 500 \text{ m} \times \text{R}0,45 = \text{R}225$$

3. Lauren works at a marketing company and often needs to put up sign arrow boards. The outside of the arrow is pointed in red (indicated in bold), as shown alongside. Calculate the total length of red line, which makes up the outside of the arrow.



First break down the complex shape into the basic shapes and then fill in all relevant lengths:



- Perimeter of rectangle
 $= 10 + 8 + 10$
 $= 28 \text{ cm}$

The formula for perimeter of a rectangle cannot be used, as there are only 3 lengths on the outside of the arrow.

- Perimeter of triangle:

$$\begin{aligned} \text{Hypotenuse}^2 &= \text{side}_1^2 + \text{side}_2^2 \text{ (Pythagoras)} \\ &= (4)^2 + (3)^2 \\ \text{Hypotenuse}^2 &= 25 \end{aligned}$$

$$\therefore \text{Hypotenuse} = \sqrt{25} = 5 \text{ cm}$$

$$\therefore \text{Perimeter of triangle} = 5 + 5 = 10 \text{ cm}$$

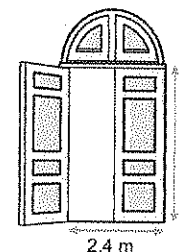
- Total perimeter of arrow = $28 \text{ cm} + 10 \text{ cm} = 38 \text{ cm}$



4. The doorway alongside has special moulding along the outside edges.

- 4.1 Find the length of moulding needed if the dimensions of the doorway are $2,4 \text{ m} \times 2,7 \text{ m}$.

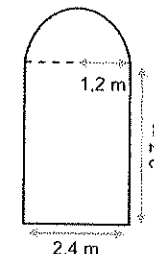
First break down the complex shape into the basic shapes and then fill in all relevant lengths.



- Perimeter of rectangle
 $= 2,7 + 2,4 + 2,7 = 7,8 \text{ m}$

- Circumference of $\frac{1}{2}$ circle
 $= \frac{1}{2} \times 2 \times \pi \times \text{radius}$
 $= \frac{1}{2} \times 2 \times 3,142 \times 1,2$
 $= 3,77 \text{ m}$

- $\therefore$ Total perimeter of doorway
 $= 7,8 \text{ m} + 3,77 = 11,57$



$$\begin{aligned} \text{Diameter} &= 2,4 \text{ m} \\ \therefore \text{Radius} &= 2,4 \div 2 \\ &= 1,2 \text{ m} \end{aligned}$$



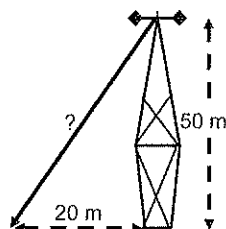
4.2 Determine the total cost of the moulding (incl. VAT), if it is sold per metre and quoted as R57,99/m excl. VAT.

- Sold per metre $\therefore 11,57 \text{ m} \approx 12 \text{ m}$
- $\text{VAT} = \frac{14}{100} \times R57,99 = R8,12$
- Price incl. VAT = $R57,99 + R8,12 = R66,11$
- $\therefore \text{Total cost of moulding} = R66,11 \times 12 \text{ m} = R793,32$

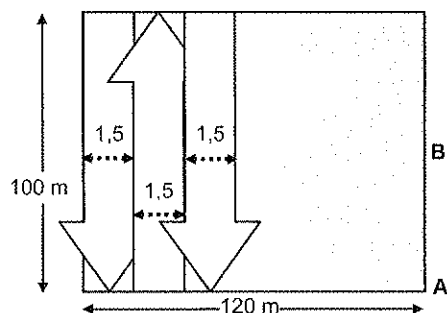
EXERCISE 1

Use the formulae on p. 134 to answer the following questions:

1. Electricity towers are tall structures that have a small base. The tower is anchored to the ground with very strong cables. If a tower is 50 m high and the anchor point is 20 m away from the base of the tower, how long must the anchor cable be?



2. The groundsman needs to mow the lawn with a big tractor-type lawnmower. The width of the cutter is 1,5 metres and he mows back and forth parallel to the width of the field, as shown alongside:



- 2.1 How far does the groundsman travel when he covers the field exactly once, assuming that his paths do not overlap?
- 2.2 The tractor pulling the cutter travels at 1,8 metres per second. How long will it take the groundsman to cut the grass? Give your answer in minutes and seconds.

3. A bus tyre has a diameter of 120 cm. The ratio of the diameter of a bus tyre to the diameter of a minibus tyre is 12 : 7.

Calculate the distance travelled by the minibus (rounded off to the nearest km) if the minibus's tyre rotated 1 862 times during the journey.

The following formula may be used:

Circumference = $2 \times \pi \times r$ where r = radius and using $\pi = 3,142$

Number of rotations = $\frac{\text{distance travelled}}{\text{circumference of tyre}}$

Area



Definition:

► The surface enclosed by the boundary lengths of a two-dimensional figure

Units of Area:

► Since area is the product of two-dimensional lengths, the units of area are mm^2 ; cm^2 ; m^2 or km^2

THE AREA OF A SQUARE, RECTANGLE, TRIANGLE AND CIRCLE

SHAPE	AREA (A)	WORKED EXAMPLES
SQUARE side = s	$A = \text{side} \times \text{side}$ $= (\text{side})^2$	 $A = (6,8)^2$ $= 46,24 \text{ mm}^2$
RECTANGLE breadth = b length = l	$A = \text{length} \times \text{breadth}$ <i>breadth = width</i>	 $A = 12,3 \times 9$ $= 110,70 \text{ cm}^2$
TRIANGLE base $\perp$ height	$A = \frac{1}{2} \times \text{base} \times \perp \text{ height}$ <i>$\perp$ means perpendicular (i.e. a height which touches a base at 90°)</i>	 $A = \frac{1}{2} \times 7,1 \times 4 = 14,2 \text{ m}^2$ $A = \frac{1}{2} \times 8 \times 12,69 = 50,76 \text{ m}^2$ <i>The hypotenuse (15 m) is not needed to calculate the area of a Δ.</i>
CIRCLE 	$A = \pi \times (\text{radius})^2$ where $\pi = 3,142$ <i>If the area of half a circle is asked, then $A = \frac{1}{2} \times [\pi \times (\text{radius})^2]$</i>	 $A = 3,142 \times (7,9)^2$ $= 196,09 \text{ cm}^2$ <i>Use the approximate value of $\pi = 3,142$ to 3 DP, not the π button on your calculator.</i> <i>Round off to 2 DP!</i>

Calculations:

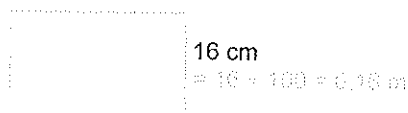
- Break the complex shape down into its basic shapes, such as a square, rectangle, triangle and/or circle.
- Add the areas of all the basic shapes together to get the area of the complex shape.
- First convert all lengths to the specified unit of the question; and then calculate the area.

Do NOT do the conversions after calculating the length!



e.g. Calculate the area of the following shape and give your answer in m^2 .

$$4 \text{ cm} = 4 \div 100 = 0,04 \text{ m}$$



$$\begin{aligned} \therefore \text{Area} &= \ell \times b \\ &= 0,04 \times 0,16 \\ &= 0,0064 m^2 \end{aligned}$$

NOTE!

- If you first calculated the area in cm^2 : $\therefore A = \ell \times b$
 $= 4 \times 16$
 $= 64 \text{ cm}^2$



- To convert cm^2 to m^2 , we need to divide by the (conversion factor)²
 $\therefore 64 \text{ cm}^2 = 64 \div (100)^2 = 0,0064 \text{ m}^2$

The conversion factor is **squared** as both lengths are converted from cm to m.

NOTE! $64 \text{ cm}^2 \neq 64 \div 100 \neq 0,64 \text{ m}^2$

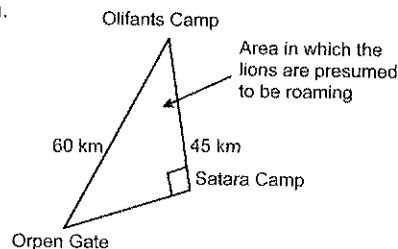
- Round-off your *final* answer to 2 decimal places unless otherwise stated
- Remember that **products** are often **sold per whole m^2/m^3** etc.; so then round-up to the nearest whole number when calculating costs.

Worked Examples

- The angle formed at Satara Camp can be presumed to be 90° . The direct distance between Olifants Camp and Satara Camp is 45 km and the direct distance between Olifants Camp and the Orpen Gate is 60 km.

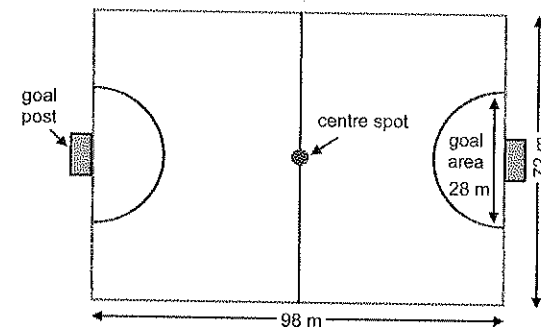
Calculate the area in which the lions are presumed to be roaming.

$$\begin{aligned} A &= \frac{1}{2} \times \text{base} \times \perp \text{height} \\ &= \frac{1}{2} \times 39,69 \times 45 = 893,03 \text{ km}^2 \end{aligned}$$



- Two weeks before the final hockey match, a concert was held on the hockey field. The stadium manager inspected the field after the concert and found that some of the lines on the field were unclear and part of the grass on the field was damaged.

The dimensions of the hockey field are: length = 98 m
breadth = 72 m



The diameter of the goal area = 28 m

- Calculate the area of ONE goal area that has to be re-grassed.

$$\begin{aligned} \text{Area of } \frac{1}{2} \text{ circle} &= \frac{1}{2} \times \pi \times (\text{radius})^2 \\ &= \frac{1}{2} \times 3,142 \times (14)^2 \\ &= 307,92 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Radius} &= \text{diameter} \div 2 \\ &= 28 \div 2 = 14 \text{ m} \end{aligned}$$

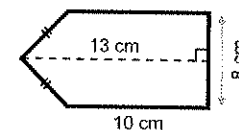


- Calculate the cost of re-grassing ONE goal area, if grass is sold at R68,20/ m^2 .

Since grass is sold per m^2 , $307,92 \text{ m}^2 \approx 308 \text{ m}^2$

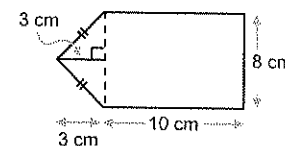
$$\therefore \text{Total cost} = R68,20 \times 308 \text{ m}^2 = R21\,005,60$$

- Lauren works at a marketing company and often needs to put up sign arrow boards. The outside of the arrow is pointed in red (indicated in bold), as shown alongside:



- Calculate the area of the sign arrow board.

First break down the complex shape into the basic shapes and then fill in all relevant lengths:



- Area of rectangle = $\ell \times b$
 $= 10 \times 8$
 $= 80 \text{ cm}^2$

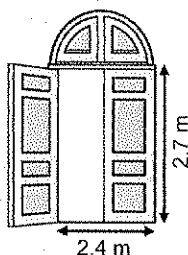
- Area of triangle = $\frac{1}{2} \times \text{base} \times \perp \text{height}$
 $= \frac{1}{2} \times 8 \times 3$
 $= 12 \text{ cm}^2$

- $\therefore \text{Total area of sign arrow board} = 80 + 12 = 92 \text{ cm}^2$

- 3.2 If the cardboard from which the sign arrow board is cut measures 13 cm by 8 cm, determine how many m^2 of cardboard is wasted when cutting the sign arrow from it.

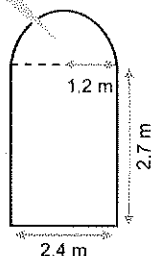
- Area of cardboard = $\ell \times b$
 $= 13 \times 8$
 $= 104 \text{ cm}^2$
- Area of sign arrow = 92 cm^2 [from Question 3.1]
- Wasted area of cardboard = $104 - 92$
 $= 12 \text{ cm}^2$

4. Calculate the area of wood used to make the doorway shown alongside:



Diameter = 2,4 m
 $\therefore$ Radius = $2,4 \div 2$
 $= 1,2 \text{ m}$

First break down the complex shape into the basic shapes and then fill in all relevant lengths.



- Area of rectangle = $\ell \times b$
 $= 2,4 \times 2,7$
 $= 6,48 \text{ m}^2$
- Area of semicircle = $\frac{1}{2} \times \pi \times r^2$
 $= \frac{1}{2} \times 3,142 \times (1,2)^2$
 $= 2,26 \text{ m}^2$
- $\therefore$ Total area of doorway = $6,48 + 2,26 = 8,74 \text{ m}^2$

- 4.2 Calculate the number of tins and cost of the varnish that is needed to varnish the doorway. The varnish is sold in 1 l tins at R79,99/tin, with a spread rate of 250 ml/m^2 .

- Area of doorway = $8,74 \text{ m}^2 \approx 9 \text{ m}^2$
- Volume of varnish needed = $9 \text{ m}^2 \times 250 \text{ ml}$
 $= 2\,250 \text{ ml}$
- $\therefore 2\,250 \text{ ml} = 2\,250 \div 1\,000 = 2,25 \text{ l}$
- No. of tins needed = $3 \times 1 \text{ l tins}$
- Cost of varnish = $R79,99 \times 3 \text{ tins} = R239,97$

EXERCISE 2

Use the formulae on p. 136 to answer the following questions:

1. Mr Handyman wants to build a kennel for his puppy, Canin. The measurements for the kennel are given alongside.

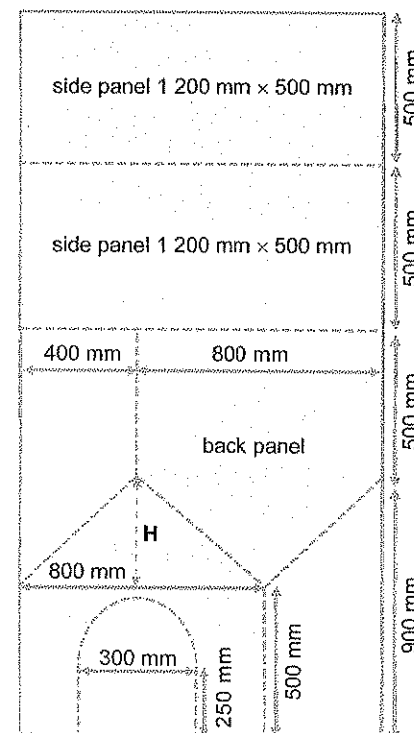
- 1.1 Determine the *dimensions* of the large plywood sheet that Mr Handyman will need in order to cut out all the panels shown on the diagram? Give your answer in both mm and in m.

- 1.2 Calculate the area of the side panels in m^2 .

- 1.3 If the height of the kennel is 900 mm and the height of the side panel is 500 mm, find the length of H on the diagram.

- 1.4 Calculate the area of the back panel of the kennel in m^2 .

- 1.5 Calculate the area of the door (in m^2) that needs to be cut out from the front panel.

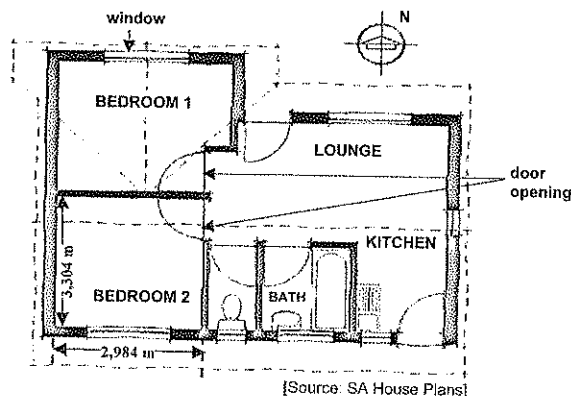


HINT: The door is made up of a rectangle and a semicircle.

- 1.6 Calculate the area of the front panel in m^2 (the front panel is the panel with the door).
- 1.7 Calculate the total area of the kennel in m^2 (the shaded area on the drawing).
- 1.8 Calculate the area of the sheet of plywood that will not be used (the unshaded area on the drawing).



2. The diagram below is the floor plan of Mrs Wong's new home.



- The height of the inside walls of all rooms, from the floor to the ceiling is 2 400 mm.
- The dimensions of the windows in each bedroom are 160 cm × 130 cm.
- The height of a door opening is 2,14 m.

2.1 The area of a door opening is 9% more than the area of a bedroom window. Calculate the width of a door opening in metres.

2.2 Mrs Wong wants to paint the inside walls of the two bedrooms. The inside walls of bedroom 1 have a total area of 28,44 m².

2.2.1 Calculate the total inside wall area of bedroom 2.

2.2.2 Mrs Wong estimated that the paint for both bedrooms will cost less than R500,00. She intended using paint that covers 4 m² per litre and which is sold in 5 l containers at a price of R169,99 per container. Verify, showing ALL calculations, whether her estimation was correct.

Total Surface Area (TSA)

Definition:

- ▶ The total exterior area of all the exposed surfaces of a three-dimensional object



The only difference between total surface area (TSA) and area, is that TSA refers to 3-D objects; while area refers to 2-D objects.

Units of Total Surface Area:

- ▶ Since TSA is the sum of all the areas (and area is the product of two-dimensional lengths), the units of TSA are mm²; cm²; m² or km².

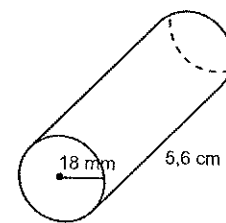
THE TSA OF A RECTANGULAR PRISM AND A CYLINDER

SHAPE	TOTAL SURFACE AREA (TSA)	WORKED EXAMPLES
RECTANGULAR PRISM length = l breadth = b height = h	$\text{TSA} = (2 \times \text{length} \times \text{breadth}) + (2 \times \text{length} \times \text{height}) + (2 \times \text{breadth} \times \text{height})$ $\text{breadth} = \text{width}$	 $\begin{aligned} \text{TSA} &= (2 \times 4 \times 1,5) + (2 \times 4 \times 9) + (2 \times 1,5 \times 9) \\ &= (12 + 72 + 27) \\ &= 111 \text{ cm}^2 \end{aligned}$
CYLINDER radius = r height = h	$\text{TSA} = [2 \times \pi \times (\text{radius})^2] + [2 \times \pi \times \text{radius} \times \text{height}]$ where $\pi = 3,142$ Use the approximate value of $\pi = 3,142$ ∴ don't use the π button on your calculator.	 $\begin{aligned} \text{TSA} &= (2 \times 3,142 \times 7,1 \times 7,1) + (2 \times 3,142 \times 7,1 \times 8) \\ &= 316,77644 + 356,9312 \\ &= 673,71 \text{ cm}^2 \end{aligned}$

Calculations:

Check that all measurements are in the same units. First convert all units into those specified by the question, and then calculate the TSA.

Calculate the TSA of the following cylinder and give your answer in cm².



$$\begin{aligned} 18 \text{ mm} &= 18 : 10 \\ &= 1,8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{TSA} &= (2 \times \pi \times r^2) + (2 \times \pi \times r \times h) \\ &= [2 \times 3,142 \times (1,8)^2] + [2 \times 3,142 \times 1,8 \times 5,6] \\ &= 20,36016 + 63,34272 \\ &= 83,70 \text{ cm}^2 \end{aligned}$$

Round off your *final* answer to 2 decimal places; unless otherwise stated.

NOTE! • If you first calculated the TSA in mm^2 :



$$\begin{aligned}\therefore \text{TSA} &= (2 \times \pi \times r^2) + (2 \times \pi \times r \times h) \\ &= [2 \times 3,142 \times (18)^2] + [2 \times 3,142 \times 18 \times 56] \\ &= 2\,036,016 + 6\,334,272 \\ &= 8\,370,29 \text{ mm}^2\end{aligned}$$

$$5,6 \text{ cm} = 5,6 \times 10 = 56 \text{ cm}$$

- Then, to convert mm^2 to cm^2 , we need to **divide by the (conversion factor)²**

The conversion factor is **squared** as both lengths are converted from mm to cm.

$$\therefore 8\,370,29 \text{ mm}^2 = 8\,370,29 \div (10)^2 = 83,70 \text{ cm}^2$$

$$\text{NOTE! } 8\,370,29 \text{ mm}^2 \neq 8\,370,29 \div 10 \neq 837,03 \text{ cm}^2$$

Remember that **products** are often sold per whole m^2/m^3 etc.; so then round up to the nearest whole number when calculating costs.

Worked Examples

1. Calculate the TSA of the following shapes:

- 1.1 Give your answer in cm^2

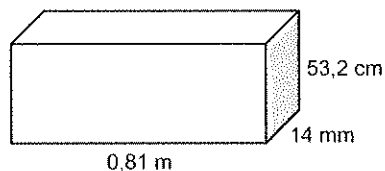
- First convert all measurements to cm:

$$\ell = 0,81 \text{ m} = 0,81 \times 100 = 81 \text{ cm}$$

$$b = 14 \text{ mm} = 14 \div 10 = 1,4 \text{ cm}$$

$$h = 53,2 \text{ cm}$$

- $\text{TSA} = (2 \times \ell \times b) + (2 \times \ell \times h) + (2 \times b \times h)$
 $= (2 \times 81 \times 1,4) + (2 \times 81 \times 53,2) + (2 \times 1,4 \times 53,2)$
 $= 226,8 + 8618,4 + 148,96 = 8994,16 \text{ cm}^2$



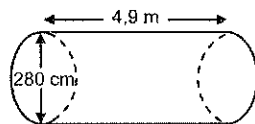
- 1.2 Give your answer in m^2

- First convert all measurements to m:

$$r = 280 \text{ cm} \div 2 = 140 \text{ cm}$$

$$\therefore 140 \text{ cm} \div 100 = 1,4 \text{ m}$$

- $\text{TSA} = (2 \times \pi \times r^2) + (2 \times \pi \times r \times h)$
 $= [2 \times 3,142 \times (1,4)^2] + (2 \times 3,142 \times 1,4 \times 4,9)$
 $= 12,31664 + 43,10824 = 55,42 \text{ m}^2$

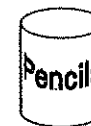


Only round off the final answer to 2 DP!

2. Thandiwe wants to make a new pencil holder. She has a choice of an open cylindrical holder or an open rectangular holder. She wants to cover the outer surface area (excluding base) of the holder to match the table cloth on her desk.

A cylindrical holder with:

radius = 5 cm and
height = 15 cm



$$\begin{aligned}\text{Lateral surface area of a cylinder} \\ &= 2\pi \times \text{radius} \times \text{height, and using } \pi \\ &= 3,142\end{aligned}$$

A rectangular holder with:

length = 10 cm,
breadth = 8 cm and
height = 15 cm



$$\begin{aligned}\text{Lateral surface area of a rectangular prism} \\ &= 2 \times (\text{length} + \text{breadth}) \times \text{height}\end{aligned}$$



Which container will require the least amount of table cloth fabric?

Lateral surface area of a cylinder

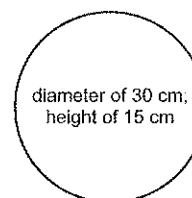
$$\begin{aligned}&= 2\pi \times \text{radius} \times \text{height} \\ &= 2 \times 3,142 \times 5 \times 15 \\ &= 471,3 \text{ cm}^2\end{aligned}$$

Lateral surface area of rectangular prism

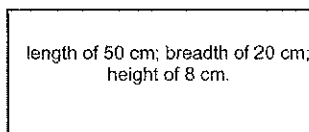
$$\begin{aligned}&= 2 \times (\text{length} + \text{breadth}) \times \text{height} \\ &= 2 \times (10 + 8) \times 15 \\ &= 2 \times 18 \times 15 = 540 \text{ cm}^2\end{aligned}$$

$\therefore$ The cylinder will require the least table cloth fabric.

3. Ronwyn and Bronwyn are twins. They plan to celebrate their 21st birthday by having a big party. Ronwyn has decided that she wants a round cake (fruit cake covered with marzipan), while Bronwyn has decided to have a rectangular cake (chocolate cake covered with caramel).



RONWYN'S ROUND CAKE



BRONWYN'S RECTANGULAR CAKE



$$\begin{aligned}\text{TSA of round cake} \\ &= (\pi \times \text{radius}^2) \\ &\quad + (2 \times \pi \times \text{radius} \times \text{height})\end{aligned}$$

$$\begin{aligned}\text{TSA of rectangular cake} \\ &= (\text{length} \times \text{breadth}) + (2 \times \text{length} \\ &\quad \times \text{height}) + (2 \times \text{breadth} \times \text{height})\end{aligned}$$



Which cake will cost more to cover - with either marzipan (at R0,04/cm²) or with caramel (at R0,05/cm²)?

- **TSA of exposed surfaces of a round cake**
 $= [3,142 \times (15)^2] + [2 \times 3,142 \times 15 \times 15]$
 $= 706,75 + 1\,413,9$
 $= 2\,120,65 \text{ cm}^2$

- $\therefore$ **Total cost of marzipan**
 $= 2\,120,65 \times R0,04$
 $= R84,83$

- **TSA of exposed surfaces of a rectangular cake**
 $= (50 \times 20) + (2 \times 50 \times 8) + (2 \times 20 \times 8)$
 $= 1\,000 + 800 + 320$
 $= 2\,120 \text{ cm}^2$

- $\therefore$ **Total cost of caramel** $= 2\,120 \times R0,05 = R106$
- **Therefore, Bronwyn's rectangular cake with caramel will cost more to cover.**

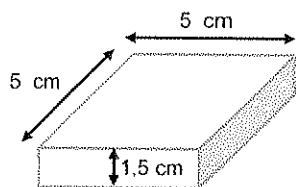


$$\begin{aligned} \text{Radius} &= \frac{\text{diameter}}{2} \\ &= \frac{30}{2} \\ &= 15 \text{ cm} \end{aligned}$$

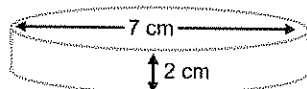
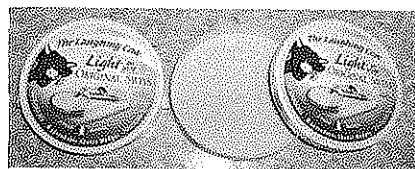
EXERCISE 3

Use the formulae on p. 139 to answer the following questions:

1. Cambrieni cheese comes in a square container with a side length of 5 cm and a depth of 1,5 cm. Laughing Cow cheese comes in a cylindrical container with a diameter of 7 cm and a depth of 2 cm; as shown below.



Consider the cheese box as one single container.



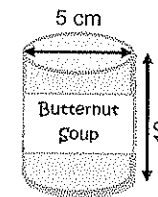
Which container will require the least amount of cardboard for the cheese box?

2. Mason owns a tin soup company. They make the tins and fill it with different flavours of soup. The standard tin has a diameter of 5 cm and a height of 12 cm. The label that is stuck onto the tin has a height of 8 cm.

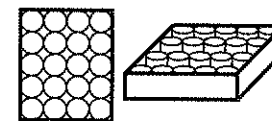
- 2.1 Calculate the area of sheet metal needed to make a soup tin.

- 2.2 Calculate the total surface area of the label.

Use the formula: **TSA = $2 \times \pi \times \text{radius} \times \text{height}$**
 where $\pi = 3,142$



- 2.3 Tins of butternut soup are tightly packed into boxes, with 4 tins fitting in the width of the box, and 5 tins fitting in the length of the box as shown in the diagrams. What are the dimensions (length and breadth) of the box?

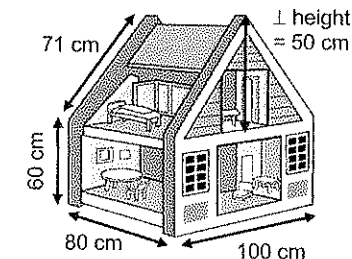


- 2.4 If the box has the same height as the soup tin, and it is open at the top, calculate the total surface area of the box.

- 3.1 Calculate the total surface area of the doll's house, including the roof, that will be painted.

Hint: Break the surfaces down into basic shapes and use the following area formulae:

Area of a rectangle = $\text{length} \times \text{breadth}$
Area of a triangle = $\frac{1}{2} \times \text{base} \times \perp \text{height}$



- 3.2 Calculate the total cost of paint, if paint is sold in 500 ml tins, which cost R27,99/tin and has a spread rate of 10 cm²/ml.

Volume

Definition:

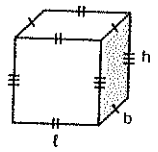
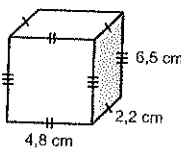
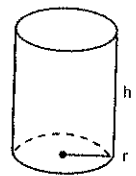
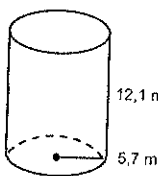
- The three-dimensional space occupied by a gas, a liquid or a solid substance.
- The word 'capacity' also refers to volume.

Units of Volume:

- The units of a liquid volume include: ml; l or kl.
- The units of a solid volume include: mm³; cm³; m³ or km³.

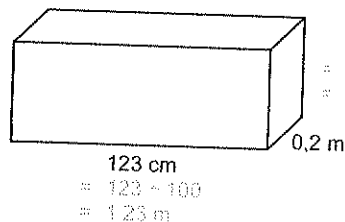


THE VOLUME OF A RECTANGULAR PRISM AND A CYLINDER

SHAPE	VOLUME	WORKED EXAMPLES
RECTANGULAR PRISM  length = l breadth = b height = h	$V = \text{length} \times \text{breadth} \times \text{height}$	 $V = 4,8 \times 2,2 \times 6,5$ $= 68,64 \text{ mm}^3$
CYLINDER  radius = r height = h	$V = \pi \times (\text{radius})^2 \times \text{height}$ where $\pi = 3,142$ You must work with the approximate value of $\pi = 3,142$ $\therefore$ don't use the π button on your calculator.	 $V = 3,142 \times (5,7)^2 \times 12,1$ $= 1\,235,21 \text{ m}^3$

Calculations:

- Check that all measurements are in the same units, otherwise first convert all units into those specified by the question, and then calculate the volume.
- Calculate the volume of the following rectangular prism and give your answer in m^3 .



$$\begin{aligned}
 \therefore V &= l \times b \times h \\
 &= 1,23 \times 0,2 \times 3,45 \\
 &= 0,8487 \text{ m}^3 \\
 &= 0,85 \text{ m}^3
 \end{aligned}$$

- Round off your *final* answer to 2 decimal places; unless otherwise stated.

NOTE! • If you first calculated the volume in cm^3 :



$$\begin{aligned}
 \therefore V &= l \times b \times h \\
 &= 123 \times 20 \times 345 \\
 &= 848\,700 \text{ cm}^3
 \end{aligned}$$

- 0,2 m
 $= 0,2 \times 100 = 20 \text{ cm}$
- 3 450 mm
 $= 3\,450 \div 10 = 345 \text{ cm}$

- Then, to convert cm^3 to m^3 , we need to **divide by the (conversion factor)**³
 [since three dimensions are converted from cm to m]
 $\therefore 848\,700 \text{ cm}^3 = 848\,700 \div (100)^3 = 0,8487 \text{ m}^3$

NOTE! $848\,700 \text{ cm}^3 \neq 848\,700 \div 100 \neq 8\,487 \text{ m}^3$

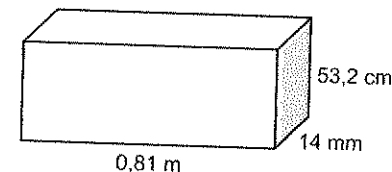
- Remember that **products** are often **sold per whole m^2/m^3 etc.**; so then round up to the nearest whole number when calculating costs.

Worked Examples

- Calculate the volume of the following shapes:

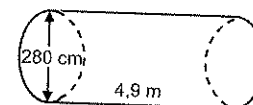
- Give your answer in cm^3

- First convert all measurements to cm:
 $l = 0,81 \text{ m} = 0,81 \times 100 = 81 \text{ cm}$
 $b = 14 \text{ mm} = 14 \div 10 = 1,4 \text{ cm}$
 $h = 53,2 \text{ cm}$

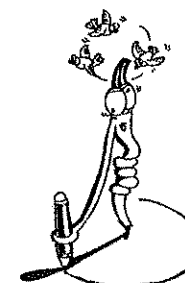


$$\begin{aligned}
 V &= l \times b \times h \\
 &= 81 \times 1,4 \times 53,2 = 6\,032,88 \text{ cm}^3
 \end{aligned}$$

- Give your answer in m^3



- First convert all measurements to m:
 $r = 280 \text{ cm} \div 2 = 140 \text{ cm}$
 $\therefore 140 \text{ cm} \div 100 = 1,4 \text{ m}$
- $V = \pi \times r^2 \times h$
 $= 3,142 \times (1,4)^2 \times 4,9 = 30,18 \text{ m}^3$

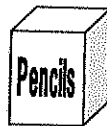


2. Thandiwe wants to make a new pencil holder. She has a choice of an open cylindrical holder or an open rectangular holder. She wants to cover the outside of the holder to match the table cloth on her desk.

A cylindrical holder with:
radius = 5 cm and
height = 15 cm



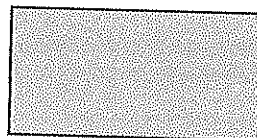
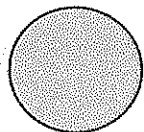
A rectangular holder with:
length = 10 cm,
breadth = 8 cm and
height = 15 cm



Which container has the greater volume?

- Volume of cylinder $= \pi \times r^2 \times h$
 $= 3,142 \times (5)^2 \times 15$
 $= 1178,25 \text{ cm}^3$
- Volume of rectangular holder $= \ell \times b \times h$
 $= 10 \times 8 \times 15$
 $= 1200 \text{ cm}^3$
- Therefore, the rectangular holder has the greater volume.

3. Ronwyn and Bronwyn are twins. They plan to celebrate their 21st birthday by having a big party. Ronwyn has decided that she wants a round cake, while Bronwyn has decided to have a rectangular cake, as shown in the pictures below.



RONWYN'S ROUND CAKE

diameter of 30 cm;
height of 15 cm

Fruit cake covered
with marzipan

BRONWYN'S RECTANGULAR CAKE

length of 50 cm;
breadth of 20 cm; height of 8 cm.

Chocolate cake covered
with caramel

Which cake will cost less to make, if the fruit cake costs 3 cents/cm³; while the chocolate cake costs R0,02/cm³.

- Volume of round cake $= \pi \times r^2 \times h$
 $= 3,142 \times (15)^2 \times 15$
 $= 10\,604,25 \text{ cm}^3$



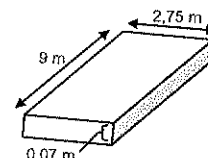
$$\begin{aligned} \text{Radius} &= \frac{30}{2} \\ &= 15 \text{ cm} \end{aligned}$$

- Cost of fruit cake
 $= 10\,604,25 \times R0,03$
 $= R318,13$
- Volume of rectangular cake $= \ell \times b \times h$
 $= 50 \times 20 \times 8$
 $= 8\,000 \text{ cm}^3$
- Cost of chocolate cake $8\,000 \times R0,02 = R160$
- Therefore, Bronwyn's rectangular chocolate cake will cost less to make.

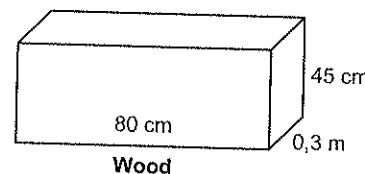
EXERCISE 4

Use the formulae on p. 142 to answer the following questions:

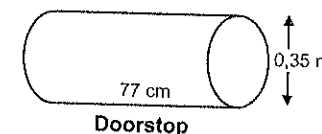
1. The rectangular long jump pit at the school is 2,75 m wide and 9 m long and is filled with sand.



- 1.1 Calculate the volume of sand needed to fill the long jump pit to a depth of 0,07 m.
- 1.2 Calculate the cost of filling the sand-pit to 95% of its volume, if sand costs R129,95/bag and bags are sold in volumes of 0,75 m³/bag.
2. Lazarus would like to carve a cylindrical doorstop from a piece of wood that he bought. The dimensions of the wood and the doorstop are shown below:



Wood



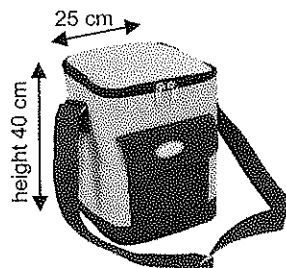
Doorstop

- 2.1 Calculate the volume of wood that would remain, after carving the doorstop.
- 2.2 Would Lazarus have enough wood left over to compress it into ten wooden blocks for his child, with dimensions of 12 cm $\times$ 50 cm $\times$ 6 cm?

3. Rosco was given a cooler bag for his birthday. The label on the bag says that it has a capacity of 26 litres.

3.1 If $1 \text{ litre} = 1\,000 \text{ cm}^3$, convert 26 litres to cm^3 .

3.2 Use your previous answer to calculate the breadth of the cooler bag if the volume of a rectangular prism is calculated from the formula: $V = \ell \times b \times h$



Rosco wants to take his cooler bag to the cricket to keep his cooldrinks chilled. He offers to keep his friends' cooldrinks cold as well, but he is not sure how many tins can fit into the cooler bag.

- 3.3 If the diameter of a tin of cooldrink is 5 cm, how many tins of cooldrink can fit along the length of the cooler bag?
- 3.4 How many tins can Rosco fit along the breadth of the cooler bag?
- 3.5 If the height of a tin is 16 cm, how many tins can Rosco stack on top of each other and still be able to close the cooler bag?
- 3.6 Using your answers to Questions 3.3, 3.4 and 3.5, calculate how many tins of cooldrink Rosco could fit into the cooler bag.
- 3.7 Would you get the same answer for Question 3.6 if you divided the volume of the cooler bag by the volume of a tin of cooldrink? Explain.
- 3.8 Prove your answer to Question 3.7 by finding the volume of the cooler bag divided by the volume of a tin of cooldrink.

Test Your Understanding



Use all the formulae given for the following, on the pages indicated below:

- * Perimeter (see p. 134)
- * Area (see p. 136)
- * Total Surface Area (see p. 139)
- * Volume (see p. 142)

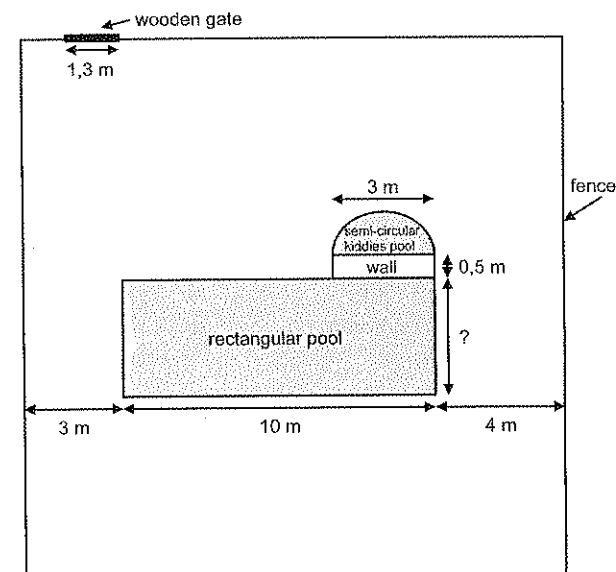
or as stated in the questions:

1. The diagram alongside shows the layout of a swimming pool area. It is made up of a rectangular part with a shallow semicircular kiddies pool attached. The two pools are separated by a wall. The entire pool area, which is in the shape of a square, is surrounded by a fence. There is a wooden gate 1,3 m wide in the fence.

- 1.1 Determine the cost of the fencing for the swimming pool area if the fencing comes in 5 m rolls that cost R389,95 each. The gate does not form part of the fencing.

- 1.2 The area around the pool, yet within the fence, is to be paved with bricks. Calculate the breadth of the main rectangular pool if the area needed to be paved is approximately 244 m^2 when rounded to the nearest square metre.

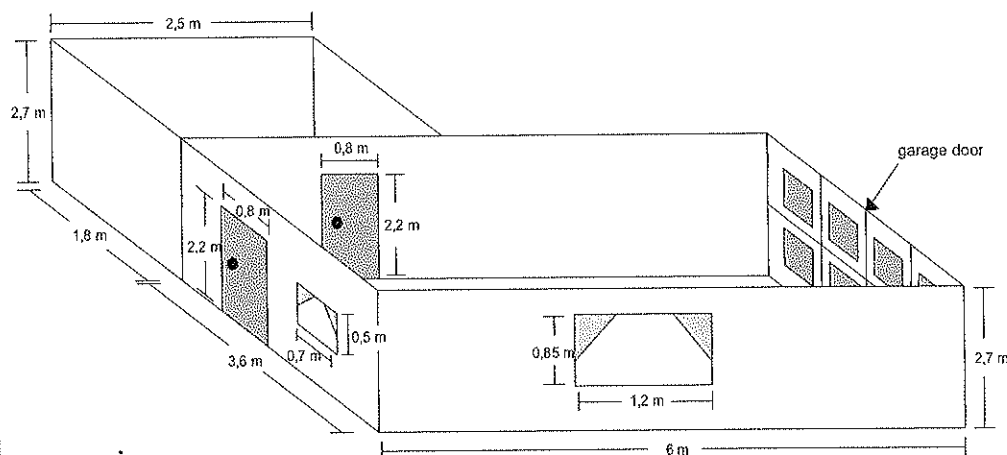
TOP VIEW OF THE SWIMMING POOL AREA



- 1.3.1 The volume of the main rectangular pool is 61 m^3 . Calculate the total volume of both the kiddies pool (which has a uniform depth of 45 cm) and the main rectangular pool, and round off your answer to the nearest whole number.
- 1.3.2 Calculate the amount of water (to the nearest kilolitre) needed to fill both the pools to 95% capacity.

$$1 \text{ m}^3 = 1 \text{ kilolitre}$$

2. The picture below shows a 3-dimensional representation of a garage and store room. (Note: The picture is not drawn to scale)



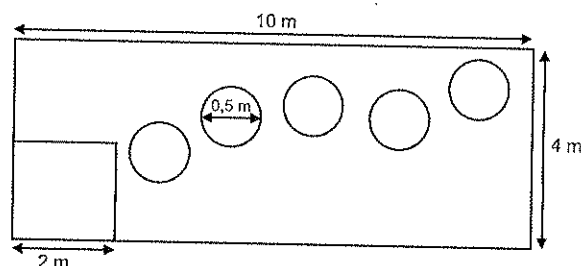
The table below shows the coverage ratio for the undercoat paint and acrylic paint that Bob's Builders will use to paint the inside walls of a garage and store room (excluding the garage door, other doors and windows).

Type of Paint	Paint Coverage (m ² per litre)	Paint Tin Sizes	Cost of Paint
Undercoat	9	2 l	R65,00
		5 l	R112,50
Acrylic	7,5	1 l	R48,00
		5 l	R82,00
		10 l	R147,00

Use this table and the picture on p. 144 to calculate how many of each size paint tin Bob's Builders will need to buy to paint one layer of undercoat and two layers of acrylic paint on the walls.

3. The Grade 12's of a certain school are designing a new rectangular garden as a farewell gift for their school when they matriculate. The garden will consist of a square pond at one end, and circular stone tiles forming a walkway to the pond. The area surrounding the pond and walkway will be grassed.

The design of the garden is indicated below:



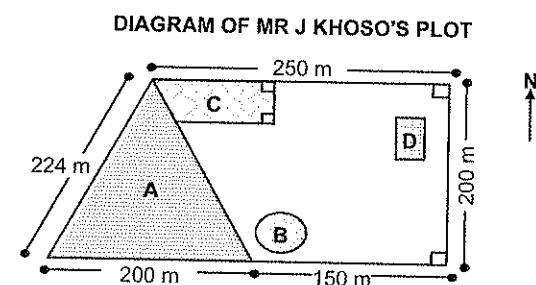
- 3.1 The Grade 12 committee needs to order the stone tiles and grass for the new garden.
- 3.1.1 Calculate the area that one circular stone tile will cover.
- 3.1.2 Calculate the number of square metres of grass that will need to be ordered from the local nursery for the garden.
- 3.1.3 If the cost of grass is R25 per $\frac{1}{2}$ m², calculate the cost of the grass for the new garden.
- 3.2 The pond at the bottom of the garden will be filled with Koi fish. According to the local pet shop the pond should be at least 150 cm deep.
- 3.2.1 Calculate the volume of the pond in cubic metres (m³).
- 3.2.2 If 1 m³ = 1 000 litres, how much water will be needed to fill the pond to 90% of its capacity?

- 3.2.3 The local fish shop recommends that for every 200 litres of water you have two Koi fish. What is the maximum number of fish that may be ordered for the pond?

- 3.2.4 Waterproofing paint will be needed to paint the interior of the fish pond. If 1 litre of paint covers 10 m², how many litres will be needed to waterproof the pond? The pond only needs one coat of paint.

- 3.3 The garden will need to be watered for one hour a week. To save money they decide to get a rainwater tank. The water tank will supply 20 litres of water per minute. How much water, in litres, will the garden receive once a week from the tank?

4. Mr J Khoso owns a plot, as shown in the diagram below (not drawn to scale). His house (D) is on the eastern side of the plot. Also on the plot is a cattle kraal (A), a circular water tank (B), and a vegetable garden (C).



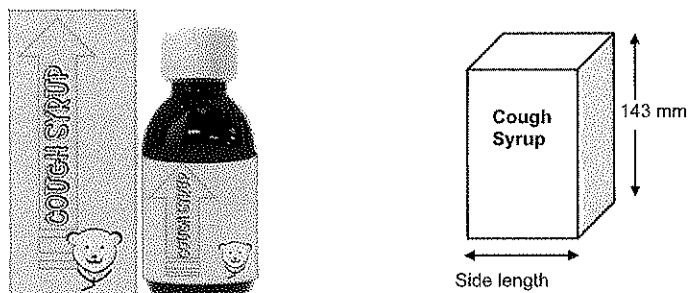
Key	Dimensions
A cattle kraal	⊥ height = 200 m, base (south) = 200 m, slanting side = 224 m
B water tank	radius = 10 m
C vegetable garden	parallel sides = 100 m and 125 m distance between parallel sides (height) = 50 m
D house	length = 25 m, breadth = 8 m

- 4.1 Determine the perimeter of Mr Khoso's plot.
- 4.2 Calculate the volume of water in the circular water tank, if the height of the water in the tank is 2 m.
- 4.3 Determine the area of the cattle kraal.
- 4.4 Calculate the total area of Mr Khoso's plot. Use the formula:

$$\text{Area of a trapezium} = \frac{1}{2} \times (\text{sum of the parallel sides}) \times \perp \text{ height}$$

5. Triggers Enterprises was awarded the tender for making rectangular cardboard boxes to package bottles of cough syrup. Each bottle is packed in a cardboard box with a square base, as shown below.

- › The diameter of the base of the bottle is 58 mm and the height of the box is 143 mm.
- › The length of the side of the base of the box must be approximately 105% of the diameter of the base of the bottle.
- › The height of the box is approximately 102% of the height of the bottle.
- › The design of the open cardboard box is in Question 5.3.



The following may be used:

Area of circle = $\pi \times (\text{radius})^2$, and using $\pi = 3,142$

Area of opened cardboard box = $4(A + D) + 2(B + C) + E$

The following conversions may be useful:

$1 \text{ cm}^2 = 100 \text{ mm}^2$; $1 \text{ m}^2 = 10\,000 \text{ cm}^2$

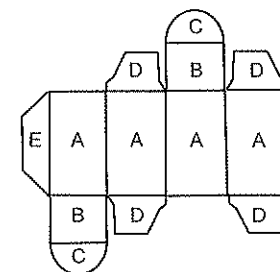
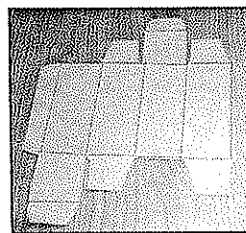
- 5.1 Calculate the height of the bottle to the nearest millimetre.
- 5.2 In order to minimise the cost of cardboard required for the box, the following guideline is used:

The difference between the areas of the base of the cardboard box and the base of the bottle should not be more than 11 cm^2 .

Determine whether the dimensions of this cardboard box satisfy the above guideline. Show ALL appropriate calculations.

- 5.3 To ensure that the box is strong enough, the cardboard used has a mass of 240 grams per m^2 (g/m^2).

The layout of the opened cardboard box is shown below.



Picture of opened box

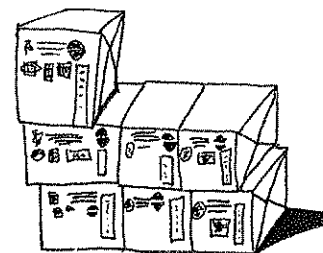
Diagram of layout of opened box

- Section C is semicircular.
- The area of each section D = $1\,832 \text{ mm}^2$.
- The area of section E = $2\,855 \text{ mm}^2$.

- 5.3.1 Calculate the total mass of the cardboard needed for one box, to the nearest gram.

- 5.3.2 The total cost of the cough syrup includes the cost of the cardboard box. Use the following formula to calculate the cost of a boxed bottle of cough syrup:

Total cost = $\text{R}16,00 + (\text{mass of cardboard box in kg}) \times \text{R}20,00 \text{ per kg}$



5 MAPS, PLANS AND REPRESENTATIONS

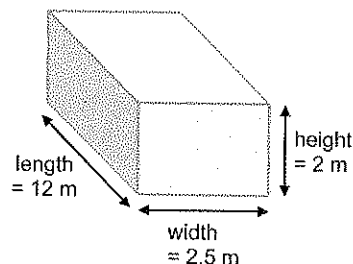
UNIT

1

SCALE

Dimensions

- Dimensions are used to describe the size and shape of an object.
- Length and width (or breadth) are the dimensions of a two-dimensional figure.
- Length, width and height are the dimensions of a three-dimensional figure.
- Example: The dimensions of a shipping container are $12\text{ m} \times 2,5\text{ m} \times 2\text{ m}$.

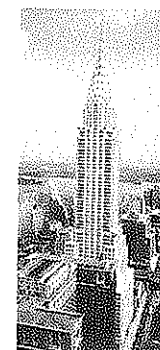


Scale

- Scale drawings represent the actual size and shape of an object on paper.
- Each dimension of the actual object is either **reduced** (for very large objects e.g. a building) or **enlarged** (for very small objects e.g. a screw) by a certain ratio, called the **scale factor**.

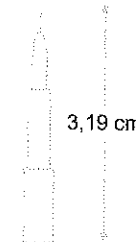
- Example:** The Chrysler Building is a famous skyscraper in New York and measures 319 m. If we were to reduce the actual height ($319\text{ m} = 31\,900\text{ cm}$) by a scale factor of 10 000, then the height of the scale drawing would be 3,19 cm; as shown below:

Actual height of
Chrysler Building:



319 m

Scale drawing of
Chrysler Building
1 : 10 000



3,19 cm

Numerical scales

- Numerical scales (number scales) are always written in the form:

map : reality

e.g. 1 : 100

This means that for every 1 unit on the map, it is equivalent to 100 units in reality.

1 : 100 - the 100 is called the **scale factor**.



In this unit, the term 'map' is used as a general term for a map, plan, representation, diagram or drawing.

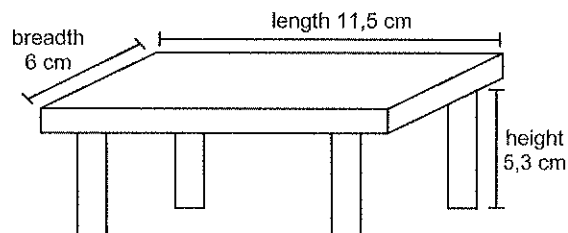
- Note:** Scales are written in the form of ratios (e.g. 1 : 100), which implies that you are comparing equivalent units. Therefore, no particular units are given in scales, because if we measure 1 cm, it is equivalent to 100 cm in reality; or if we measure 1 mm, it is equivalent to 100 mm in reality.

- To determine the actual length, we would use the following general formula:

$$\text{Actual length (reality)} = \text{map length} \times \text{scale factor}$$



Example: Monwa wants to build a table and draws the following plan of the table. The drawing is to a scale of 1:20. Calculate the actual dimensions of the table, in metres.



$$\text{Actual length} = \text{map length} \times \text{scale factor}$$

$$\begin{aligned} \therefore \text{Length of actual table} &= 11,5 \text{ cm} \times 20 \\ &= 230 \text{ cm} = 2,3 \text{ m} \\ \therefore \text{Breadth of actual table} &= 6 \text{ cm} \times 20 \\ &= 120 \text{ cm} = 1,2 \text{ m} \\ \therefore \text{Height of actual table} &= 5,3 \text{ cm} \times 20 \\ &= 106 \text{ cm} = 1,06 \text{ m} \end{aligned}$$



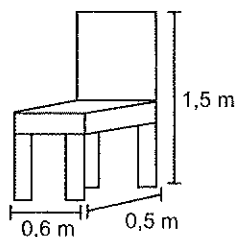
$$\begin{aligned} &\text{m} \quad \text{cm} \\ &\div 100 \\ &1 \text{ cm} = 0,01 \text{ m} \end{aligned}$$

- To determine the map length, we would use the following general formula:

$$\text{Map length} = \text{actual length} \div \text{scale factor}$$



Example: Monwa wants to make a replica of his dad's favourite chair to use at his new table. He did a quick drawing and wrote down the actual measurements of the chair. (This drawing is not drawn to scale.) If Monwa wants to draw an accurate scale drawing of the chair (also to a scale of 1:20), calculate the measurements on the drawing, in cm.



$$\text{Map length} = \text{actual length} \div \text{scale factor}$$

$$\begin{aligned} \therefore \text{Length of chair on drawing} &= 0,6 \text{ m} \div 20 \\ &= 0,03 \text{ m} \\ &= 3 \text{ cm} \end{aligned}$$



$$\begin{aligned} &\times 100 \\ &\text{m} \quad \text{cm} \\ &1 \text{ m} = 100 \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{Breadth of chair on drawing} &= 0,5 \text{ m} \div 20 \\ &= 0,025 \text{ m} \\ &= 2,5 \text{ cm} \\ \therefore \text{Height of chair on drawing} &= 1,5 \text{ m} \div 20 \\ &= 0,075 \text{ m} \\ &= 7,5 \text{ cm} \end{aligned}$$

- To determine the scale of the map, we would use the following general formula:

$$\text{Scale of map} = \frac{\text{map length}}{\text{actual length}}$$

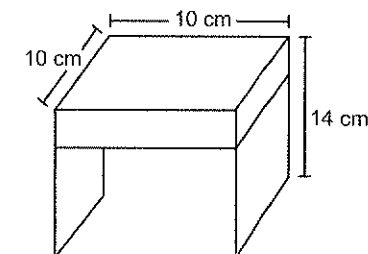


This fraction must be written in a ratio

i.e. **Scale of map = map length : actual length**

Example: Monwa's mom asks him to build her a small coffee table and she gives him this scale drawing:

If the actual table must have a height of 0,7 m and a length and breadth of 0,5 m each, determine the scale of the drawing.



Ratio scale - must be in the same units!

First convert all the measurements to cm.

$$\therefore \text{Height of actual table} = 0,7 \text{ m} \times 100 = 70 \text{ cm}$$

$$\begin{aligned} \text{Breadth and length of actual table} &= 0,5 \text{ m} \times 100 \\ &= 50 \text{ cm} \end{aligned}$$



$$\text{Scale} = \frac{\text{map length}}{\text{actual length}}$$

$$\begin{aligned} \therefore \text{Scale of map (height)} &= \frac{14 \text{ cm}}{70 \text{ cm}} \\ &= \frac{1}{5} \end{aligned}$$

$$\therefore \text{Scale is } 1 : 5$$

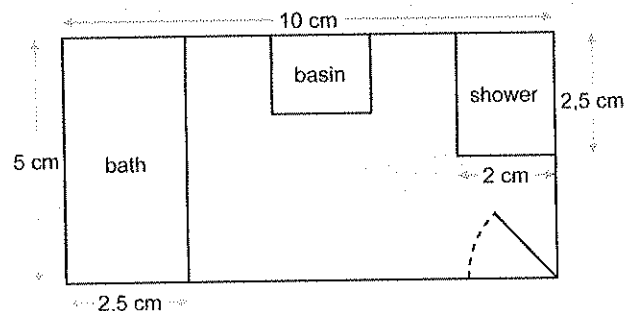
$$\text{OR} \quad \text{Scale of map (length and breadth)} = \frac{10 \text{ cm}}{50 \text{ cm}} = \frac{1}{5}$$

$$\therefore \text{Scale is } 1 : 5$$

It is not necessary to calculate the scale for all dimensions, as it is assumed that the same scale factor would be used for all dimensions.

Worked Examples

1. Bob drew the basic layout of his bathroom, as shown below:
(Diagram not drawn to scale.) The scale is 1 : 40.



- 1.1 What are the actual dimensions of the bath, in metres?

$$\begin{aligned} \text{Actual length} &= \text{map} \times \text{scale factor} & \text{Actual breadth} &= 2,5 \text{ cm} \times 40 \\ &= 5 \text{ cm} \times 40 & &= 100 \text{ cm} \\ &= 200 \text{ cm} & &= 1 \text{ m} \\ &= 2 \text{ m} \end{aligned}$$

$\therefore$ Dimensions of bath: $2 \text{ m} \times 1 \text{ m}$

- 1.2 If the dimensions of the basin in reality are $0,5 \text{ m} \times 0,3 \text{ m}$; determine the length and breadth of the basin on the map (in centimetres).

$$\begin{aligned} \text{Map length} &= \text{actual length} \div \text{scale factor} \\ &= 0,5 \div 40 \\ &= 0,0125 \text{ m} \\ &= 1,25 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Map breadth} &= 0,3 \div 40 \\ &= 0,0075 \text{ m} \\ &= 0,75 \text{ cm} \end{aligned}$$

- 1.3 Calculate the cost of tiling Bob's bathroom floor, if each actual tile measures $30 \text{ cm} \times 30 \text{ cm}$ and cost R64,99 per tile. Additionally, the tiler charges R112/ m^2 for his labour.

Bathroom

$$\begin{aligned} \text{Actual length} &= \text{map} \times \text{scale factor} & \text{Actual breadth} &= 5 \text{ cm} \times 40 \\ &= 10 \text{ cm} \times 40 & &= 200 \text{ cm} \\ &= 400 \text{ cm} & &= 2 \text{ m} \\ &= 4 \text{ m} \end{aligned}$$

$$\therefore \text{Actual area of bathroom} = 4 \text{ m} \times 2 \text{ m} = 8 \text{ m}^2$$

Bath

$$\text{Actual length} = 2 \text{ m}$$

$$\text{Actual breadth} = 1 \text{ m}$$

$$\begin{aligned} \therefore \text{Actual area of bath} &= 2 \text{ m} \times 1 \text{ m} \\ &= 2 \text{ m}^2 \end{aligned}$$

Basin

$$\text{Actual length} = 0,5 \text{ m}$$

$$\text{Actual breadth} = 0,3 \text{ m}$$

$$\begin{aligned} \therefore \text{Actual area of basin} &= 0,5 \text{ m} \times 0,3 \text{ m} \\ &= 0,15 \text{ m}^2 \end{aligned}$$

Shower

$$\begin{aligned} \text{Actual length} &= 2,5 \text{ cm} \times 40 \\ &= 100 \text{ cm} \\ &= 1 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Actual breadth} &= 2 \text{ cm} \times 40 \\ &= 80 \text{ cm} \\ &= 0,8 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{Actual area of shower} &= 1 \text{ m} \times 0,8 \text{ m} \\ &= 0,8 \text{ m}^2 \end{aligned}$$

Area of bathroom floor:

$$\begin{aligned} &= \text{area of bathroom} - \text{area of bath} - \text{area of basin} - \text{area of shower} \\ &= 8 \text{ m}^2 - 2 \text{ m}^2 - 0,15 \text{ m}^2 - 0,8 \text{ m}^2 \\ &= 5,05 \text{ m}^2 \end{aligned}$$

Area of tile:

$$\text{Dimension} = 30 \text{ cm} = 0,3 \text{ m}$$

$$\begin{aligned} \therefore \text{Area} &= l \times b \\ &= 0,3 \text{ m} \times 0,3 \text{ m} \\ &= 0,09 \text{ m}^2 \end{aligned}$$

Number of tiles needed

$$\begin{aligned} &= \text{area of bathroom floor} \div \text{area of tile} \\ &= 4,24 \text{ m}^2 \div 0,09 \text{ m}^2 \\ &= 47,11 \\ &\approx 48 \text{ tiles} \end{aligned}$$

Total cost of tiling

$$\begin{aligned} &= \text{cost of tiles} + \text{cost of labour} \\ &= (48 \text{ tiles} \times \text{R}64,99) + (4,24 \text{ m}^2 \times \text{R}112) \\ &= \text{R}3\,119,52 + \text{R}474,88 \\ &= \text{R}3\,594,40 \end{aligned}$$

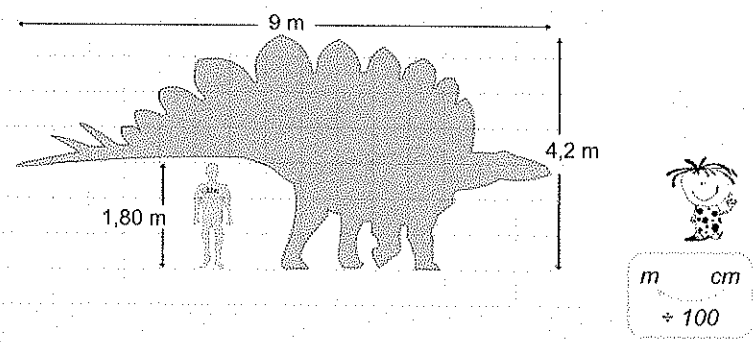
REMEMBER!

First convert to metres before calculating the area, since all our calculations are in m^2 !

REMEMBER!

We need to round up, in order to have enough tiles!

2.1 Determine the scale factor used to illustrate the difference in size between a dinosaur and a man, assuming 1 block = 1 cm.



Actual length of dinosaur = 9 m

Map length of dinosaur
= 15 blocks $\times$ 1 cm
= 15 cm
= 0,15 m

$\therefore$ Scale is 1 : 60

OR

Actual height of dinosaur = 4,2 m

Map height of dinosaur
= 7 blocks $\times$ 1 cm
= 7 cm
= 0,07 m

$\therefore$ Scale is 1 : 60

OR

Actual height of man = 1,8 m

Map height of man
= 3 blocks $\times$ 1 cm
= 3 cm
= 0,03 m

$\therefore$ Scale is 1 : 60

$\therefore$ Scale of map (length of dinosaur)
 $= \frac{\text{map length}}{\text{actual length}}$
 $= \frac{0,15 \text{ m}}{9 \text{ m}}$
 $= \frac{1}{60}$

$\therefore$ Scale of map (height of dinosaur)
 $= \frac{0,07 \text{ m}}{4,2 \text{ m}}$
 $= \frac{1}{60}$

$\therefore$ Scale of map (height of man)
 $= \frac{0,03 \text{ m}}{1,8 \text{ m}}$
 $= \frac{1}{60}$

REMEMBER!

Only do **one** calculation, as it is assumed that all drawings used the same scale factor.

2.2 How many times larger is the dinosaur in comparison to the man, in terms of its height?

Number of times larger = $\frac{\text{actual height of dinosaur}}{\text{actual height of man}} = \frac{4,2 \text{ m}}{1,8 \text{ m}} = 2,33 \text{ times}$

EXERCISE 1

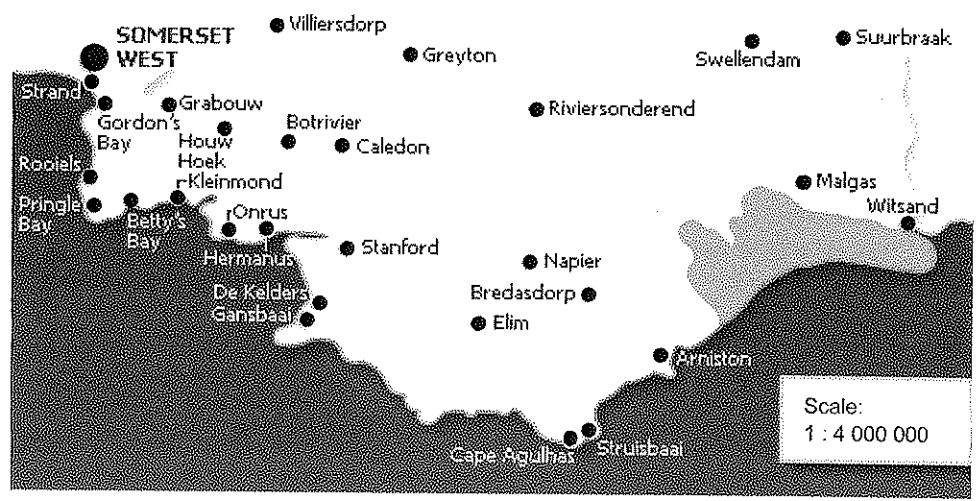
1. If a house plan is drawn using the scale 1 : 50; determine the missing plan and actual dimensions in the table below:

Feature	Measurement on the plan (cm)	Actual measurement (metres)
Height of the roof	A	1,5 m
Height of the doors	4 cm	B
Width of the doors	C	0,8 m
Height of big windows	2,4 cm	D

2. The blue crane is South Africa's national bird. It appears to scale on the 5c coin. They grow to about 1,2 m in height. The height of the blue crane on the 5c coin is 15 mm. What is the scale of the blue crane on the coin to the actual height of the blue crane?

3. Jaydon, Nathi and Trevor (3 friends from Somerset West) want to go away for a couple of days once they have written their matric examinations. They have two options: they can either stay in Nathi's parents' house in Arniston, or they can rent a house in Hermanus (which is closer). Below is a map of the Helderberg and Overberg region:

HELDERBERG & OVERBERG REGION



Scale:
1 : 4 000 000

[source: <http://www.stayinsa.co.za/themaps/Overberg1>]

3.1 Use the map on p. 150 to determine the distance (as the crow flies):

3.1.1 from Somerset West to Arniston.

3.1.2 from Somerset West to Hermanus.

As the crow flies?
The shortest and most direct route. In this case, literally from dot to dot.

3.2.1 In the end the three boys decide on the house in Arniston. The Citi Golf that they are going to use to drive there has an average petrol consumption of 7,4 ℓ/100 km. Determine, using your answer in Question 3.1.1, the approximate number of litres of petrol they will use driving to Arniston and back.

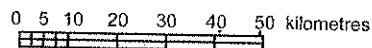
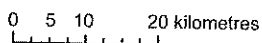
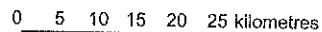
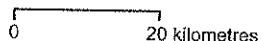
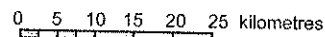
3.2.2 Use your answer in Question 3.2.1, to calculate the amount that each boy should budget for petrol costs if it costs R9,79 per litre.

Bar scales



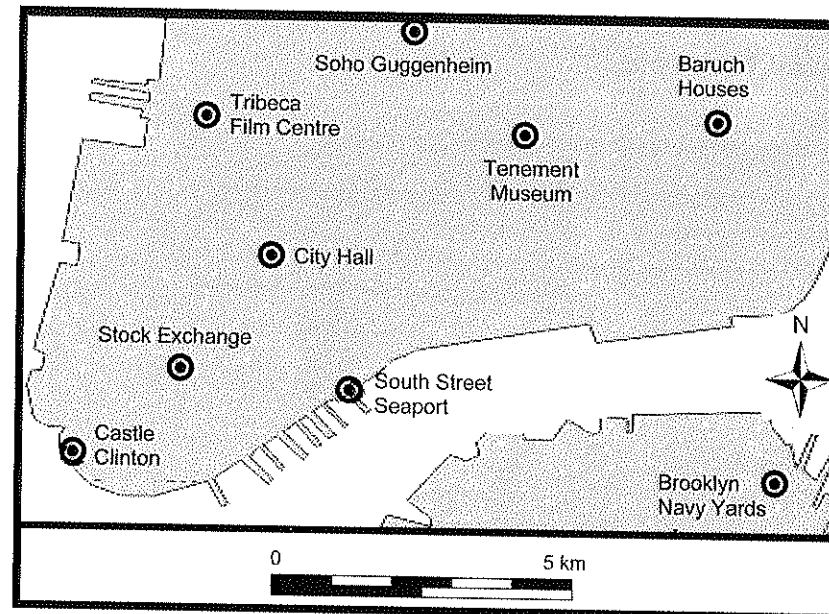
Bar scales are also known as **linear scales**.

Examples of bar scales:



- By measuring the map distance between two points and comparing this distance with the bar scale of the map, you are able to work out the distance between the two points in reality.
- In contrast to a number scale, where no units are included, the units are a very important part of the bar scale. This is because the bar scale shows a very specific relationship between the measured length of the lines of segments on the bar scale and actual length.

Example: Rati stays in Baruch Houses and has a job interview at the Tribeca Film Centre. Use the map and bar scale alongside to determine how far Rati needs to travel from her home to her job interview. Give your answer in km.



Step 1: Use your ruler to accurately measure the length of the bar scale.
4 cm on the map represents 5 km in reality.

Step 2: Write the bar scale measurements in the form of a numerical scale; i.e. 1 :

$$4 \text{ cm} : 5 \text{ km}$$

$$4 \text{ cm} : 500\,000 \text{ cm}$$

Divide both sides by the number on the left to get the ratio
1 :

$$\frac{4}{4} : \frac{500\,000}{4}$$

$$1 : 125\,000$$

Convert into the smallest unit, i.e. cm
 $\times 1\,000 \times 100$
km m cm

Once both measurements are in the same units, we drop the units and only work with the ratio numbers.

Step 3: Use your ruler to accurately measure the distance between Baruch Houses and Tribeca Film Centre on the map.
6,8 cm on the map

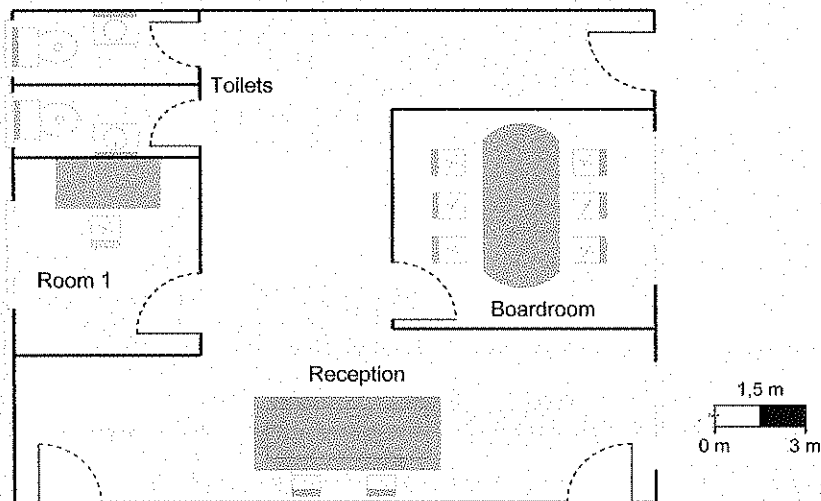
Step 4: Use the numerical scale (from Step 2) and the distance between the two points on the map (from Step 3); to calculate the actual distance. (Check that the units are the same in your answer and the question.)

$$\begin{aligned} \text{Actual distance} &= \text{map distance} \times \text{scale factor} \\ &= 6,8 \text{ cm} \times 125\,000 \\ &= 850\,000 \text{ cm} \\ &= 8,5 \text{ km} \end{aligned}$$

$$\begin{array}{ccc} \text{km} & \text{m} & \text{cm} \\ & \div 1\,000 & \div 100 \end{array}$$

Worked Example

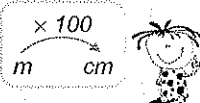
A 2-dimensional floor plan of an office is given below:



1. Determine the dimensions of the office in reality.

Step 1: 0,6 cm on the map represents 1,5 m in reality.

$$\begin{aligned} \text{Step 2: } 0,6 \text{ cm} &: 1,5 \text{ m} \\ 0,6 \text{ cm} &: 150 \text{ cm} \\ \frac{0,6}{0,6} &: \frac{150}{0,6} \\ &\approx 1 : 250 \end{aligned}$$



REMEMBER!
Although we use 'map' in the general formula, in this module it may refer to a plan/diagram/representation. In this example it refers to the plan above.

Round up to the nearest whole number, if necessary.

Step 3: Length of office on map is 8,5 cm
Breadth of office on map is 6,5 cm

$$\begin{aligned} \text{Step 4: Actual length of office} &= \text{map length} \times \text{scale factor} \\ &= 8,5 \text{ cm} \times 250 \\ &= 2\,125 \text{ cm} = 21,25 \text{ m} \\ \text{Actual breadth of office} &= 6,5 \text{ cm} \times 250 \\ &= 1\,625 \text{ cm} \\ &= 16,25 \text{ m} \end{aligned}$$

$\therefore$ Dimensions of office: 21,25 m $\times$ 16,25 m

$$\begin{aligned} &\text{m} \quad \text{cm} \\ &\div 100 \end{aligned}$$



2. Calculate the cost of carpeting the boardroom, if the carpet costs R147,50 per m².

Scale is 1 : 250 (Determined in Q1.1)

Step 3: Length of boardroom on map = 3,5 cm
Breadth of boardroom on map = 2,9 cm

Step 4: Actual length of boardroom = 3,5 cm $\times$ 250
= 875 cm
= 8,75 m

Actual breadth of boardroom = 2,9 cm $\times$ 250
= 725 cm
= 7,25 m

$$\begin{aligned} \therefore \text{Area of boardroom} &= l \times b \\ &= 8,75 \text{ m} \times 7,25 \text{ m} \\ &= 63,44 \text{ m}^2 \\ &\approx 64 \text{ m}^2 \end{aligned}$$

REMEMBER!
Round up since the cost is per whole number!

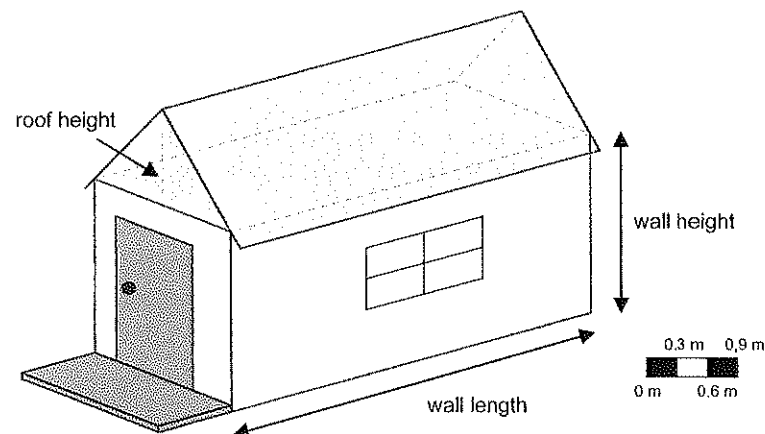


$\therefore$ Cost of carpeting boardroom = 64 m² $\times$ R147,50 = R9 440,00

EXERCISE 2

The plan has been drawn to scale and a bar scale is attached to the picture.

3-DIMENSIONAL SCALE DRAWING OF A GARDEN SHED



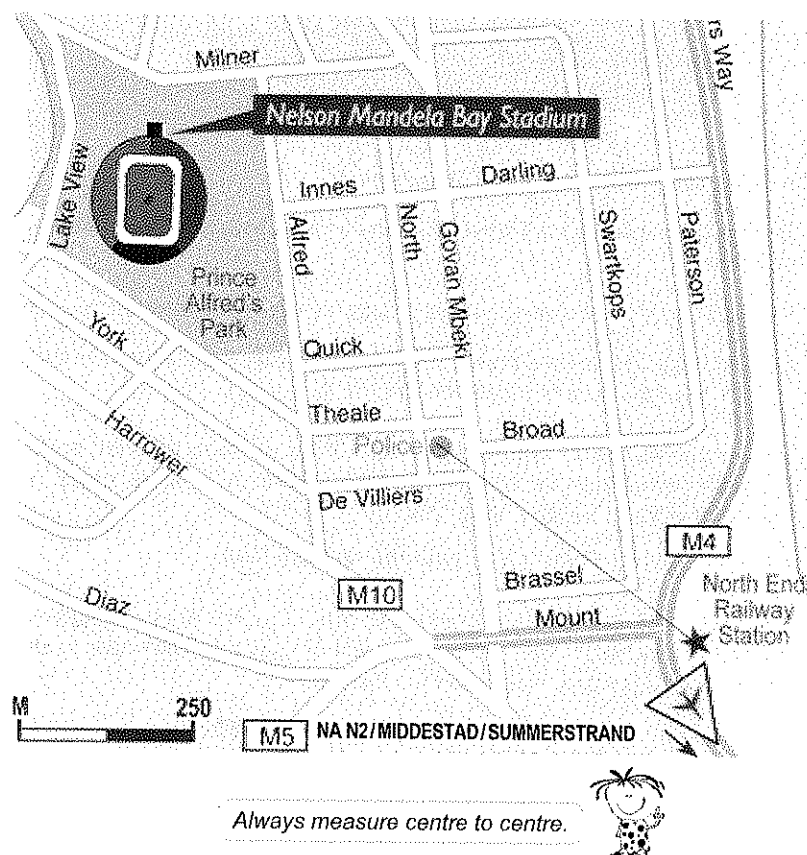
Use the plan to answer the following questions:

- 1.1 Use the bar scale to complete the following:
1 cm on the plan = _____ cm in actual length

1.2 Use the bar scale to complete the following table.

Dimension	Measurement on plan (cm)	Actual measurement (m)
Wall length		
Wall height		
Roof height		
Window length		
Window height		

2. Jabulani is going to see Kaizer Chiefs vs Ajax play soccer at the Nelson Mandela Bay Stadium in Port Elizabeth. He will take a train and get off at the North End Railway Station (indicated with ★) and take a taxi to the Police Station (indicated with ●), to pick up his friend who is a policeman. Together, they will walk to the entrance of the Nelson Mandela Bay Stadium (indicated with ■); in order to watch the soccer match. Jabulani looks at the map below:

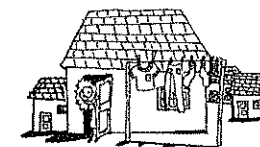


- 2.1.1 Calculate the distance (in km) as the crow flies from the North End Railway Station to the Police Station.
- 2.1.2 If the taxi drives at an average speed of 10 km/h due to the heavy traffic, determine how long it will take Jabulani to cover the distance calculated in Question 2.1.1. Give your answer in minutes.
- 2.2.1 Determine the distance (in km) as the crow flies from the Police Station to the entrance to the Nelson Mandela Bay Stadium.
- 2.2.2 If they leave the Police Station at 17:38 and it takes them 23 minutes to walk to the stadium, will they be on time for the start of the match at 18:00?

Advantages and Disadvantages of each Type of Scale



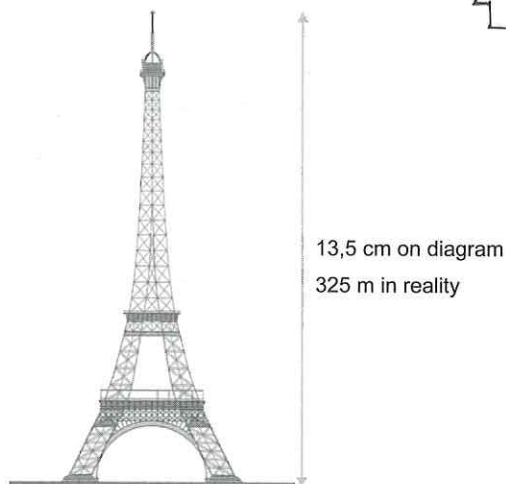
TYPE	ADVANTAGES	DISADVANTAGES
Numerical scales (Number scales) e.g. 1 : 5 000	- Number scales are more accurate than bar scales, therefore lengths and distances in reality are more accurate. - Number scales are more convenient when working at a very small scale, e.g. 1 : 20 000 000. - When using a number scale, you may work in any unit you choose (mm or cm) and then convert to the most appropriate unit (m or km). - The number scales provided with models give us a clear idea of the relationship between the model and the actual object, e.g. 1 : 20 means that the model is 1/20th of the size of the actual object.	- Number scales require calculations to determine the distances or lengths in reality. - With digital printing, number scales become inaccurate and useless if there is any resizing of the map or plan.
Bar scales (Linear scales) e.g. 	- Bar scales are quick and relatively easy to use. - You may be able to determine lengths and distances in reality without doing calculations. - With digital printing, bar scales are resized along with the image and so are still accurate.	- Measurements (using a ruler, string or markings) obtained using bar scales tend to be less accurate. - Bar scales may still require calculations to determine the distances or lengths in reality.



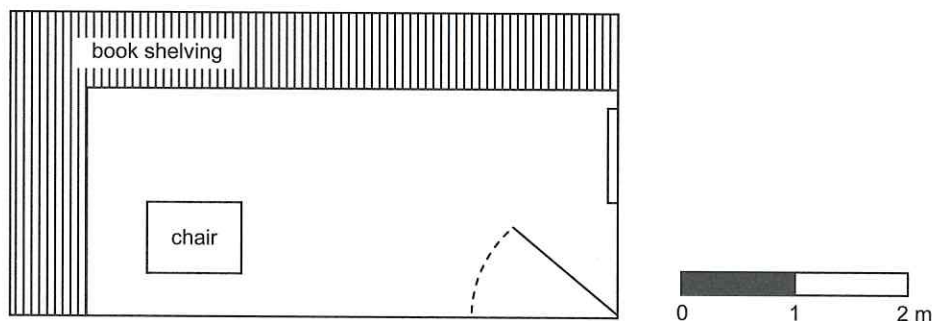
Test Your Understanding



1. Calculate the scale of this drawing of the Eiffel Tower, if it measures 325 m in reality and 13,5 cm on the diagram.
(Diagram not drawn to scale.)



- 2.1 Judy measures the distance between Pietermaritzburg and Durban on a map as 18 cm. If the map is drawn to a scale of 1 : 500 000, calculate the actual distance from Pietermaritzburg to Durban (in km).
- 2.2 If, on the same map, the actual distance from Durban to Harrismith is 295 km, calculate the distance (in cm) between these two places on the map.
3. Mrs Pennyfarthing gave the re-decorators a scaled diagram of how she would like her new library fitted, as shown in the diagram below:



- 3.1 Determine the actual dimensions of Mrs Pennyfarthing's new library.
- 3.2 Calculate the cost of carpeting Mrs Pennyfarthing's library, if the carpet covers the whole floor (including under the chair, but not under the book shelving) and costs R98,20/m².

4. The Naidoo family lives in Pietermaritzburg. Part of a map of South Africa is given below:

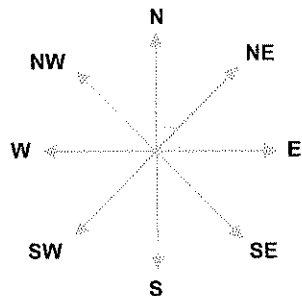


Use the map to answer the following questions:

- 4.1 The family travelled from Pietermaritzburg to Johannesburg using the route as indicated.
- 4.1.1 Measure the distance, in centimetres, on the map between these two cities.
- 4.1.2 Hence, use the scale given on the map to calculate the actual distance, in kilometres, between these two cities.
- 4.2 The car travelled at an average speed of 110 km/h. They departed at 08:15 and planned to arrive in Johannesburg at 14:30. Determine whether they arrived at their destination at the predicted time.
- Use the formula: **Distance = average speed × time**
- 4.3 The family left Pietermaritzburg with a full tank of petrol. Along the way they stopped at a petrol station to refuel at a cost of R455,40. The capacity of the tank is 60 litres and the cost of fuel is R10,12 per litre.
- 4.3.1 Before refuelling, the fuel gauge indicated that the tank was half full. Verify, showing ALL calculations, whether the fuel gauge is accurate.
- 4.3.2 If the car's fuel consumption was 9 litres per 100 km, determine how far they were from Johannesburg when they refuelled.

Maps

- A **map** is a two-dimensional representation of an area of the earth's surface e.g. country map; street map; building map, etc.
- A **plan** is a more detailed representation of a smaller area; often showing landmarks or objects, e.g. house plan; cinema seating plan; sports stadium, etc.
- Maps and plans are often drawn using **scales** (but not always), in order to allow calculations of actual lengths and areas.
- **Relative position** is used when describing your position or directions to someone; in relation to surrounding landmarks.
 - ▷ Key words include: left; right; up; down; in front of; behind; and compass directions (i.e. north, south, east and west)
- The earth has been divided into 4 **compass directions** (also known as 'cardinal points'), namely: north (N), south (S), east (E), west (W).
 - ▷ These four compass directions have been further divided for greater accuracy, namely: north-east (NE); south-east (SE); south-west (SW) and north-west (NW); as shown below.
 - ▷ Compass directions are vitally important for pilots, cartographers (people who draw maps); architects, etc.
- A **numbering system** is often used within complexes (e.g. shopping malls; housing estates; office blocks etc.) and allows people to easily locate a shop/house/office, without having to know the relative position of surrounding landmarks.



Worked Examples

1. Joshua has just moved into Northdale and lives in Orient Road. He will be attending Esther Payne Smith Senior Secondary School.

- 1.1 What other schools can be found on this map?

Rosefern and Haythorne High

- 1.2 Where would Joshua end up if he followed these directions?

Exit Orient Street;
turn right into
Bombay Street;
turn left into
Mysore Street;
turn left into
Dahlia Street;
turn right into
Muniredy Street
and the entrance
is on the right.

Rosefern School

- 1.3 Which directions would get him to his new school?

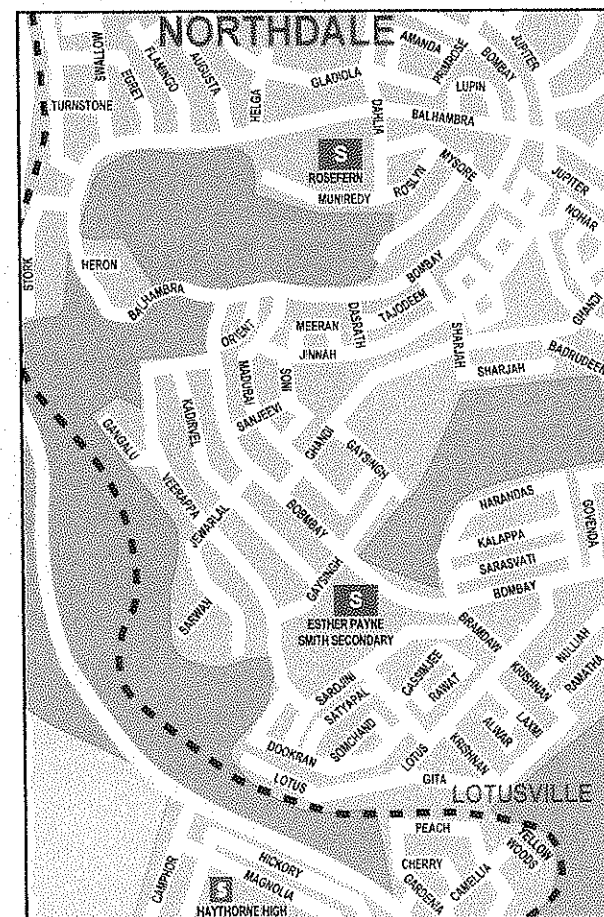
Exit Orient Street
by turning left into
Bombay Street;
turn right into
Gaysingh Street;
the school is on
the left hand side
of the road.

If the entrance to the school is in Sarojini Street, then Joshua will need to:
Turn left from Gaysingh Street into Veerappa Street and turn left again
into Sarojini Street.

- 1.4 If the scale on the map is 1 : 50 000, determine the distance (in km) as the crow flies, between Rosefern School and Esther Payne Smith Secondary School.

Distance on the map is 5,9 cm

$\therefore \text{Actual distance} = 5,9 \text{ cm} \times 50\,000 = 295\,000 \text{ cm} = 2,95 \text{ km}$



2. Study the cinema theatre layout below and answer the following questions:

2.1 Manny and Phil go to the movies and would like to sit in seats K14 and K15. Are these seats available?

K14 is available, but K15 is booked.

2.2 Does this cinema make provision for wheelchair patrons? If so, which row?

Yes, wheelchair patrons are catered for. They would sit in Row D.

2.3 What percentage of all the seats in the cinema are already booked?

$$\% = \frac{\text{booked seats}}{\text{total no. of seats}} \times 100 = \frac{12}{228} \times 100 = 5,26\%$$

		SCREEN																															
A		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19													
B		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19													
C		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19													
D						1	2	3	4	5	6	7	8	9	10	11																	
		AISLE																															
E	A	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16																															
F	A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16												A				
G	A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16												I				
H	A	S	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16												S			
J	A	L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16												L			
K	A	E	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20												E			
L		1	2	3	4	5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20																											
M		1	2	3	4	5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20																											
N		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24								
		Available Booked Unavailable Wheelchair																															

Available

Booked

Unavailable

Wheelchair

3. Cindy goes shopping with her mom at Morningside Shopping Mall in Johannesburg. Study the upper floor plan provided opposite and answer the questions that follow.

3.1 How many bathrooms are there on the upper floor?

2 bathrooms

3.2 Are there stairs down to the lower floor?

No, there is only an escalator to take you to the lower floor, but there is a symbol indicating Lifts/Stairs.

3.3 Describe where the two entrances to the mall are.

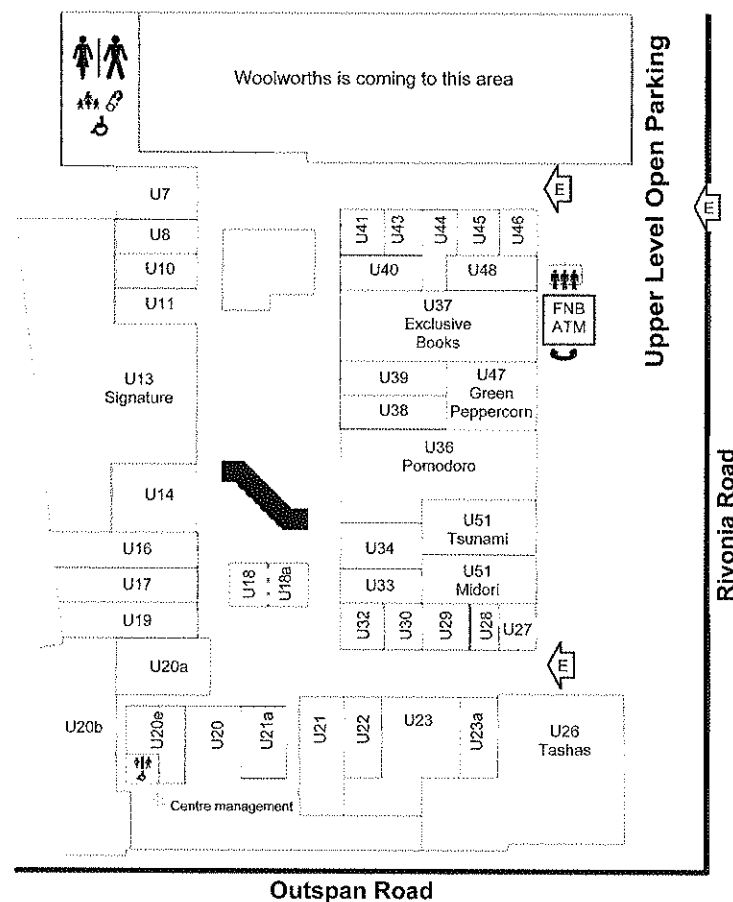
- The one entrance is on the side of the shopping centre that faces Rivonia Road, near the corner that borders Outspan Road, and between Shop U27 and U26 (Tashas).*
- The second entrance is directly opposite the entrance to the shopping centre grounds from Rivonia Road; and between Shop U46 and where Woolworths will be located.*

- 3.4 Describe the relative position of Shop U13 (Signature).

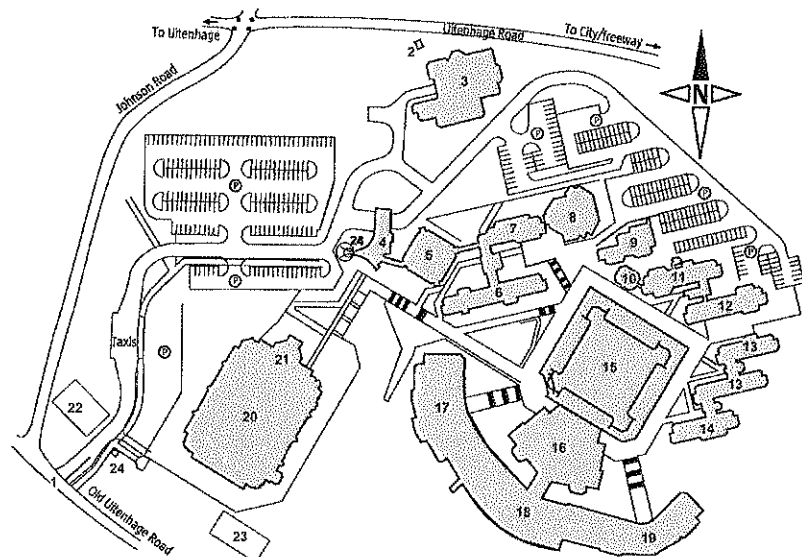
Shop U13 lies between shop U11 and U14; and opposite Shops U37; U39; U38 and U36.

- 3.5 Cindy was in the bathroom with baby changing facilities and her mom was in Exclusive Books. Describe Cindy's route to meet her mom in Exclusive Books.

As Cindy leaves the bathroom, she must turn right and walk past shops U7; U8; U10 and U11 or past Shops U41 and U40. Exclusive Books will then be on her left-hand side after Shop U11 or U40.



4. Below is a plan of the Nelson Mandela Metropolitan University Vista Campus, in Port Elizabeth:



Key

- | | | |
|--------------------------|---------------------|---------------------------------|
| (P) Parking | 9 Psychology Centre | 18 Lecture Hall B |
| 1 Entrance | 10 Chemical Store | 19 Lecture Hall C |
| 2 Weather Station | 11 Chemistry | 20 Yebo Gogo NMMU Indoor Centre |
| 3 Technical Services | 12 Physics | 21 Cafeteria |
| 4 Main Entrance & Clinic | 13 Education | 22 Cricket Nets |
| 5 Registration | 14 Geography | 23 Sport Fields |
| 6 Academic Offices | 15 Library | 24 Security |
| 7 Administration | 16 Study Centre | |
| 8 Conference Hall | 17 Lecture Hall A | |

- 4.1 Describe how to get to the entrance of the University from the City/freeway. Be sure to use compass directions.

Travel west on Uitenhage Road; then turn left at a set of robots into Johnson Road. Travel in a south-westerly direction and turn left into Old Uitenhage Road. The entrance will be on your left-hand side.

- 4.2 Which number represents the Sport Fields? 23

- 4.3 How many lecture halls are there? 3

- 4.4 In which direction is the library, if you were standing at the Yebo Gogo NMMU Indoor Centre?

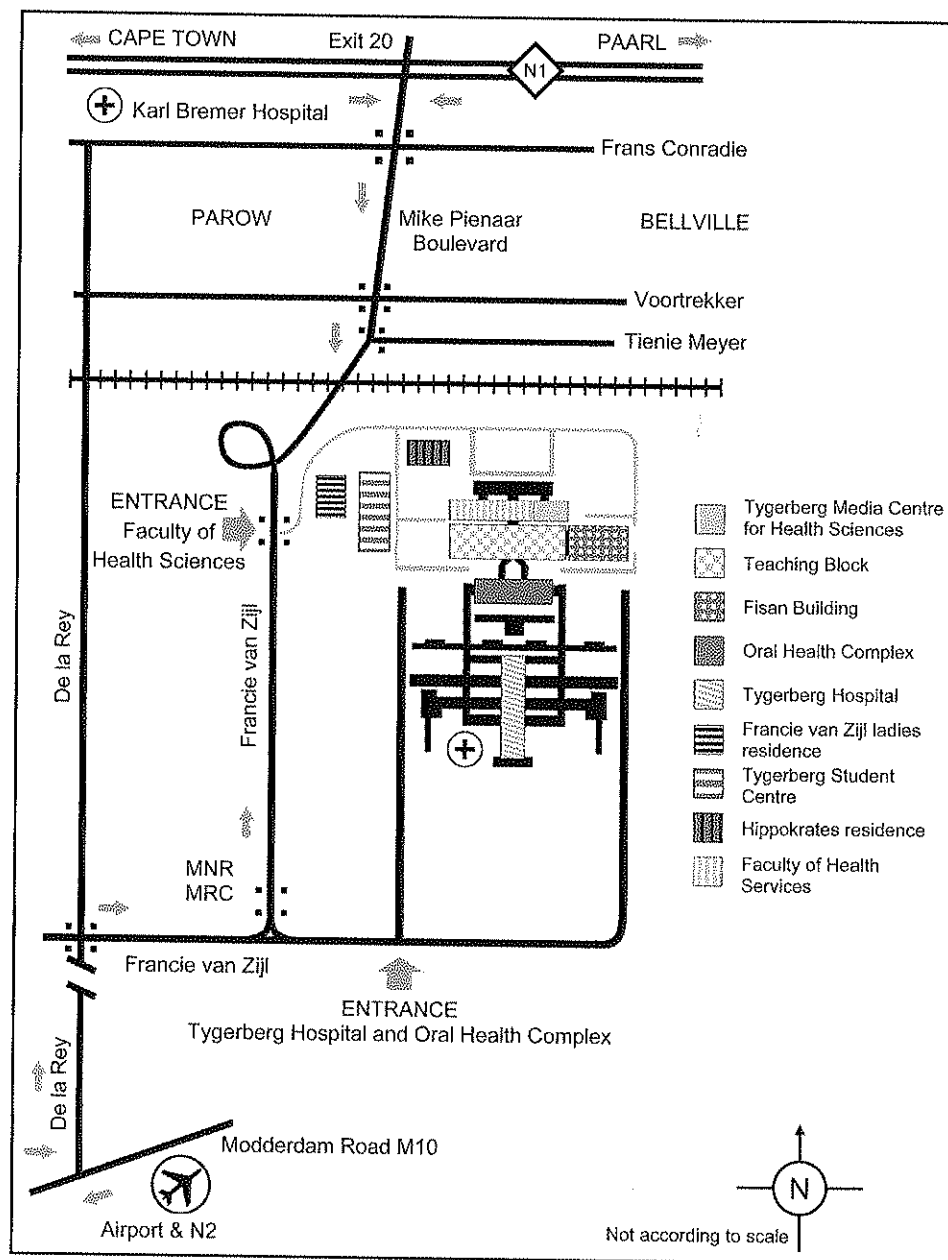
North-easterly direction

- 4.5 Describe the relative position of the Technical Services building.

The technical services building (3) is on the northern side of the university campus and borders Uitenhage Road. The building is flanked by open ground on its left and by parking bays on its right.

EXERCISE 1

Lily is a student doing a nursing degree at the Faculty of Health Sciences at Tygerberg Hospital. The following map shows the location of the faculty:



- Describe how to enter the Faculty of Health Sciences if you are coming from Paarl.
- In which direction is Tygerberg Hospital relative to the Fisan Building?
- Lily stays in Francie van Zijl ladies residence. Describe the relative position of this residence.

Travel Maps and Plans



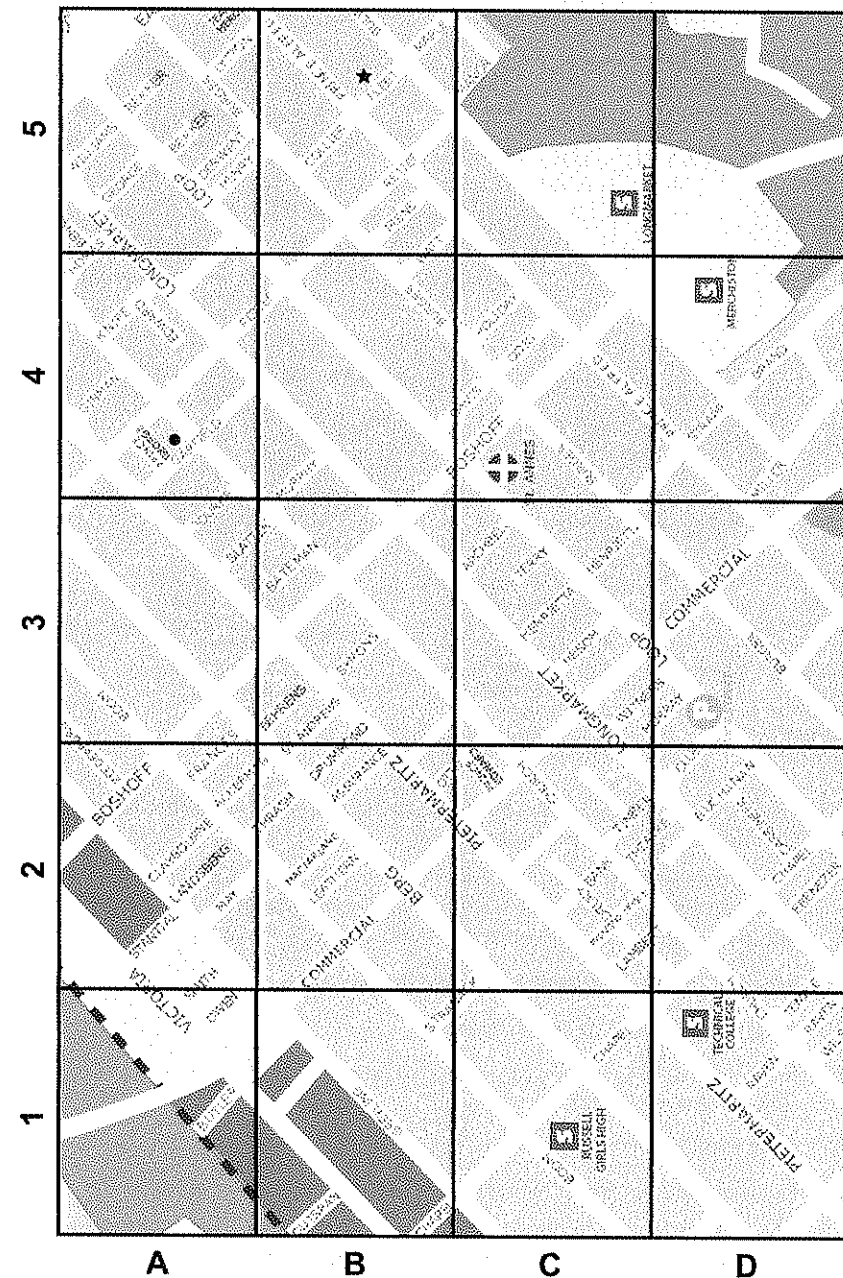
- When travelling from place to place, one might need to use national, provincial, suburban and/or street maps - each map showing more and more detail as one narrows the particular area of interest.
- Grid reference maps** label the area using a grid with rows and columns, each labelled alphabetically or numerically.
 - Grids make finding places easier because a grid reference number (e.g. A4) locates the area of interest within just one block.
- A **strip map/chart** is a linear map representing the approximate distances between major towns along a main road, as well as the general direction in which the major town lies relative to the main road.
- Distance tables** indicate the approximate distance between towns in a table - therefore, there is no need to calculate distances using a scale on a map.
- Transport maps** indicate the transport routes that buses/trains/taxis takes, together with its stops (to pick up and drop off passengers) along the route.

Worked Examples



- Study the grid street map of the suburbs in Pietermaritzburg provided:
 - What is the name of the school in block C1?
Russell Girls High School
 - What is the grid reference for St. Annes Hospital? *C4*
 - What is the grid reference for the intersection of Burger and Commercial Street?
D3
 - What is the biggest landmark in block C5? *Longmarket School*
 - Simon lives at 2 Fleet Street (indicated by a star ★) and his friend lives at 12 Garfield Road (●). Give directions for Simon to travel to his friend.
Simon must travel up Fleet Street and turn left into Prince Alfred. He must then take the first road right, into Retief Street and travel up. Simon must then turn left into church Road and his friend's house will be on the right-hand corner of Garfield and Church Road.

GRID STREET MAP OF SUBURBS OF PIETERMARITZBURG



2. Piet and his son decide to travel from their home town of Vanrhynsdorp in South Africa, to Keetmanshoop in Namibia, for a hunting expedition. Study the strip chart below and answer the questions that follow:

- 2.1 What do the distances on the right of the N7 indicate versus the distances on the left?

The distances on the right of the N7 indicate the distances travelling from Cape Town to Keetmanshoop (i.e. looking up the strip chart/N7).

The distances to the left of the N7 indicate the distances travelling from Keetmanshoop to Cape Town (i.e. looking down the strip chart/N7).

- 2.2 How far is it from Cape Town to Keetmanshoop?
995 km

- 2.3 How far must Piet and his son travel to reach Keetmanshoop?
 $995 \text{ km} - 300 \text{ km} = 695 \text{ km}$

REMEMBER: Piet is 300 km from Cape Town (his starting point).

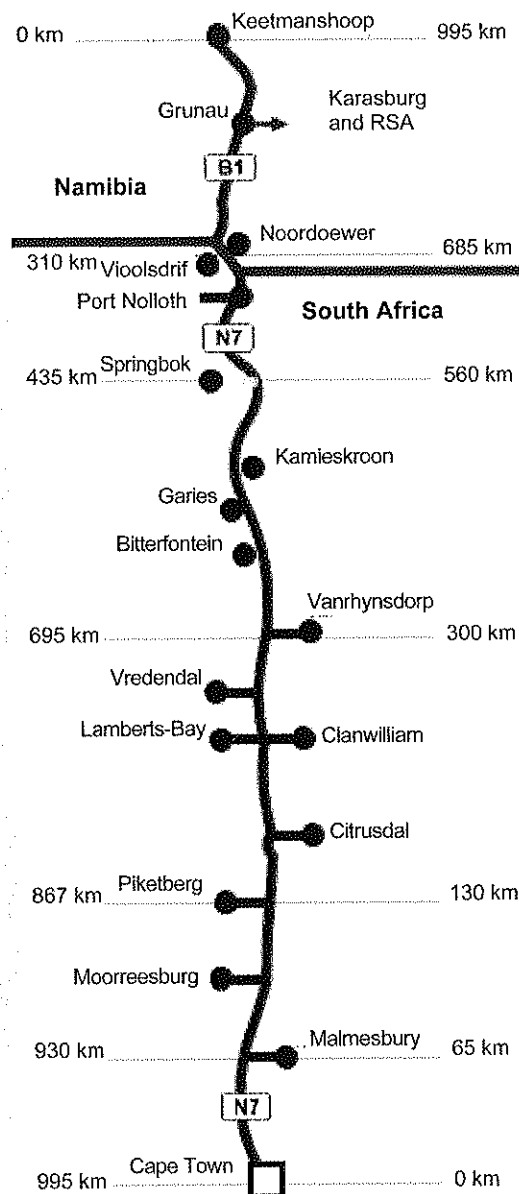
- 2.4 How many towns would Piet and his son pass while travelling through South Africa?

6

All the towns from Vanrhynsdorp to Vioolsdrif.

- 2.5 On their way back from Keetmanshoop, Piet allows his son to drive. However, his son misses the turn-off for Vanrhynsdorp. What is the next town that he will encounter?

Vredendal



3. Sipive and his family are spending some time on holiday at the different camps in the Kruger National Park. Study the distance table provided:

Gates and Camps Distances	Berg-en-Dal	Crocodile Bridge	Letaba	Lower Sabie	Malelane	Mopani	Numbi Gate	N'wanetsi	Olifants	Orpen	Pafuri Gate	Paul Kruger Gate	Phalaborwa Gate	Pretoriuskop	Punda Maria	Satara	Shingwedzi	Skukuza
Berg-en-Dal	-	149	234	113	12	281	97	180	219	213	453	83	285	92	415	165	344	172
Crocodile Bridge	149	-	196	34	141	243	130	142	181	175	415	88	246	125	377	127	306	77
Letaba	234	196	-	182	228	47	218	94	32	117	218	173	51	211	176	69	109	162
Lower Sabie	113	34	162	-	105	209	95	108	147	141	380	63	213	90	342	83	271	43
Malelane	12	141	228	105	-	272	94	170	210	204	444	74	277	85	408	156	333	64
Mopani	281	243	47	209	272	-	263	141	86	184	172	220	74	258	130	116	63	209
Numbi Gate	97	130	216	95	94	283	-	162	201	195	434	65	267	9	396	147	325	54
N'wanetsi	180	142	94	108	170	141	162	-	79	63	312	119	145	156	274	25	203	108
Olifants	219	181	32	147	210	86	201	79	-	102	250	158	83	195	212	54	147	147
Orpen	213	175	117	141	204	184	195	63	102	-	335	152	167	184	297	48	228	137
Pafuri Gate	453	415	218	380	444	172	434	312	250	335	-	392	246	438	76	287	109	380

- 3.1 How far is Crocodile Bridge from Mopani?
243 km

- 3.2 How far is Olifants from Paul Kruger Gate?
158 km

- 3.3 If Sipive leaves Orpen gate at 07:15 and travels at 40 km/h; what time will he and his family arrive at Skukuza Camp?

Distance from Orpen to Skukuza = 137 km

$$\therefore \text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\therefore \text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{137 \text{ km}}{40 \text{ km/h}}$$

$$= 3,425 \text{ h}$$

$$= 3 \text{ h} + (0,425 \text{ h} \times 60 \text{ min})$$

$$= 3 \text{ h} + 25,5 \text{ min} \approx 3 \text{ h } 26 \text{ min}$$

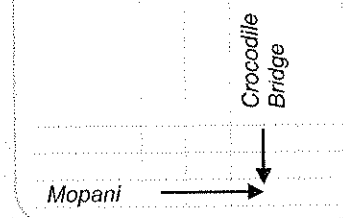
$$\therefore 07:15$$

$$+ 3:26$$

$$10:41$$

$\therefore$ Sipive and his family will arrive at Skukuza Camp at 10:41.

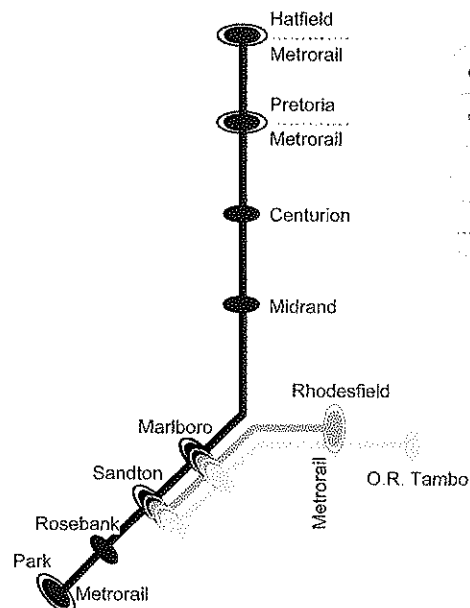
Find Crocodile Bridge on the top row and travel down that column until it intersects with the row of Mopani
i.e.



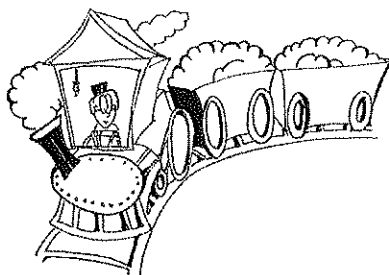
EXERCISE 2

1. Olga is flying to O.R. Tambo Airport in Johannesburg for a work conference. She will be staying in Sandton, but will also need to travel to Centurion for one meeting. Study the Gautrain transport map below and answer the following questions:

GAUTRAIN TRANSPORT MAP



1. North-south Commuter All Trains stop at every station.
2. East-west Commuter All trains stop at every station, use REAR two coaches only.
3. Airport Use FRONT two coaches only, demarcated:
- Metrorail Interchange Change here for Metrorail lines.



- 1.1 Which numbered line should Olga take from O.R. Tambo Airport to her hotel in Sandton?
- 1.2 Which is the most northerly station?
- 1.3 How many stops will Olga make on the Gautrain, on her way from Sandton to Centurion?
- 1.4 Which line only goes to Rhodesfield?
- 1.5 Which stop is south of Centurion?
- 1.6 Calculate Olga's total Gautrain transport costs if it costs R135 from O.R. Tambo Airport one-way, and another R49 per trip from Sandton to Centurion.

2. Below is a distance table for major cities in South Africa. Use the table to answer the questions that follow.

	Pretoria			
	Port Elizabeth		1 120	
	Polokwane		1 393	273
	Nelspruit		320	1 373
	Mafikeng		589	565
	Kimberley		360	832
	Johannesburg		467	273
	East London		992	750
	Durban		667	598
	Cape Town		1 660	1 042
	Bloemfontein		998	667

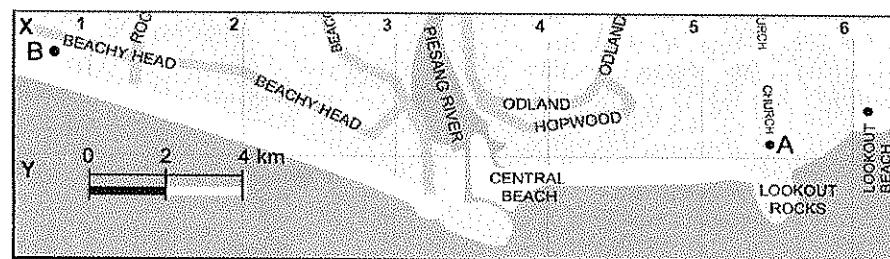
Distances given are by the best and most practical routes from centre to centre, and not necessarily the shortest

- 2.1 What is the distance from Port Elizabeth to Durban?
- 2.2 Calculate the distance from Johannesburg to Nelspruit via Pretoria.

'via' means 'by way of' - so via Pretoria means starting from Johannesburg, driving to Pretoria and then from Pretoria to Nelspruit.



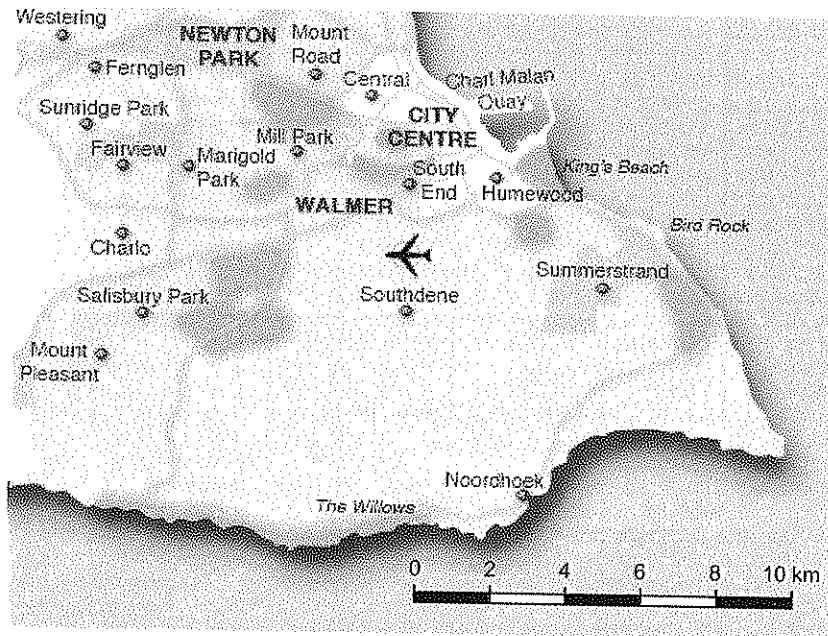
- 2.3 Zama and Phyllis are taking a road trip from Johannesburg to Cape Town. They could leave Johannesburg and travel via Kimberley or via Bloemfontein. Which is the shortest route and how much shorter is the route?
- 2.4 Determine the average speed of travelling from East London to Mafikeng, if the journey takes 12 and a half hours.
3. Four friends, Erin, Jack, Paula and Ian, hired a house in Plettenberg Bay. Below is a map of Plettenberg Bay:



- 3.1 The group is staying in a house on Church Street, labelled A. What is the grid reference of their house on the map?
- 3.2 They want to meet up with other friends that are at Lookout Beach.
- 3.2.1 Measure the distance (in cm) from their house to Lookout Beach.
- 3.2.2 Determine, using the scale, the actual distance to Lookout Beach. State your answer in metres and kilometres.

- 3.3 There is a party at a house on Beachy Head Road, labelled **B** on the map. Erin decides to call Good Fellows to take them home after the party. Good Fellows is a company that will meet you at your location, and then drive you and your car home safely so that you do not drink and drive. The company charges a R50 once-off fee and then R3,00 per km. What will each of the four friends pay to get home safely?
4. Amy and Eric stay in East London and are going on honeymoon to Port Elizabeth. They will be staying in a hotel in Humewood and intend to take a day trip out to Addo Elephant National Park and surrounds.
- Study the maps below, as well as the distance table (see p. 160) and the Annual Running Costs of a Car table alongside to answer the questions below.

MAP OF PORT ELIZABETH SUBURBS



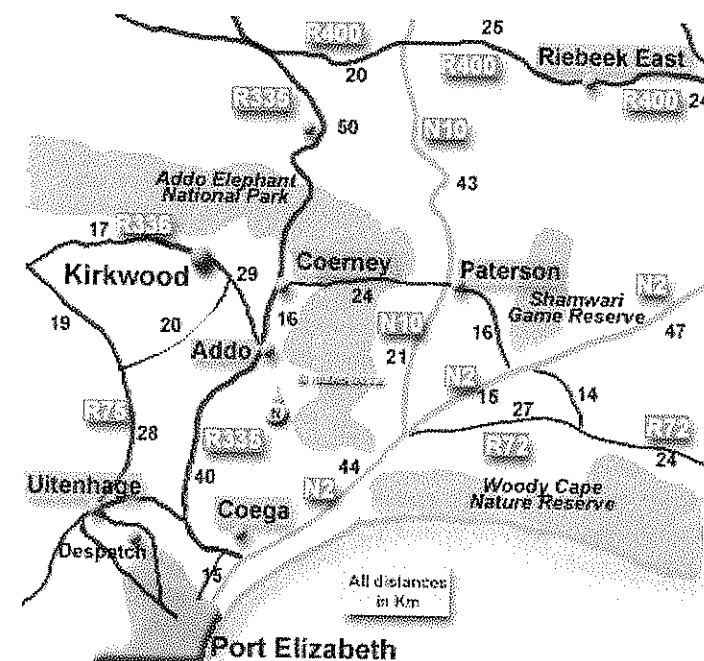
- 4.1 How far is Port Elizabeth from East London?
- 4.2 How long will it take Amy and Eric to travel from East London to Port Elizabeth, if they travel at 80 km/h? Give your answer in hours and minutes.
- 4.3 Estimate the total car cost for the trip there and back, if Amy and Eric honeymoon for 1 week and also estimate to travel another 500 km in and around Port Elizabeth. They own a Toyota Tazz which is worth R145 000.
- 4.4 Amy and Eric would like to visit friends after supper in Summerstrand. Do you think it would be feasible to walk there and back after supper? Give a reason for your answer.

- 4.5 Eric wants to take Amy to Addo Elephant Park for the day. He plans his route as follows: Port Elizabeth - Coega - Addo - Coerney - Paterson - and back onto the N2 back to Port Elizabeth. Calculate the total distance of their round-trip.
- 4.6 In which direction is Woody Cape Nature Reserve, from Addo Elephant National Park?

ANNUAL RUNNING COSTS OF A CAR

Value of vehicle	Fixed cost (R/annum)	Fuel cost	Maintenance cost
0 - 60 000	R19 310	81,4 c/km	26,2 c/km
60 001 - 120 000	R38 333	86,1 c/km	29,5 c/km
120 001 - 180 000	R52 033	90,8 c/km	32,8 c/km
180 001 - 240 000	R65 667	98,7 c/km	39,4 c/km
240 001 - 300 000	R78 192	113,6 c/km	46,3 c/km
300 001 - 360 000	R90 668	130,3 c/km	54,4 c/km
360 001 - 420 000	R104 374	134,7 c/km	67,7 c/km
420 001 - 480 000	R118 078	147,7 c/km	70,5 c/km
exceeding 480 000	R118 078	147,7 c/km	70,5 c/km

MAP OF PORT ELIZABETH AND SURROUNDS



Test Your Understanding



1. The following seating plan was used for Zoey's first semester examination. Zoey's examination number was 211141112.

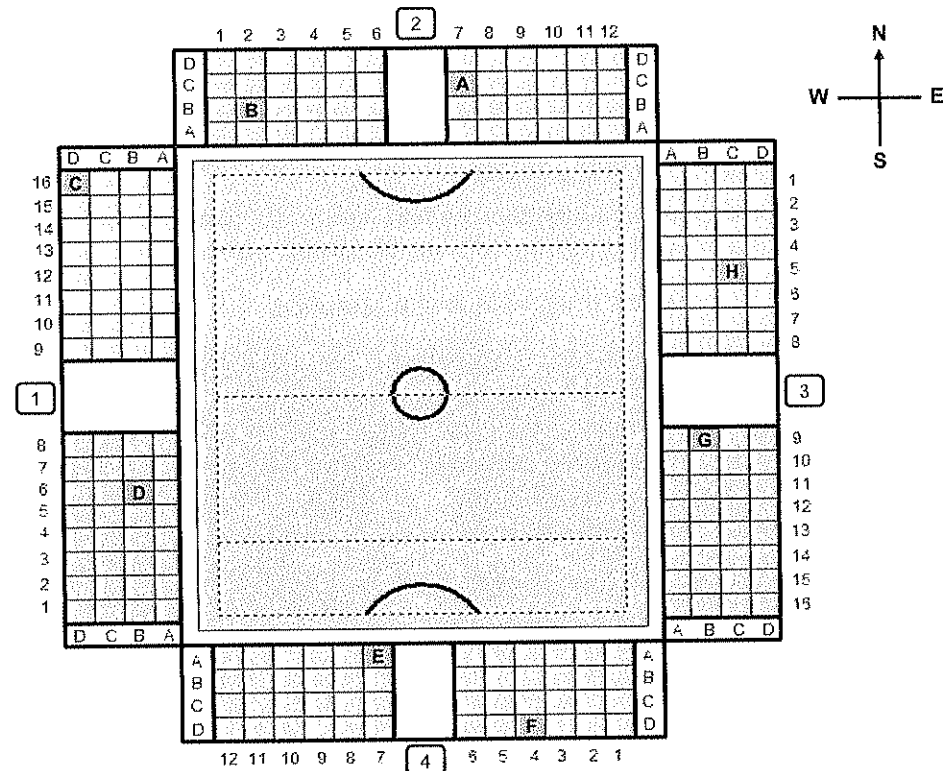
EXAMINATION SEATING PLAN

DOOR	Invigilator's table			
	Column 1	Column 2	Column 3	Column 4
Row 1	211141101	211141116	211141117	211141132
Row 2	211141102	211141115	211141118	211141131
Row 3	211141103	211141114	211141119	211141130
Row 4	211141104	211141113	211141120	211141129
Row 5	211141105	211141112	211141121	211141128
Row 6	211141106	211141111	211141122	211141127
Row 7	211141107	211141110	211141123	211141126
Row 8	211141108	211141109	211141124	211141125

Emergency exit

- 1.1 Identify the row and column in which Zoey sat.
 - 1.2 How many students sat in the same column, but behind Zoey?
 - 1.3 Determine the general direction of the emergency exit from where Zoey sat.
 - 1.4 Examination rules state that each candidate must occupy an area of at least $0,75\text{m}^2$. Calculate the minimum total area occupied by the students in this examination venue.
2. In preparation for the 2010 soccer world cup in South Africa, the Msunduzi Municipality renovated the local soccer stadium and upgraded the seating. More seats were added and there was a new seat numbering system. (see map provided)
- The features of the new numbering system are:
- ▶ There are 4 stands in the stadium - North, East, South and West stands. Each stand is named according to its position.
 - The North stand faces south and is positioned at the very top of the stadium.
 - The South stand faces north and is positioned at the bottom end of the stadium.
 - The East stand faces west and is positioned to the left of the North stand.
 - The West stand faces east and is positioned to the right of the North stand.
 - ▶ Each row of seats is labelled with the letter A, B, C or D and each column of seats is numbered (columns 1 - 16 or 1 - 12).

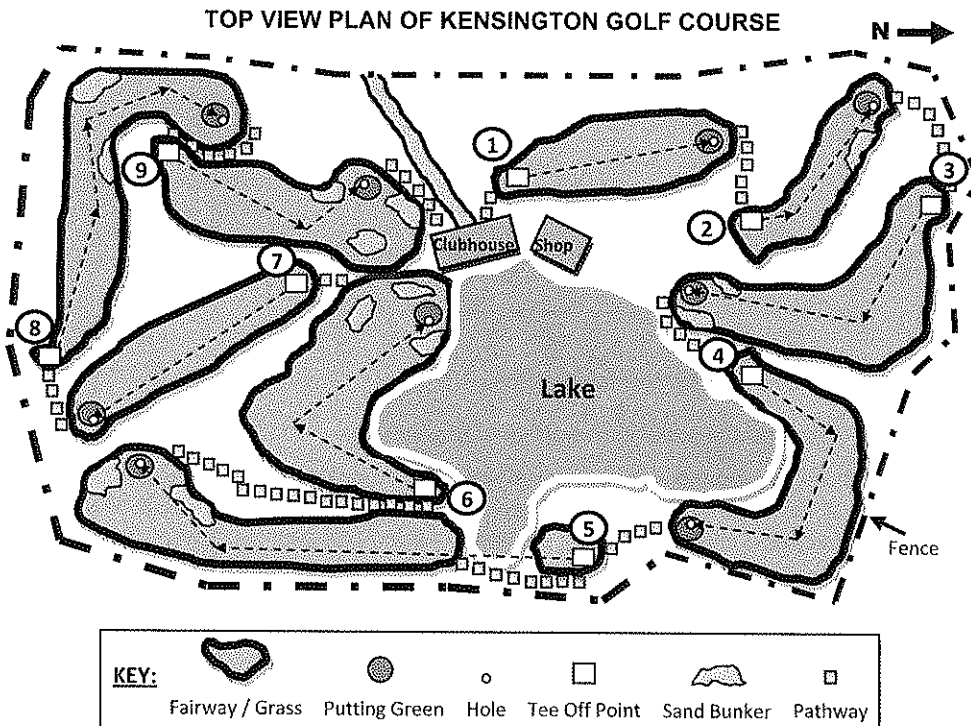
- ▶ Each seat is labelled according to the stand, row and column in which the seat is positioned. e.g., a seat in the first row and 3rd column of the North stand is numbered North A3. See the Msunduzi soccer stadium seating plan below:



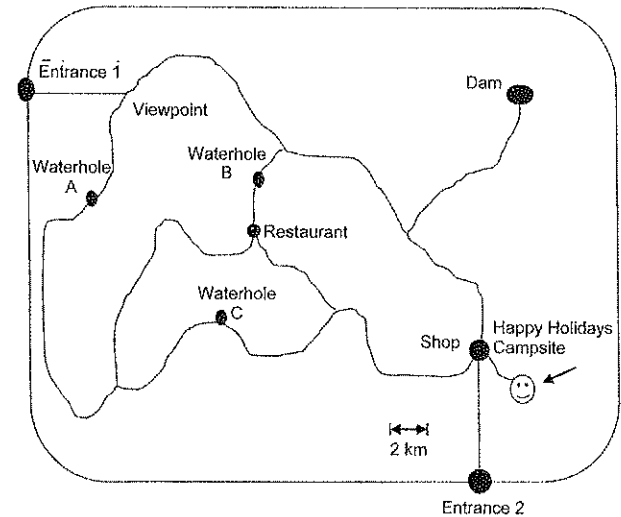
- 2.1 Use the compass provided to determine which stand, 1, 2, 3 and 4, is the North, East, South and West stand.
- 2.2 If the sun rises in the east and sets in the west, in which stand will the spectators be looking into the sun during a late afternoon game?
- 2.3 Use the soccer stadium's seat numbering system to write down the location of the seats marked A - H.



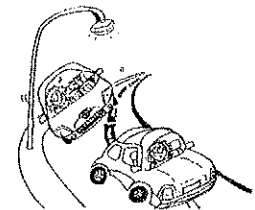
3. A top view plan of the Kensington Golf Course is given below. Use the diagram to answer the following questions.
- 3.1 How many 'holes' are there at the Kensington Golf Course?
 - 3.2 Which hole do you think will be the hardest to play? Explain your answer.
 - 3.3 Which hole has the longest distance from the tee off point to the hole? Explain how you arrived at your answer.
 - 3.4 The dotted lines show the recommended path that a golfer should aim to hit the ball from the tee off point to the hole on the putting green. In which compass direction will you be hitting if you tee off from Hole 5 and hit in the same direction as the first segment of the dotted line?
 - 3.5 On a particular day the wind is blowing very strongly from the west. This wind will affect the direction in which a golf ball flies.
 - 3.5.1 If you had to tee off at Hole 1 and hit the ball in the same direction as the dotted line, explain what might happen to your ball.
 - 3.5.2 If you had to tee off at Hole 8 and hit the ball in the same direction as the first segment of the dotted line, explain what might happen to your ball.
 - 3.6 Hole 1 is a 250 m course. This means that the distance as measured along the dotted line from the edge of Tee Off Point 1 to the hole in the green on Hole 1 is 250 m. Use this information to determine the approximate length of Hole 7.



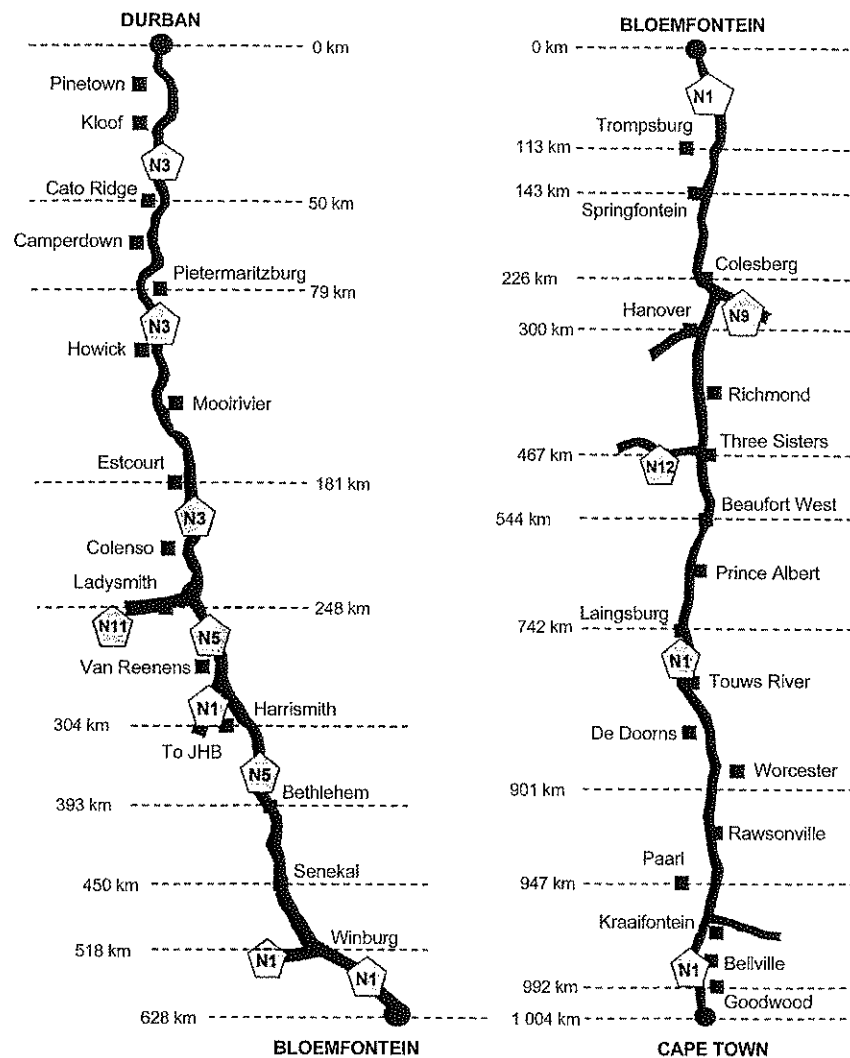
4. Study the map of the game reserve below and answer the following questions.



- 4.1 Determine your destination if you follow these directions:
 - Depart from the campsite and travel in a north-westerly direction until you reach an intersection.
 - At the intersection take the second left exit.
 - Continue along this road until you reach another intersection. At this intersection turn right.
 - Continue along this road for approximately 4 km and you will arrive at your destination.
- 4.2 The diagram on the map which represents the scale is indicated by . Use this to determine a numerical scale for the map.
- 4.3 Use the scale found in Question 4.2 to determine the distance from Entrance 2 to the shop. Give your answer in kilometres.
5. Vuyani is preparing for a car trip from Durban to Cape Town. To plan the route, he uses the strip charts on p. 164.
 - 5.1 Calculate the distance from:
 - 5.1.1 Durban to Pietermaritzburg
 - 5.1.2 Senekal to Springfontein
 - 5.1.3 Durban to Cape Town



STRIP CHARTS - DURBAN TO CAPE TOWN



[Source: Adapted from: Map Studio, Engen Road Maps of Southern Africa (pages 84 & 87)]

5.2 Vuyani decides to keep a log book of part of the trip (provided alongside).

5.2.1 What is the distance from Harrismith to Winburg?

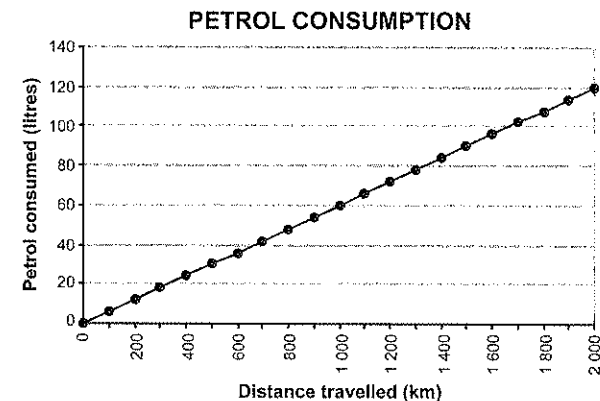
5.2.2 How long did it take for Vuyani to travel from Pietermaritzburg to Bethlehem?

5.2.3 Why do you think Vuyani stopped for an extended period of time in Bloemfontein?

VUYANI'S LOG BOOK

Approximate Time of Day	City/Town	Distance Travelled (km)
06:00	Durban	0 km
06:25	Cato Ridge	50 km
06:45	Pietermaritzburg	79 km
07:30	Estcourt	181 km
08:30	Harrismith	304 km
09:15	Bethlehem (arrive)	393 km
09:45	Bethlehem (leave)	393 km
10:30	Senekal	450 km
11:05	Winburg	518 km
12:00	Bloemfontein (arrive)	628 km
12:45	Bloemfontein (leave)	628 km
13:45	Trompsburg	741 km
14:00	Springfontein	771 km
14:45	Colesberg	854 km

5.3 Vuyani consulted the following graph relating to the petrol consumption of his car while travelling on a freeway. The graph shows the number of litres of petrol that the car uses dependent on the distance travelled.



A car's petrol consumption is determined by the number of litres of petrol used per 100 km travelled.

5.3.1 Calculate the petrol consumption of Vuyani's car.

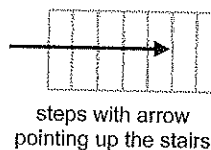
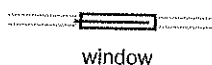
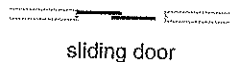
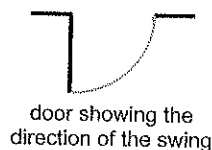
5.3.2 Vuyani's car has a 45 litre petrol tank. This means that when the petrol tank is full, it can hold 45 litres of petrol. How many times will Vuyani have to stop to put petrol in on the trip from Durban to Cape Town if he starts with a full tank?

5.4 On day 1 of the trip, Vuyani arrives in Colesberg with a $\frac{1}{4}$ tank of petrol.

5.4.1 How much petrol does Vuyani have in the tank when he arrives in Colesberg?

5.4.2 Should he fill up in Colesberg or will he have enough petrol to reach Three Sisters? Explain your answer.

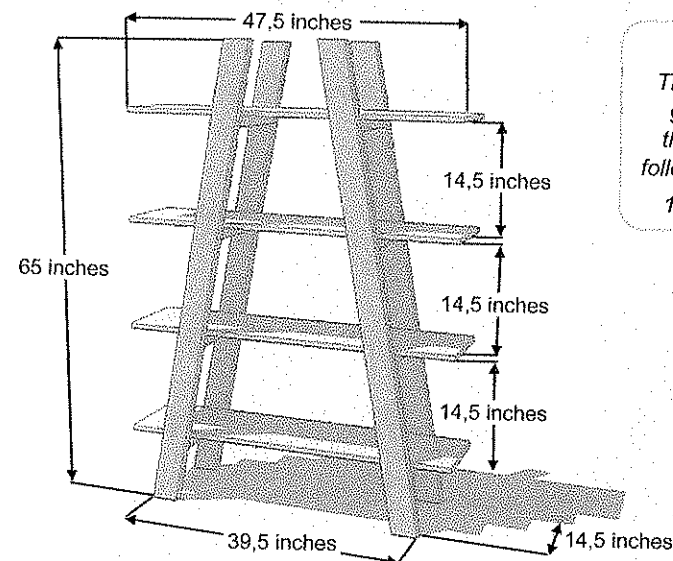
- A **floor plan** shows a two-dimensional **view** of a building/structure **from above** (i.e. bird's eye view); excluding the roof of a building.
 - It provides the following information:
 - the size and shape of each room (incl. lengths and widths of rooms and walls)
 - the position of exterior and interior walls, doors, windows and fixtures, such as: toilets, basins, built-in cupboards, showers, etc.
 - It does *not* provide information regarding the height of the walls, nor the height of the structure/building.
 - Commonly accepted drawing conventions are used on floor plans, including the following symbols:



- **Elevation plans** are two-dimensional pictures of the **outside of a building or structure**.
 - It provides information regarding the height of the structure/building and external features e.g. walls, doors and windows.
 - It does *not* provide information about sizes of rooms or interior fittings.
 - Elevation plans are named using compass directions, i.e. the 'north' elevation is the side of the house that faces the compass direction 'north'.
- **Design plans** are drawings to show how a building/structure is made and how it functions or looks.
 - It often provides manufacturing and/or assembly details, such as measurements and attachment points of individual components.
 - It is commonly used for manufacturing of furniture and clothing.

Worked Examples

1. Phil designs an outside pot plant stand that will house all his seedling plants; while ensuring maximum sunshine exposure. Study Phil's pot plant stand design plan below in order to answer the questions that follow:



NOTE!

The design plan is given in inches, therefore use the following conversion:
1 inch = 2,54 cm



See Module 4 for more imperial and metric conversions.

- 1.1 How many horizontal shelves will Phil have to put his pot plants on?
4
- 1.2 How many wooden planks will Phil need to make his stand?
8 planks [4 horizontal and 4 vertical]
- 1.3 What is the total height of the stand? Give your answer in metres.
 $65 \text{ inches} = 65 \times 2,54 \text{ cm} = 165,1 \text{ cm} = 1,651 \text{ m}$
- 1.4 If Phil's seedlings measured 27 cm in height, would they fit into the shelves? Show all calculations to support your answer.

$$\begin{aligned} \text{Height between shelves} &= 14\frac{1}{2} \text{ inches} \\ &= 14,5 \times 2,54 \text{ cm} = 36,83 \text{ cm} \end{aligned}$$

∴ The plants would definitely fit into the shelves.

- 1.5 If the diameter of each pot plant was 12 cm; how many pot plants would fit on the top shelf?

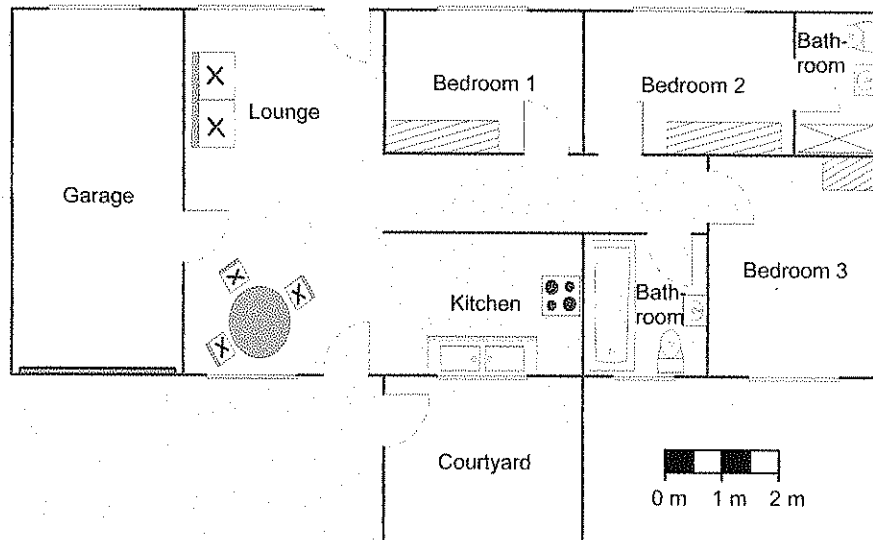
$$\begin{aligned}\text{Top shelf} &= 47,5 \text{ inches} \\ &= 47,5 \times 2,54 \text{ cm} \\ &= 120,65 \text{ cm}\end{aligned}$$

$$\begin{aligned}\therefore \text{Number of pots} &= \frac{120,65 \text{ cm}}{12 \text{ cm}} \\ &= 10,05 \\ &\approx 10 \text{ plants}\end{aligned}$$

NOTE! You have to round down, otherwise a pot plant would fall off!



2. Study the floor plan of the house below and answer the following questions:



- 2.1 How many doors does this house have?
9 doors



represents a door!

- 2.2 Describe all the features of the kitchen.

- The kitchen has a stove () and a kitchen sink ().
- There is no door leading into the kitchen, as it is open plan with the lounge.
- The kitchen also has a window overlooking the courtyard.

- 2.3 What are the similarities and differences in fixtures between the bathroom and the en suite bathroom attached to bedroom 2?

- Both bathrooms have a toilet () and a basin ().
- The difference is that the bathroom has a bath (indicated by); while the en suite bathroom has a shower (indicated by).

- 2.4 Calculate the dimensions of the house in reality (excluding the garage), in metres.

1,5 cm on the map represents 2 m in reality

$$\begin{aligned}\therefore 1,5 \text{ cm} &: 2 \text{ m} \\ 1,5 \text{ cm} &: 200 \text{ cm} \\ \frac{1,5}{1,5} &: \frac{200}{1,5} \\ 1 &: 133,33 \\ \approx 1 &: 134\end{aligned}$$

REMEMBER! Convert a bar scale into a numerical scale in order to determine actual lengths.



Always round up.



Length of house on plan = 9,2 cm

$$\begin{aligned}\therefore \text{Actual length of house} &= 9,2 \times 134 \\ &= 1\,232,8 \text{ cm} \\ &= 12,33 \text{ m}\end{aligned}$$

Breadth of house on plan = 4,8 cm

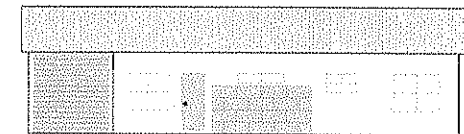
$$\begin{aligned}\therefore \text{Actual breadth of house} &= 4,8 \text{ cm} \times 134 \\ &= 643,2 \text{ cm} \\ &= 6,43 \text{ m}\end{aligned}$$

$\therefore$ Dimensions of the house are: 12,33 m $\times$ 6,16 m

3. Study the Elevation Plans alongside, together with the floor plan of the house in Question 2, and answer the following questions:

ELEVATION PLANS OF A HOUSE

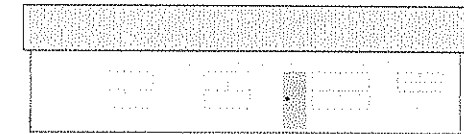
North Elevation



- 3.1 Which elevation map shows the front of the house?
Give a reason for your answer.

The North Elevation as the garage door (indicated on the left of the plan) is at the front of the house.

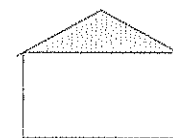
South Elevation



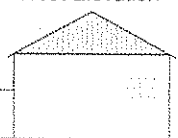
- 3.2 On which elevation map are the windows of the bedrooms of the house depicted?

South Elevation

East Elevation



West Elevation



- 3.3 What information does the floor plan tell you about the bathroom with the bath that is not visible on the elevation plans?

The width and length of the walls of the bathroom as well as the location of all of the fittings in the bathroom.

- 3.4 What information does the appropriate elevation plan tell you about the bathroom (with the bath) that is not evident on the floor plan?

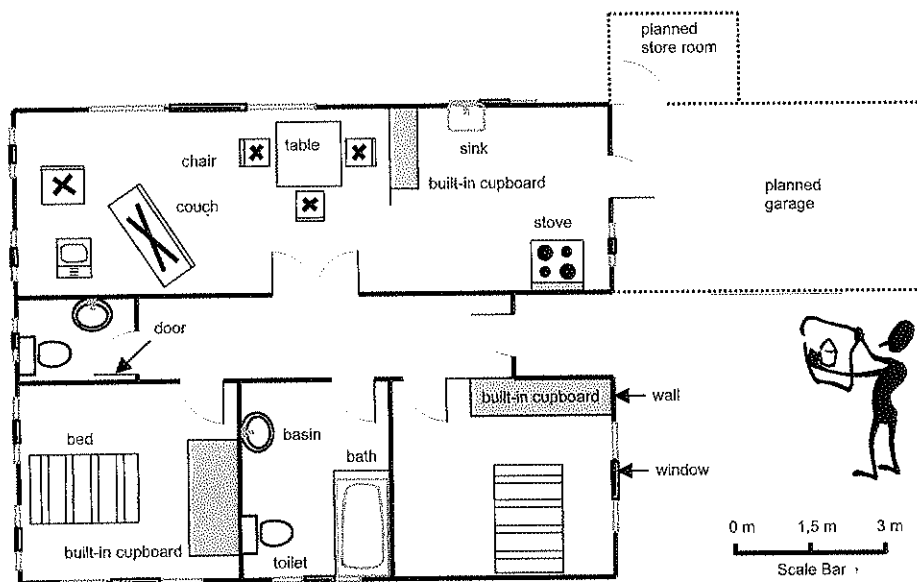
The height of the window in the bathroom wall as well as the height of that window above the ground.

Test Your Understanding



1. The first project that *Bob's Builders* is working on is renovating Mpho Dlamini's house by building a garage and store room on to the side of the house. A 2-dimensional floor plan of Mpho's house is given below. The rooms drawn with dotted lines represent the garage and store room that Mpho wants to add on to the house.

FLOOR PLAN OF MPHO'S HOUSE



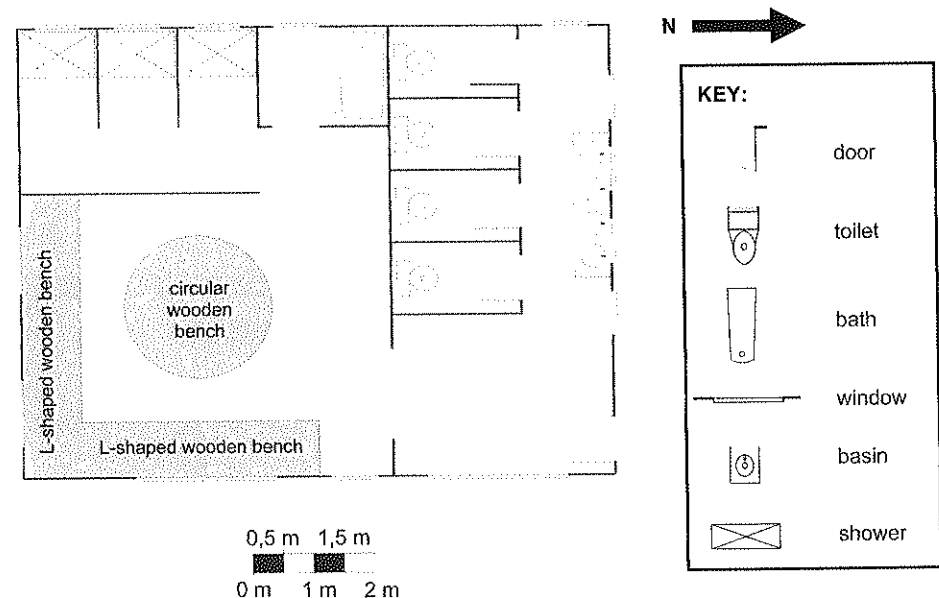
Use the floor plan to answer the following questions:

- Write a description that explains what the house looks like.
- How many windows are there in this house?
- Which door do you think is the main entrance door into this house?
- The dotted lines show where Mpho plans to build the garage and storage room. Use the given scale bar to determine the dimensions of the garage and store room in metres. Round off your answer to one decimal place.

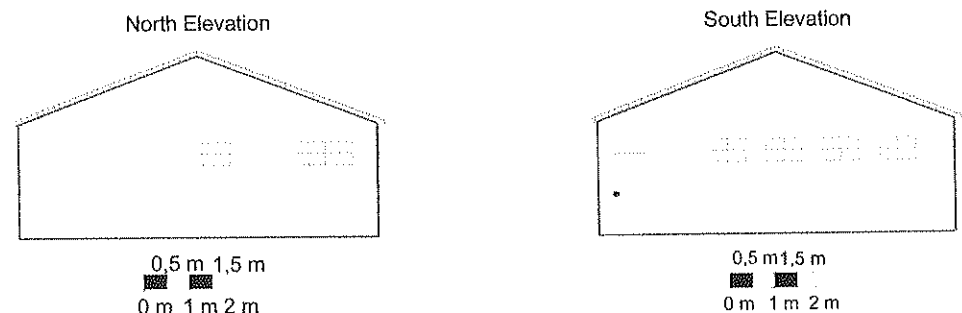
- Mpho wants to replace the carpet in the bedroom with the built-in cupboard. Calculate how much it will cost to re-carpet this bedroom if carpeting costs R129,00/1,5 m². (The carpet does not go underneath the cupboard).
- Design a house that you would like to live in and draw a floor plan of this house. Your plan does not have to be drawn to scale, but your plan must be drawn neatly. Show the positions of doors, windows and all other fittings.

2. A floor plan and elevation plans of the Lifestyle Gym's changing rooms are provided below and on p. 168.

FLOOR PLAN OF THE CHANGING ROOMS AT THE LIFESTYLE GYM



ELEVATION PLANS OF THE CHANGING ROOMS AT THE LIFESTYLE GYM



- TSA of Option A = $(2 \times 3,142 \times (2,5)^2) + (2 \times 3,142 \times 2,5 \times 11,5 \text{ cm})$
= $R219,94 \text{ cm}^2$

- $\therefore$ Cost of Option A = $219,94 \text{ cm}^2 \times R0,20$
= $R43,99$

$$\text{Radius} = \frac{1}{2} \text{ diameter}$$

$$\therefore 5 \text{ cm} \div 2 = 2,5 \text{ cm}$$

- TSA of Option B = $(2 \times 5 \times 4) + (2 \times 5 \times 11,5) + (2 \times 4 \times 11,5)$
= 247 cm^2

- $\therefore$ Cost of Option B = $247 \text{ cm}^2 \times R0,15$
= $R37,05$

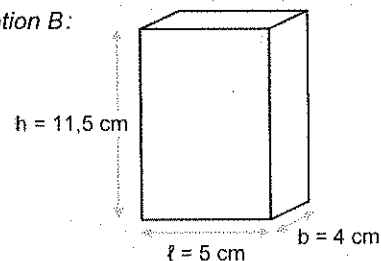
- Therefore, it is more cost-effective to use Package Option B, the rectangular prism.

2. The manufacturer wants to ship the golf balls in shipping boxes, which have the following dimensions: $1 \text{ m} \times 1,05 \text{ m} \times 1,2 \text{ m}$.

Using the most cost-effective Packaging Option found in Question 1.3 to determine how many of those golf ball boxes can fit into the shipping box.

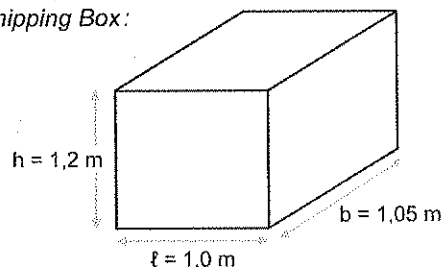
- Best Packaging Option found in Question 1.3 was

Option B:



Boxes not drawn to scale.

- Dimensions of Shipping Box:



- Convert the dimensions all into one unit - i.e. cm:
Height of shipping box = $1,2 \text{ m} \times 100 = 120 \text{ cm}$
Length of shipping box = $1 \text{ m} \times 100 = 100 \text{ cm}$
Breadth of shipping box = $1,05 \text{ m} \times 100 = 105 \text{ cm}$

Practical Method:

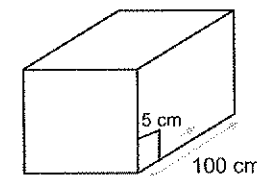
Determine how many golf ball boxes will fit across the:

length - 20 boxes

(i.e. $100 \text{ cm} \div 5 \text{ cm} = 20$)

breadth - 26 boxes

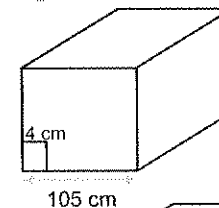
(i.e. $105 \text{ cm} \div 4 \text{ cm} = 26,25$)



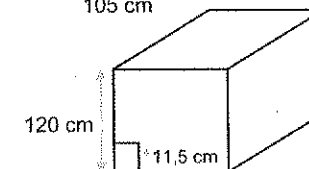
There will be a bit of extra space along the breadth, but we won't be able to squeeze 27 boxes along the breadth $\therefore$ round down your answer.

height - 10 boxes

(i.e. $120 \text{ cm} \div 11,5 \text{ cm} = 10,43$)



Again, there will be some space at the top, but not enough for 11 boxes!



Therefore, the total number of boxes that will fit in the shipping box

$$= \text{no. of boxes along length} \times \text{no. of boxes across breadth} \times \text{no. of boxes that fit in height}$$

$$= 20 \times 26 \times 10$$

$$= 5\,200 \text{ boxes}$$

Mathematical Method:

$$\begin{aligned} \text{Determine the volume of a box of golf balls} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 5 \text{ cm} \times 4 \text{ cm} \times 11,5 \text{ cm} \\ &= 230 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Determine the volume of the shipping box} &= 100 \text{ cm} \times 105 \text{ cm} \times 120 \text{ cm} \\ &= 1\,260\,000 \text{ cm}^3 \end{aligned}$$

$\therefore$ Total no. of golf ball boxes that can fit into the shipping box

$$= \frac{\text{volume of shipping box}}{\text{volume of golf ball box}} = \frac{1\,260\,000 \text{ cm}^3}{230 \text{ cm}^3} = 5\,478,26 \approx 5\,478 \text{ boxes}$$

PROBLEM!! Why is there a difference between the Practical Method answer and the Mathematical Method answer?

In the Practical Method, we discovered that there was a little bit of extra space along the breadth and height of the shipping box. This space could not be used, as it was too small to fit another box.

By comparison, the Mathematical Method just calculates volumes, which does not consider the fact that a golf ball box cannot be cut in half or pieces, to squeeze them into the little gaps!

Therefore, the Mathematical Method does not work here, since we cannot use pieces of a golf ball box to fill the shipping box.

SOLUTION!! Be very careful when using the Mathematical Method if whole items are asked to fill a space. Rather then use the Practical Method. However, if you are working with a product that can be split up e.g. a liquid or sand, then you can use the Mathematical Method.

Test Your Understanding



1. Shoe boxes in which shoes will be packed have the following dimensions:

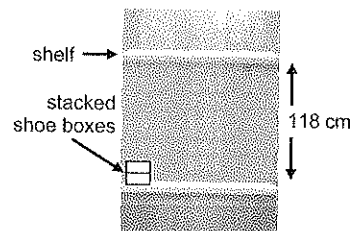
Length = 27,5 cm Breadth = 15 cm Height = 11,9 cm



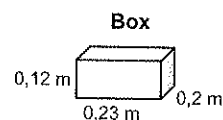
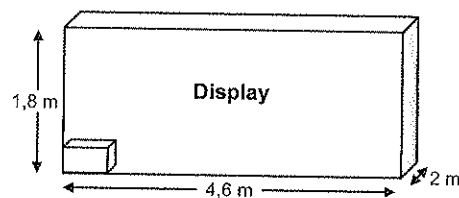
- 1.1 Calculate the volume of a shoe box.

Volume = length × breadth × height

- 1.2 The shoe boxes are packed on shelves. The height between the shelves is 118 cm. Determine the maximum number of shoe boxes that can be stacked on top of each other in a single pile between two shelves.



2. Use both the Practical Method and the Mathematical Method to determine how many boxes a shopkeeper should order, in order to build a display, as shown alongside. Evaluate both your Practical Method and Mathematical Method answers and explain why they are either the same or different.



The boxes that will be ordered have the accompanying dimensions:

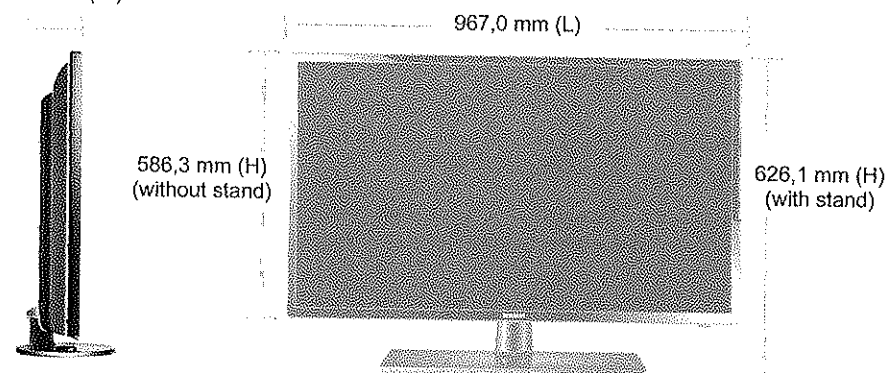
3. The Samsung Multisystem TV is shown below.

- 3.1 Determine the size of the box that will house the TV, if one needs to make allowance for an extra 20% per dimension to accommodate Styrofoam to prevent breakages. Give your final dimensions in cm.

- 3.2 Calculate the cost of the cardboard to make the box, if the cardboard costs R0,02 per cm². **TSA = (2 × ℓ × h) + (2 × ℓ × h) + (2 × ℓ × h)**

- 3.3 Using the dimensions of the TV box found in Question 3.1, determine how many boxes can fit into a storeroom with the following dimensions: 3 m × 2,5 m. Assume that the boxes cannot be stacked on top of each other, nor on their side.

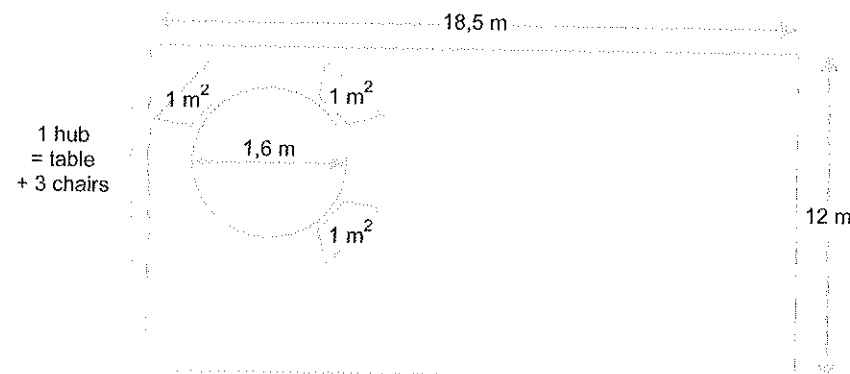
107,1 mm (W)

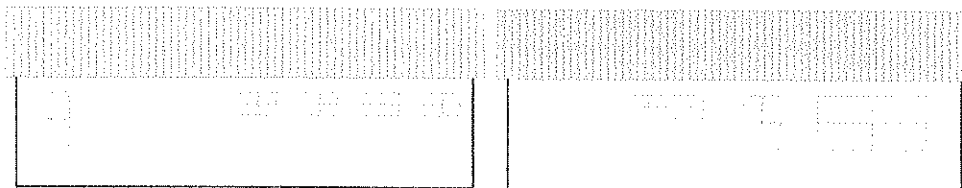


199,9 mm (W)

4. An office, measuring 18,5 m by 12 m has redesigned its working space, by creating circular hubs for 3 office workers. If each hub consists of a circular table (with a diameter of 1,6 m), plus 3 chairs (each taking up a space of 1 m²); determine how many hubs can fit into the office. Also allow 15% of the office space to be open; in order for workers to be able to walk between the hubs.

Area = ℓ × b Area = πr² where π = 3,142





- 2.1 What information does the floor plan tell you about the changing rooms that you cannot see on the elevation plans?
- 2.2 Use the floor plan and the bar scale provided on the plan to determine the actual length and width (in metres) of the room that contains the bath. Round off your answer to one decimal place.
- 2.3 Do you think the changing rooms have been well designed? Give a reason to explain your answer.

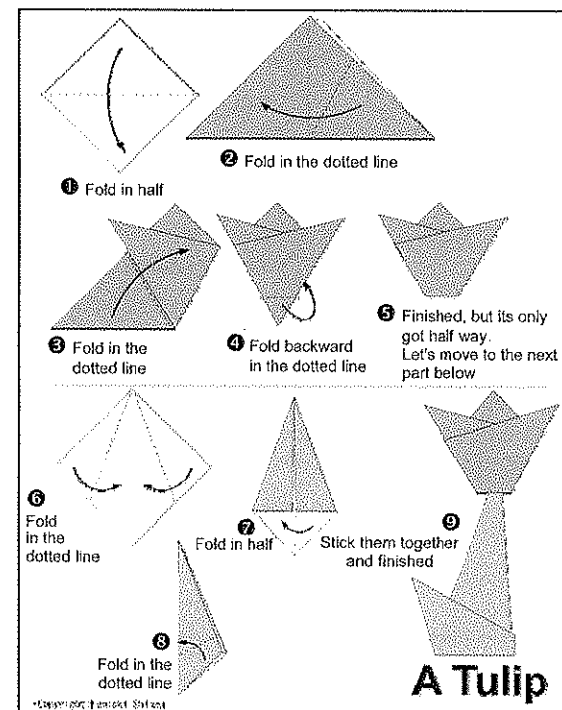
UNIT

4

INSTRUCTIONS AND ASSEMBLY DIAGRAMS

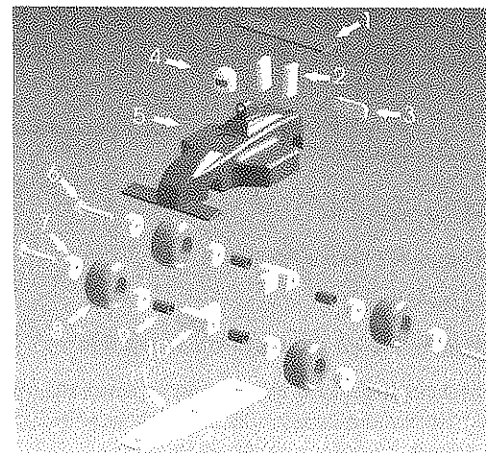
- With new inventions and progress in technology, we are often required to follow technical instructions. Sometimes the instructions are in the form of diagrams (or pictures), words, or a combination thereof. It is important for us to be able to follow instructions and also to develop instructions for other people to follow.
- Be sure to take note of the following points when writing your own set of instructions and/or diagrams:
 - Use short, clear sentences
 - Use precise, descriptive words
 - Often numbering, arrows and dotted lines help to show measurement, direction and ordering
 - Diagrams should be clear, large and easily understandable
 - If possible, colourful diagrams are very effective

- Example:
How to make an Origami Tulip:



Worked Examples

1. Siphokazi has bought his son a racing car for Christmas. He is shocked when he opens the box, only to find a box full of parts and a diagram, as shown below:



1. wing
2. wing strut (2 needed)
3. exhaust
4. helmet
5. body
6. $2 \frac{1}{2}$ " long $\frac{5}{16}$ " bolt (4 needed)
7. wheel bearing
8. skateboard wheel (4 needed)
9. wheel spacer (4 needed)
10. wheel/axle housing (2 needed)
11. belly pan



1.1 How many wheel bearings should Siphokazi have in the box?

8

1.2 Write out a set of instructions to help Siphokazi assemble his son's racing car.

Step 1: Separate the racing car parts into two groups:

Group A: Parts labelled 1 - 5

Group B: Parts labelled 6 - 11

Step 2: From Group A, take the body of the car 5 and attach the following:

(i) Place the helmet 5 into the circular seat in the middle of the body of the car (5).

(ii) Attach the exhaust (3) to the back of the body (5).

(iii) Take the two wing struts (2) and position them vertically at the back of the body (5).

(iv) Attach the wing (1) on top of the wing struts.

Step 3: From Group B, take one wheel/axle housing (10) and attach the wheel spacer (9) to the left of the wheel/axle housing (10) and another one wheel spacer (9) to the right of the wheel/axle housing (10).

Step 4: From Group B, take one wheel bearing (7) and attach it to the left of the wheel spacer (9) and another wheel bearing (7) to the right of the other wheel spacer (9).

Step 5: From Group B, take one skateboard wheel (8) and attach it to the left of the wheel bearing (7) and another skateboard wheel (8) to the right of the other wheel bearing (7).

Step 6: From Group B, take one wheel bearing (7) and attach it to the left of the skateboard wheel (8) and another wheel bearing (7) to the right of the other skateboard wheel (8).

Step 7: From Group B, take one $2\frac{1}{2}$ " long 5/16" bolt (6) and attach it to the left of the wheel bearing (7) and another $2\frac{1}{2}$ " long 5/16" bolt (6) to the right of the other wheel bearing (7).

Step 8: Repeat steps 3 - 7 for the back set of wheels.

Step 9: Attach both sets of front and back wheels to the bottom of the body (5).

Step 10: Attach the belly pan (11) to the underside of the racing car body (5).

2. Henrietta would like to print her own pictures for her T-shirts. She finds this website www.makeyourowngarments.com, where they describe how to design your own T-shirts by ironing your image to the T-shirt. Match the written instructions with the diagrams below; in the correct order.



INSTRUCTIONS

A: Preheat the iron for 3 minutes on the hottest cotton setting.

B: Use on light/pastel colour fabrics only.

C: Peel away transfer paper whilst warm.

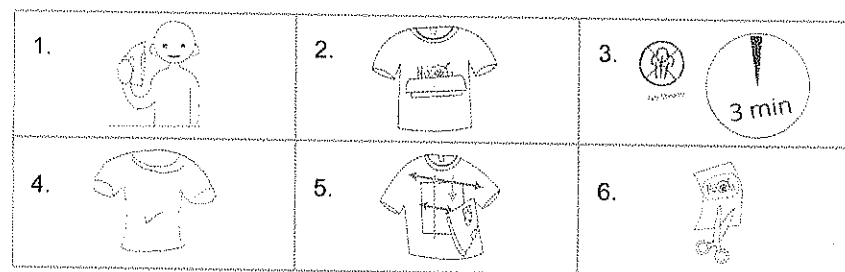
D: Choose a smooth hard surface at waist level.
Place the transfer sheet face down on your garment.

E: Trim around or contour cut the image.
Note: This paper has a clear film background.

F: Iron over it with firm pressure in a smooth and constant smooth motion, ensuring that all areas, including the edges, have been well ironed.

The iron should move slowly over the transfer left to right as well as up and down until all areas have been covered at least once!

MYOG Diagrams



Step 1: Instruction B - Diagram 4

Step 2: Instruction E - Diagram 6

Step 3: Instruction A - Diagram 3

Step 4: Instruction D - Diagram 1

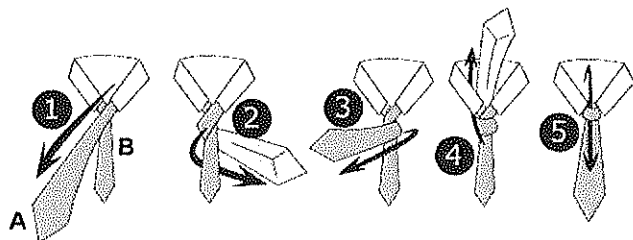
Step 5: Instruction F - Diagram 5

Step 6: Instruction C - Diagram 2

Test Your Understanding



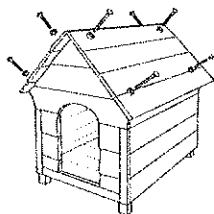
1. Devan is starting school and needs to learn how to knot his school tie. Write out a set of instructions for Devan, using the diagrams below:



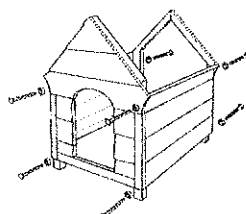
2. Piet has bought a dog kennel and needs to assemble it, using the diagrams below. However, the order of the diagrams and instructions have got muddled in the box.

Diagrams:

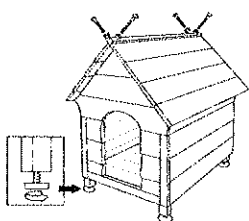
A:



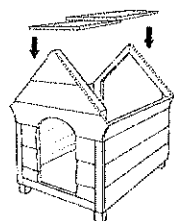
B:



C:



D:



Instructions:

1. Secure ridge cap and adjust feet as needed
 2. Drop in floors
 3. Secure front and back panels
 4. Attach roof
- 2.1 Evaluate diagrams A - D and determine how many bolts are indicated (/).
 - 2.2 Explain why the actual number of bolts needed differs from the number indicated in the diagram.
 - 2.3 Help Piet assemble the dog kennel by matching the diagrams to the instructions, in the correct order.

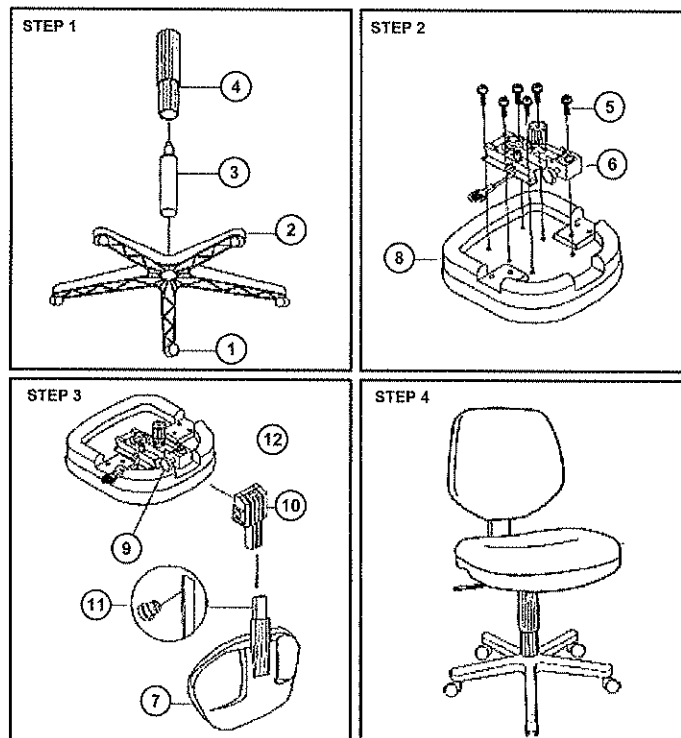
3. Study the assembly instructions for 'Comfort' Classic Chair No. CH290 by ALVIN:



Assembly Instruction for 'COMFORT' Classic Chair No. CH290

PART LIST

KEY	QTY	DESCRIPTION	KEY	QTY	DESCRIPTION	KEY	QTY	DESCRIPTION
1	5	Caster	5	6	Mechanism Screw	9	1	Backrest Height Adj. Knob
2	1	Base	6	1	Mechanism	10	1	Mechanism Plastic Bellow
3	1	Seat Post	7	1	Backrest	11	1	Plastic Stopper
4	1	Seat Post Cover	8	1	Seat			



Alvin & Company, Inc. Bloomfield, Ct Grand Prairie, TX www.alvinco.com

- 3.1 How many backrest height adjustment knobs are needed?
- 3.2 Which part comes in a quantity of six?
- 3.3 Write a set of instructions to match the diagram steps for the assembly of the 'Comfort' classic chair.



- When investigating the most cost effective and most appropriate way to package cans and/or boxes for optimal use of space, we can use one of two ways:

Practical Method

Using 2-D diagrams or 3-D models of the scenario, to evaluate how many cans/boxes can fit into a space, using lengths, breadths, heights and diameters.

Mathematical Method

Using volume and surface area formulae to analyse the scenario.



See Module 4: Measurement for more details.

Be aware of the need to round down, as rounding up would result in the can/box not fitting in the desired space.

- When analysing the best packaging arrangement and material, keep the following in mind:

Practicality

E.g. a box that contains 48 cans that is 48 cans long and 1 can wide, would be very difficult to carry compared to a box that is 8 cans long and 6 cans wide.

Optimal use of space

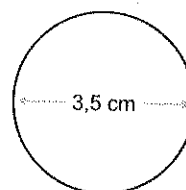
The cans must be packed properly so as to ensure that the largest number of cans fit in the box, but the box must still be able to close. If the box doesn't close properly, it will take up more room than it should, reducing the amount of available space. This will increase transport costs as less boxes can fit in the truck or shipping container.

Cost-effectiveness

Packaging materials, such as cardboard, can be expensive. Therefore you need to find the smallest container that will hold the cans.

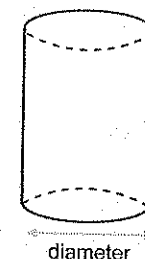
Worked Examples

- A golf ball manufacturer produces the FlyFar Golf Ball, which has a diameter of 3,5 cm. The manufacturer is investigating the best packaging arrangement - either a cylindrical container or a rectangular box - in order to minimise costs. The manufacturer wants to sell 3 golf balls at a time. Study the diagrams below and answer the questions that follow:

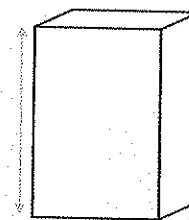


FlyFar Golf Ball

Package Option A



Package Option B



breadth = 4 cm

- Determine the minimum height of both Option A and B, if there needs to be an additional 1 cm of space at the top of the container.

$$\text{Height of 3 golf balls} = 3 \times 3,5 \text{ cm} = 10,5 \text{ cm}$$

$$\therefore \text{Total height of both Option A and B containers} \\ = 10,5 \text{ cm} + 1 \text{ cm} = 11,5 \text{ cm}$$

- Determine the minimum diameter of Option A and the minimum length of Option B, if there must be 0,75 cm space on either side of the golf ball.

$$\text{Minimum diameter of Option A and minimum length of Option B}$$

$$= 3,5 \text{ cm} + (2 \times 0,75 \text{ cm})$$

$$= 3,5 \text{ cm} + 1,5 \text{ cm} = 5 \text{ cm}$$

- Use your answers from Question 1.1 and Question 1.2 to determine which Packaging Option is most cost-effective, if the cardboard for Packaging A costs R0,20/cm² and the cardboard for Packaging B costs R0,15/cm².

Use the following formulae:

$$\text{Total surface area of cylinder} = 2 \times \pi \times (\text{radius})^2 + 2 \times \pi \times \text{radius} \times \text{height}; \\ \text{where } \pi = 3,142$$

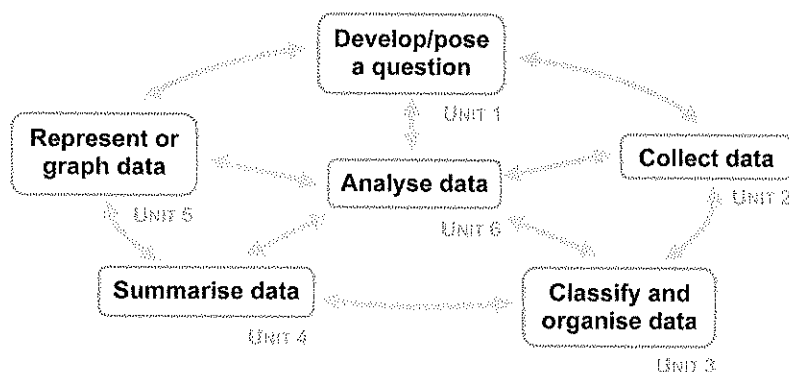
$$\text{Total surface area of rectangular prism} = (2 \times \text{length} \times \text{breadth}) \\ + (2 \times \text{length} \times \text{height}) + (2 \times \text{breadth} \times \text{height})$$

6 DATA HANDLING

Introduction



- **Data** is raw information that has been collected, without any organisation or analysis.
- **Data handling** refers to the process of collecting, organising, summarising, representing and analysing information.
 - ▶ The purpose of data handling is to make data more meaningful in order to answer questions and make informed decisions.
- **Statistics** is the branch of Mathematics that focuses on collecting, organising, representing, analysing and interpreting data.
- The **statistical process** is made up of at least 6 inter-connected stages; where each stage is analysed and dependent on the stage before it; as illustrated below:



THE STATISTICAL PROCESS

UNITS 1 - 6 will deal with each stage of the statistical process separately.

UNIT

1

DEVELOPING QUESTIONS

- The first step in the statistical process is to **develop or pose a question**.
- The phrasing of the question will determine the way in which the data is collected, organised and analysed - so be aware of bias and ambiguity in posing questions.
- When developing/posing the question, first identify the main aim or reason for the investigation (main question); followed by sub-questions which would provide additional information, e.g.:

Main question: 'What is the average monthly income of South Africans?'

- Sub-questions:**
- ① In which age category do you fall?
e.g. 18 - 20; 21 - 30; 31 - 40; 41 - 50, etc.
 - ② In which market sector/industry do you work?
e.g. agriculture; science, education, finance, etc.
 - ③ What is your job title?
e.g. accountant, fisherman, secretary, miner, etc.
 - ④ How long have you been working in this job?
e.g. 1 year, 4 months, etc.
 - ⑤ Do you have any tertiary education qualifications?
e.g. degree, diploma, etc.

Test Your Understanding

For each of the main questions given below; formulate 5 sub-questions that will enable meaningful data collection:

1. How can road accident deaths be minimised on South African roads?
2. How can the South African matric pass rate be improved?
3. Are the expenses incurred for a Matric dance justified?



COLLECTING DATA

- The data needed is sometimes already available, while other times it needs to be collected. This data is known as **raw data**, as it is unprocessed.
- There are 3 main **methods of collecting data**:
 - **Observation** - e.g. counting the number of cars passing an intersection
 - **Interview** - e.g. asking people their opinions on a budget proposal
 - **Survey** - e.g. finding out about a patron's restaurant experience, by means of a questionnaire
- The **advantages and disadvantages** of the different methods of **collecting data** are as follows:

	ADVANTAGES	DISADVANTAGES
Observation	▸ easy to record info ▸ participants don't fill in forms	▸ time consuming for observer ▸ reliant on accuracy of observer
Interview	▸ allows for discussion between researcher and interviewee ▸ researcher is present to clarify wording ▸ researcher can ask interviewee to clarify responses ▸ interviewees tend to be more honest	▸ time consuming ▸ can be expensive ▸ difficult to arrange ▸ difficult to target a large audience in a wide geographical area
Survey using a questionnaire	▸ can be completed by many people simultaneously ▸ can be distributed in different ways ▸ can be fast and easy to complete ▸ can be completed at a convenient time. ▸ can be applied to many different areas of research ▸ can target a large audience in a wide geographical area	▸ researcher is not always present to clarify wording ▸ people cannot explain their responses ▸ people may not complete the questionnaire ▸ questions can be vague or ambiguous ▸ people can be dishonest

Questionnaires



- A questionnaire is a tool used to conduct a survey.
- Questionnaires can be completed by telephone; online; in person at a particular venue (e.g. shopping mall) or by mail.

- Features of a good questionnaire include:

- ① Aims of the investigation are clearly stated.
- ② The questions match the aims of the investigation.
- ③ Instructions are provided on how to correctly answer the questions.
- ④ The questionnaire is short and simple.
- ⑤ The language is clear, unambiguous, appropriate to the context and matched to the level of the participant.
- ⑥ The questions avoid bias e.g. 'Don't you think that MacDonalds' burgers are better than Burger Kings'?
- ⑦ There are more closed-ended questions (i.e. 'Yes/No?') as opposed to open-ended questions (i.e. 'What do you think?')
- ⑧ Questions are stated positively (e.g. 'Do you like ...') as opposed to negatively stated questions (e.g. 'Don't you think...').
- ⑨ Personal information is not required unless absolutely necessary.
- ⑩ Sufficient space is allowed for responses on questionnaires, together with easy response formats such as tick-boxes.



A survey is the process of gathering information using tools, such as questionnaires.

Populations and Samples



- A **population** in statistics refers to the entire group of interest, - e.g. the population of a school would be all of the people who are part of the school, including the principal, teachers, students, parents, administrative staff, cleaning staff, ground staff, etc.
- A **sample** is a representative part/slice/subset of the population - e.g. a sample of a school would be a defined number of randomly selected pupils from each grade, which represents the whole school population in terms of male : female ratios; culture; language; race; etc.

Characteristics of a good sample. It must be:

- ① **representative** of the population in terms of male:female ratio; age distribution; race; culture; language; etc.
- ② **randomly chosen**, where each person has an equal opportunity of selection; and one selection is independent of another.
- ③ **large enough** to be an accurate representation of the population.
- ④ **free from bias**.

- **Reliability** of the data is largely dependent on fulfilling all the characteristics of a good sample as listed above.

Test Your Understanding



1. Decide on the best method of collecting data for the following investigations. Give reasons for your answer:
 - 1.1 The gender of shoppers visiting a shopping centre during lunch time.
 - 1.2 How delegates attending a workshop felt about the workshop.
 - 1.3 How you would discipline your own children.
2. Penny would love to open a coffee shop at her local shopping centre, so she decides to hand out a questionnaire at the shopping centre's entrance, in order to do market research. Draw up a questionnaire that Penny could use to gather the information she requires.
3. A researcher is interested in the effect of an electrolytic sports drink on the endurance of long distance athletes. A group of 100 athletes who are taking part in the Two Oceans Marathon is selected. Half are tested while consuming the sports drink and the other half are tested while drinking water. For this study:
 - 3.1 Identify the population
 - 3.2 Identify the sample
4. In the run up to the 2014 National Elections, a radio station asked listeners to log onto their website and take part in a poll in which they would get to indicate which party they thought would win the elections.

Do you think the information that they collected would be a true reflection of the opinions of all South Africans, or do you think this information could be biased? Explain your answer.

5. The table below shows the demographics of all the employees of a particular company, as well as a sample group of employees. The owner of the company selected the sample group of employees to determine their preferences for a Christmas party.



DEMOGRAPHICS OF EMPLOYEES			
		ALL EMPLOYEES	SAMPLE GROUP
GENDER	Male	87	10
	Female	102	5
RACE	Black	121	6
	Coloured	5	0
	Indian	7	2
	White	53	7
	Other	3	0

Do you think that the information collected from this sample group of employees will be representative of all the employees in the company? Explain your answer.

UNIT

3

CLASSIFYING AND ORGANISING DATA

Once the data has been collected, it needs to be classified and organised into a format that can be easily summarised, represented or analysed.

Classifying Data



- Data can be classified into one of two types:
 - ① **Categorical data** is generally **descriptive** in nature, as data is classified and organised into categories.
 - Data is usually observed, but not measured.
 - Examples: textures, smells, tastes, gender, eye colour, months of the year, age group, shoe sizes and country of birth.
 - Categorical data can consist of 'yes' or 'no' answers.

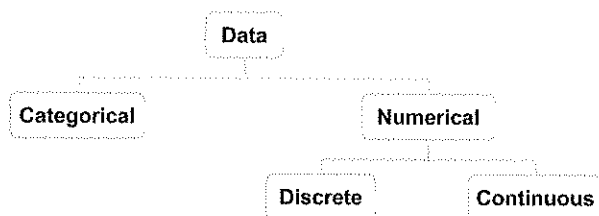
② **Numerical data** refers to data consisting of **quantities** or **numerical values**.

▶ Examples include: measurements, e.g. length, height, area, volume, mass, speed, time, temperature, rainfall, humidity, sound levels, etc.

▶ Numerical data can be further classified into **discrete** data or **continuous** data.

- **Discrete data** is a set of values that can be **counted**, e.g.
 - the number of children in a family
 - the number of cars in a parking lot
- **Continuous data** is data that you **measure**, e.g.
 - the height of a learner
 - the time taken to run a race

Therefore, data can be classified as follows:



Organising Data

- When working with a **large number of data items** or **continuous data**; it is better to **group data into class intervals** e.g. 0 - 9; 10 - 19; 20 - 29, etc.

The class intervals must be the same width i.e. each interval must have the same number of data points to be comparable.

e.g. 20 - 29	each class interval has	✓ class width the same
30 - 39	10 data points in it	
40 - 49		
vs		
20 - 30	11 data points	✗ class width different
31 - 40	10 data points	
41 - 50		

- Data can be organised using **Tally Tables** and/or **Frequency Tables**.

e.g. ①

TALLY TABLE OF VOTES FOR MR LOMBARD AS PRESIDENT

Vote	Tally
YES	+++ ++
NO	+++



This type of data is classified as **Categorical data**.

A 'tally' is a quick and easy tool that can be used to keep track of how often a data item appears in the raw data. Every stripe | represents one occurrence of a particular data item. 5 occurrences are represented by 4 vertical stripes and 1 horizontal stripe (the 5th occurrence) drawn through the other 4 stripes ++++.



e.g. ②

FREQUENCY TABLE OF THE AVERAGE VALUE OF MONTHLY POCKET MONEY IN A GRADE 12 CLASS

Rand Value (R)	Tally	Frequency
0 - 99		3
100 - 199	++++	8
200 - 299	++++ ++++	15
300 - 399		4
400 - 499		2
500 - 599		0
TOTAL		32

How many data points are there in a class interval?

Subtract values + 1
e.g. $99 - 0 + 1 = 100$
 $199 - 100 + 1 = 100$

This continuous, numerical data has been grouped into class intervals with a width of 100 data points.

This is the sum of all of the frequency values. It provides a check that each piece of raw data has been included.

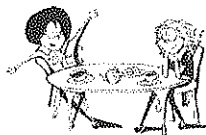


Note how Frequency tables are easier to use, as the tallies have been written in a **numerical** format; together with a total of the number of data items.

Test Your Understanding



- Classify each of the following as **categorical** or **numerical**; and then as **discrete** or **continuous** data (if data is numerical in nature).
 - The number of coffee beans in a packet of coffee.
 - The time of day when people take lunch.
 - The colour of jelly beans in a box.
 - The rate at which water flows from a hose pipe.
 - The types of tastes present in a bottle of wine.
 - The number of light bulbs in a pack.
- A sample of 15 female shoppers were asked to classify a perfume as either sweet, fresh, musky or citrus. Use their responses below to answer the following questions:



Musky	Fresh	Musky	Citrus	Musky
Musky	Sweet	Citrus	Musky	Musky
Fresh	Musky	Sweet	Sweet	Citrus

- Draw a frequency table of the Perfume Classification by shoppers.
 - What percentage of the shoppers classified the perfume as 'Musky'?
- A random sample of 40 gym members participated in a research project, whereby their gender, height and weight were recorded. Use the raw data in the table below to answer the following questions.

GENDER	HEIGHT (m)	WEIGHT (kg)
Female	1,61	58
Male	1,78	89
Male	1,95	97
Male	1,62	75
Female	1,83	70
Female	1,61	96
Male	1,85	71
Male	1,81	81
Male	2,20	110
Male	1,81	89
Male	1,70	78
Male	1,78	71
Female	1,88	74
Female	1,56	58
Female	1,70	68

GENDER	HEIGHT (m)	WEIGHT (kg)
Female	1,75	50
Male	1,67	68
Male	1,70	73
Male	1,78	70
Female	1,69	69
Male	1,58	73
Male	1,85	73
Male	1,92	94
Male	1,78	137
Male	1,68	76
Male	1,79	73
Male	1,75	70
Female	1,77	88
Female	1,65	64
Female	1,75	67

GENDER	HEIGHT (m)	WEIGHT (kg)
Female	1,68	68
Female	1,61	53
Male	1,90	90
Male	1,87	98
Female	1,57	58

GENDER	HEIGHT (m)	WEIGHT (kg)
Female	1,67	86
Male	1,76	89
Male	1,76	95
Male	1,86	115
Male	1,88	126

- Use the frequency table below to organise the data, based on height and gender.

FREQUENCY TABLE 1 - HEIGHT DISTRIBUTION BY GENDER

Height Category	Females		Males	
	Tally	No. of People (Frequency)	Tally	No. of People (Frequency)
1,50 - 1,59 m				
1,60 - 1,69 m				
1,70 - 1,79 m				
1,80 - 1,89 m				
1,90 - 1,99 m				
≥ 2 m				
		Total:		

- Use the frequency table below to organise the data, based on weight and gender.

FREQUENCY TABLE 2 - WEIGHT DISTRIBUTION BY GENDER

Weight Category	Females		Males	
	Tally	No. of People (Frequency)	Tally	No. of People (Frequency)
50 - 59 kg				
60 - 69 kg				
70 - 79 kg				
80 - 89 kg				
90 - 99 kg				
100 - 109 kg				
110 - 119 kg				
≥ 120 kg				
		Total:		

- Use the frequency tables drawn in Questions 3.1 and 3.2 to answer the following questions:

- In which height category do most of the women fall?
- How many men measured 1,9 - 1,99 m in height?
- How many women weighed less than 70 kg?
- What percentage of men weighed more than 100 kg?
- If one of the gym members measured 1,62 m and weighed 75 kg; determine the member's BMI



$$BMI = \frac{\text{weight in kg}}{[\text{height in (m)}]^2}$$

SUMMARISING DATA

Once the collected data has been classified and organised, it is important to summarise the data, in order to provide additional information regarding the trends of the data set.

Measures of Central Tendency



- A measure of central tendency is a value that provides an indication of the 'middle' or 'centre' of the data.
 - ▷ This 'middle' value provides a single value, a benchmark value, against which to measure and compare the other values in the data set.
 - ▷ It is a value that is **representative of the majority of values** in the data set.
- There are 3 measures of central tendency:
 - ① Mean
 - ② Median
 - ③ Mode

① MEAN

- It is also known as the 'average'.
- The formula used:

$$\text{Mean} = \frac{\text{sum of all values in data set}}{\text{total number of values in data set}}$$



- It can only be calculated if the data set is **numerical**.

Worked Examples

1. Johan obtained the following results for five Maths Literacy tests. Calculate his mean mark.

72% 60% 68% 63% 77%

$$\text{Mean} = \frac{\text{sum of all values in data set}}{\text{total number of values in data set}} = \frac{72 + 60 + 68 + 63 + 77}{5} = 68\%$$



2. Minny, Rodriguez, Ali, Naeem and Sepho write their driver's licence test and compare their results. Minny achieved 89%; Rodriguez 98%; Ali 88% and Naeem 90%. Determine Sepho's mark, if the mean value of all 5 marks is 92%.

$$\text{Mean} = \frac{\text{sum of all values in data set}}{\text{total number of values in data set}}$$

$$92 = \frac{89 + 98 + 88 + 90 + S}{5}$$

$$92 \times 5 = 365 + S$$

$$460 = 365 + S$$

$$\therefore S = 460 - 365 = 95$$

$\therefore$ Sepho achieved 95% for his driver's licence test.

Sepho's mark is unknown, so let's call it 'S'.



② MEDIAN

- All data values must first be arranged in **ascending order**.
- The median is the **middle value of an ordered data set**.
- It can only be calculated if the data set is **numerical**.

Worked Examples

- 1.1 Ross received the following Maths Literacy test scores:

65% 63% 22% 69% 60%

Use his results above to determine the median.

First arrange the data values in ascending order:

22% 60% 63% 65% 69%

Then find the middle number:

22% 60% 63% 65% 69%

$\therefore$ The median is 63%.



- 1.2 If Ross obtained 64% for an additional test; determine the median of this new data set.

First arrange the data values in ascending order:

22% 60% 63% 64% 65% 69%

Now when finding the middle value, it lies between two data values:

22% 60% 63% ↓ 64% 65% 69%

Therefore, to calculate the median; add these two 'middle' data values together and divide by two:

$$\therefore \text{The median} = \frac{63 + 64}{2} = 63,5\%$$



Therefore, in a data set with an **odd number of data values**, the **median** will be the **number in the middle** of all the values.

In a data set with an **even number of data values**, the median = $\frac{\text{sum of two 'middle' data values}}{2}$

③ MODE

- The mode is the **data value(s)** that **occur(s) most frequently** in a set.
- If **two values occur most frequently**, then the data set is referred to as **bimodal**.
- It can be determined for both **numerical** and **categorical** data.

Worked Example

Tameka got the following results for her Maths Literacy tests:

72% 69% 71% 72% 70%

1. Determine the mode of this data set.

The mode is 72%.

2. If Tameka forgot to include another test result of 71%, determine the mode of this new data set.

The mode is both 72% and 71%.



This is what we call a **bimodal data set** ('bi' means 'two' $\therefore$ 2 values occur most frequently.)



When to use the Mean, Median or Mode



- As all three measures of central tendency describe the 'middle' value of a set of data, you must know when to use the most suitable one.
- An **outlier** is a value in a data set that is far away from most of the other values.

e.g. 6 ; 51 ; 56 ; 57 ; 63 ; 65

6 is the outlier!

- The **mean** is best used to describe the average of a set of data that **does not have an outlier**. A particularly high or low value in a data set can 'skew' the mean.



Skewed data occurs when an outlier affects the measure of central tendency, thereby resulting in a 'middle' value that is not actually representative of the middle of the data set.

- The **median** is best used to describe the 'middle' value of a set of data that **does have an outlier**.
- The **mode** is usually used when the **data is categorical** or when asked to choose the most popular item (e.g. best seller in a clothing retail store).

Worked Example

Compare the following two data sets in terms of the measures of central tendency and comment as to which measures are the best representations of the 'middle' values of the data sets.

Set A: 60 65 70 72 76 77 78 78

Set B: 3 65 70 72 76 77 78 78

NOTE! 3 is considered an **outlier** because it is significantly smaller than the other data values.

$$\begin{aligned} \text{Set A: Mean} &= \frac{60 + 65 + 70 + 72 + 76 + 77 + 78 + 78}{8} \\ &= \frac{576}{8} \\ &= 72 \end{aligned}$$

Median: 60 ; 65 ; 70 ; 72 ; 76 ; 77 ; 78 ; 78

$$\therefore \text{Median} = \frac{72 + 76}{2} = 74$$

Mode: 78



$$\text{Set B: Mean} = \frac{3+65+70+72+76+77+78+78}{8} = \frac{519}{8} = 64,88$$

Median: 3 ; 65 ; 70 ; 72 ; 76 ; 77 ; 78 ; 78

$$\therefore \text{Median} = \frac{72+76}{2} = 74$$

Mode: 78

When evaluating the different measures of central tendency of the two data sets, the following conclusions can be made:

- In data Set A, the mean is the best measure of central tendency, as there are no outliers.
- In data Set B, the median is the best measure of central tendency, as it is not skewed by the outlier (number 3).
- The mode is unaffected by an outlier and remains the same (78) for both data Sets A and B.

Advantages and Disadvantages of the Measures of Central Tendency



MEAN

Advantages

- Useful when comparing sets of data, e.g. English test results in 2 class groups.
- Most common measure in fields such as Economics and Science.
- It is unique - there is only one answer.

Disadvantages

- Affected by outliers.

MEDIAN

Advantages

- Useful when comparing sets of data.
- It is unique - there is only one answer.
- Outliers do not affect the median as strongly as they do the mean.

Disadvantages

- The data first needs to be arranged in ascending order.
- The median is not always representative of a data set, especially when the data is not spread evenly about the median.
E.g. test marks: 51; 51; 52; 52; 53; 72; 78; 87; 93

The data items to the right of the median are more widely spread from 53, than those on the left.



MODE

Advantages

- Outliers do not affect the mode.

Disadvantages

- Not necessarily unique - may be more than one answer - 5 people may obtain 80% for a test and 5 may obtain 40%.
- When no values repeat in the data set, the mode is every value and is therefore useless e.g. 1, 2, 3, 4, 5.
- When there is more than one mode, it is difficult to interpret and/or compare. Shoe sizes 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7.
Which is the most common or average shoe size?



EXERCISE 1

- Determine the mean, median and mode of the following sets of data:
 - 24; 67; 32; 101; 48; 28; 53; 55; 53; 100; 109; 77
 - 100 421; 127 034; 169 809; 153 003; 160 421
 - 83,4; 61,98; 23,04; 66,01; 58,7; 72,39; 66,01; 66,01
- Lizelle ran the following times for the last 7 half-marathons she entered:
2h:00 min 2h:05 min 2h:00 min 2h:00 min 1h:58 min 2h:01 min 2h:45 min
 - What is the mean of her race times? [HINT! Convert data into minutes]
 - What is the median of her race times?
 - What is the mode of her race times?
 - Which measure of central tendency is a better indication of her time for a half-marathon? Explain your answer.
 - Her time of 2h:45 min seems to be out of the ordinary when compared with her other running times. Suggest possible reasons for this running time.
- Which measure of central tendency would be best suited to the following data sets? Give a reason for your answer.
 - A scientist collects the lengths of the thorax of different beetles.
 - The income of members of a town, which has one millionaire amongst them.

Measures of Spread



- A measure of spread is a value that provides an indication of how 'spread out' the data is (i.e. are all the data values very close together/clustering; or are they very far apart/dispersed?)
- There are two measures of spread that we will deal with:
 - Range
 - Interquartile Range

① RANGE

- The range of data indicates the difference between the highest and lowest values in the data set.

$$\text{Range} = \text{highest value} - \text{lowest value}$$



- If the range is small, the data is clustered together; and if the range is larger, then the data is more spread apart.

Worked Examples



- Calculate the range for this set of data:

170 ; 43 ; 99 ; 122 ; 69 ; 101 ; 125 ; 38 ; 189 ; 166

$$\begin{aligned} \text{Range} &= \text{highest data value} - \text{lowest data value} \\ &= 189 - 38 \\ &= 151 \end{aligned}$$

- Evaluate Johan and Busi's Maths Literacy test results by referring to the spread of their test results.

Johan's results: 60% ; 63% ; 68% ; 72% ; 77%

Busi's results: 64% ; 67% ; 68% ; 70% ; 71%

Range of Johan's test scores: $77\% - 60\% = 17\%$

Range of Busi's test scores: $71\% - 64\% = 7\%$

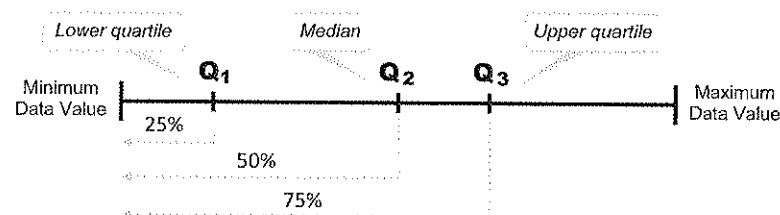
The range of Busi's test scores is smaller than the range of Johan's test scores.
Busi's results are generally closer to each other compared to Johan's.

② INTERQUARTILE RANGE

Before we can calculate the interquartile range, we first need to determine the quartiles of our data set:

Quartiles

- All data values must first be arranged in **ascending order**.
- Quartiles (derived from the word 'quarters') divide a data set into **four equal parts**.
 - The middle quartile (Q_2) is the median (or middle) of the data set.
 - The first, or lower, quartile (Q_1) indicates the first quarter of the data set [it is the median of the lower half of the data set].
 - The third, or upper, quartile (Q_3) indicates the upper quarter of the data set [it is the median of the upper half of the data set].
- This can be represented visually as follows:

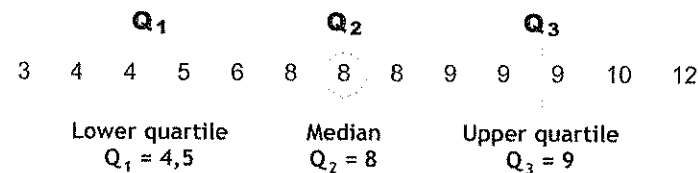


Note the following: Q_1 : 25% of the data will be on or below this value.
 Q_2 : 50% of the data will be on or below this value.
 Q_3 : 75% of the data will be on or below this value.

Worked Examples



- Finding quartiles in a data set with an odd number of values:



Number of values in data set = 13

Median/middle quartile (Q_2) = 8

First/lower quartile (Q_1) = $\frac{4+5}{2} = 4,5$



There are 6 values in the lower half of the data set (3; 4; 4; 5; 6; 8).
The lower quartile is the median of this lower half of the data set.



$$\text{Third/upper quartile } (Q_3) = \frac{9+9}{2} = 9$$

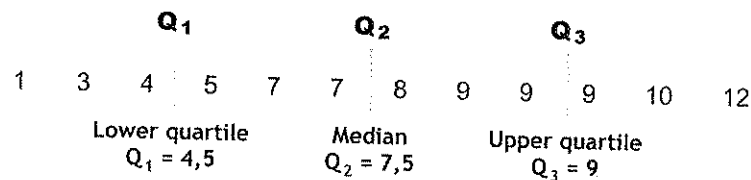
There are 6 values in the upper half of the data set (8; 9; 9; 9; 10; 12).
The upper quartile is the median of this upper half of the data set.



Therefore:

- 25% of the data is on or below 4,5 (the lower quartile Q_1)
- 50% of the data is on or below 8 (the median Q_2)
- 75% of the data is on or below 9 (the upper quartile Q_3).

2. Finding quartiles in a data set with an even number of values:



INTERQUARTILE RANGE (IQR)

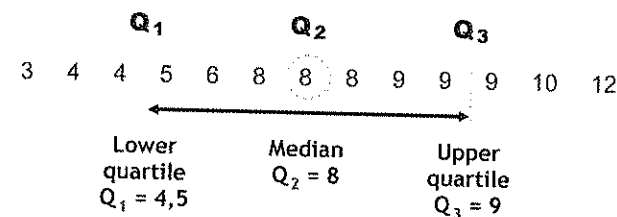
- The IQR is the difference between the upper quartile (Q_3) and the lower quartile (Q_1):
$$IQR = Q_3 - Q_1$$
- Since the Q_3 is the 75% mark and Q_1 is the 25% mark, the IQR indicates the middle 50% of the data (i.e. $75\% - 25\% = 50\%$).
- If the IQR is a low number - the middle 50% of the data is close together/clustering. If the IQR is a high number - the middle 50% of the data is far apart/spread out.

Worked Examples



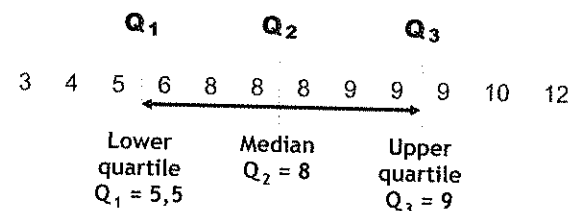
6

1. Finding the interquartile range in a data set with an odd number of values:



The interquartile range: $IQR = 9 - 4,5 = 4,5$

2. Finding the interquartile range in a data set with an even number of values:



The interquartile range: $IQR = 9 - 5,5 = 3,5$



Advantages and Disadvantages of the Measures of Spread



RANGE

Advantage

- Quick and easy to calculate.

Disadvantage

- Affected by outliers.

INTERQUARTILE RANGE

Advantages

- Gives you an indication of the spread of 50% of the data values.
- Not affected by outliers.

Disadvantages

- The data first needs to be arranged in ascending order.
- Time consuming to calculate, as you first need to determine Q_1 and Q_3 .

BOX-AND-WHISKER PLOTS

- Also known as **Box Plots**.
- The box-and-whisker plot is a **visual representation** of the '**5-Number Summary**' of a data set.

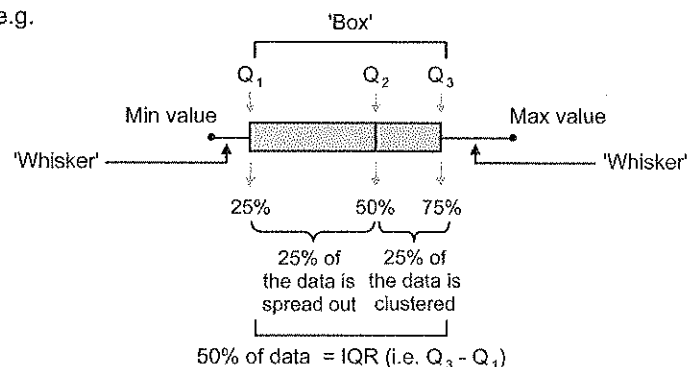
► The 5-Number Summary is made up of the following data values:

1. Minimum value
2. Q_1 (lower quartile)
3. Q_2 (median)
4. Q_3 (upper quartile)
5. Maximum value

e.g.

NOTE!

You are only expected to **interpret** the box-and-whisker plot. You will **not** be expected to draw the plot.



Worked Example



Given the following box-and-whisker plot below, interpret all the key '5-Number Summary' values with respect to the spread of the data.



'5-Number Summary' values:

1. Minimum value = 4
2. Q_1 (lower quartile) = 5.5
3. Q_2 (median) = 8
4. Q_3 (upper quartile) = 9
5. Maximum value = 12

- the values in the 3rd quartile are very close together (smallest quartile)
- the values in the 4th quartile are most spread out (biggest quartile).
- $IQR = Q_3 - Q_1 = 9 - 5.5 = 3.5$
- ∴ The IQR is a low number so the data values are clustered/close together.

The box plot consists of 4 parts (**quartiles**):

- 1st quartile - from the minimum value to the start of the box
- 2nd quartile - from the start of the box to somewhere inside the box (not necessarily in the middle)
- 3rd quartile - from inside the box to the end of the box
- 4th quartile - from the end of the box to the maximum value

The smaller a quartile on the box plot, the closer its values. The bigger a quartile, the more spread out its values.



Percentiles



- The word 'percentile' is derived from the word 'percent', or 'percentage', since percentiles divide the data set into one hundred parts. i.e. The 20th percentile is the value on or below which 20 percent of the data set lies. The 80th percentile is the value on or below which 80 percent of the data set lies.
- The 25th percentile is equivalent to the first quartile (Q_1); the 50th percentile is equivalent to the median or second quartile (Q_2); and the 75th percentile is equivalent to the third quartile (Q_3).

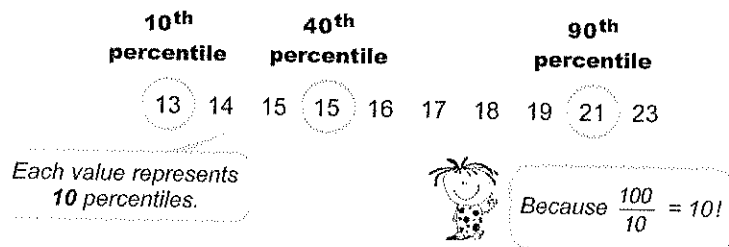
IMPORTANT NOTE: You are only expected to **interpret** percentiles. You will **not** be expected to calculate them from a data set.



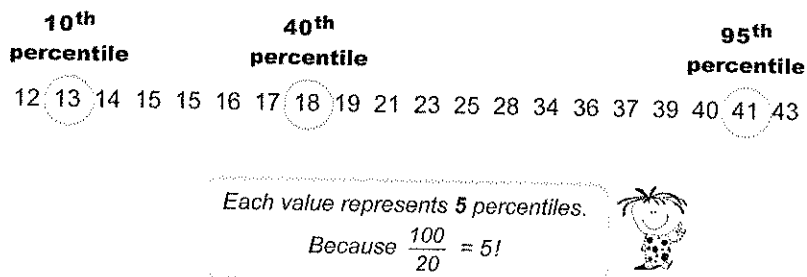
Worked Examples



1. If there are 10 values in the data set, every value represents 10% (of the data) as in the example below:



2. If there are 20 numbers in the data set, each number represents 5% as in the example below:

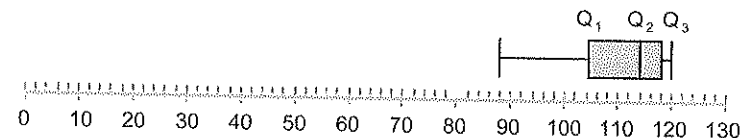


EXERCISE 2

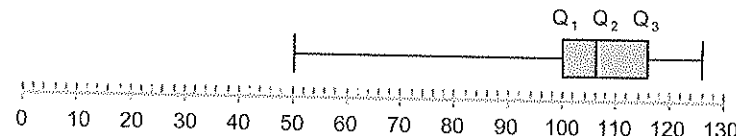
- Determine the range and interquartile range of the following sets of data:
 - 5; 7; 8; 11; 14; 15; 16; 19; 21; 25
 - 35; 17; 8; 21; 4; 15; 16; 12; 11; 25
 - 44; 35; 17; 11; 32; 29; 14; 47; 23; 41; 29; 31; 22; 45; 50

2. Mrs Ndhlovu draws the following box-and-whisker plot for the results of Paper 1 and Paper 2 in her Maths Literacy class:

Paper 1:



Paper 2:

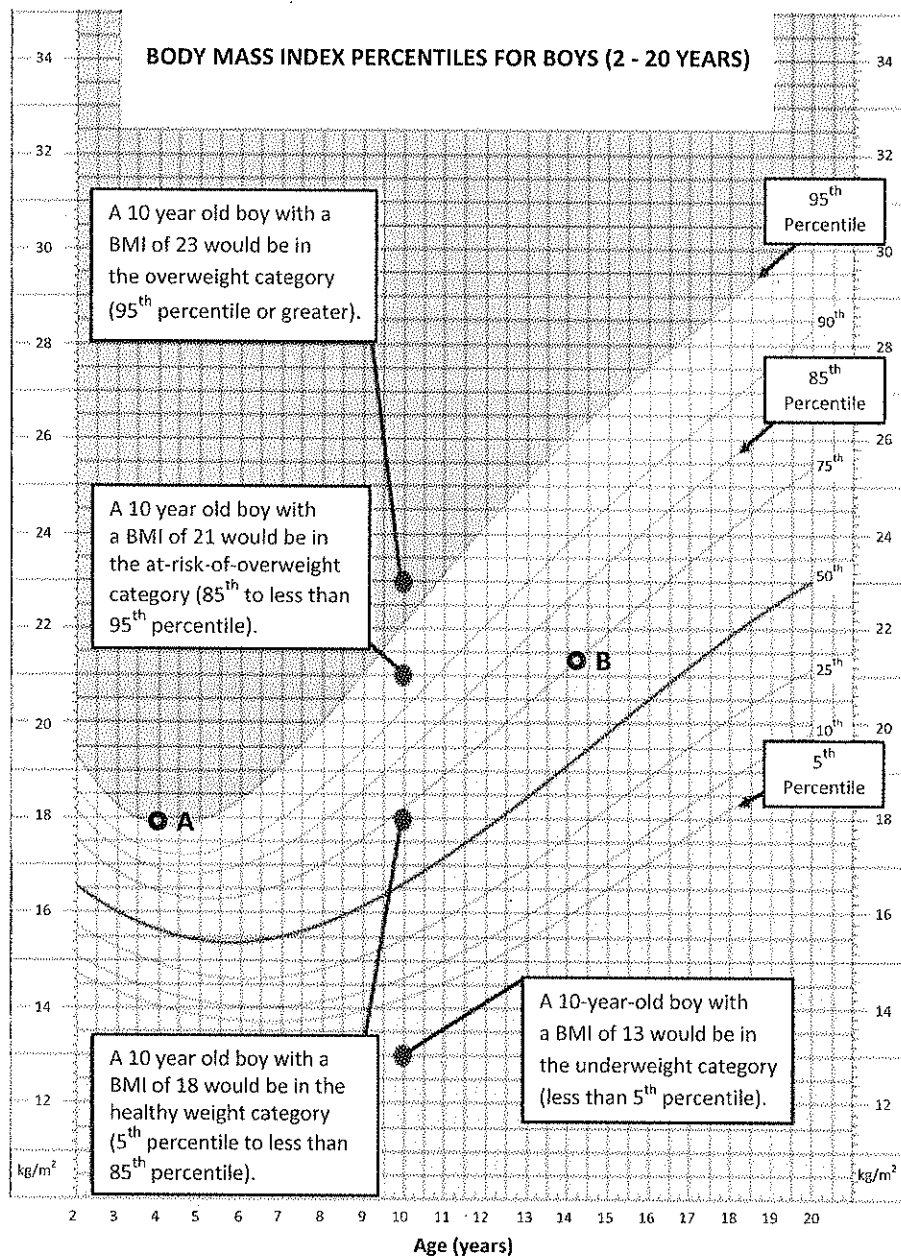


Use these plots above to answer the following questions:

- What was the lowest mark obtained for Paper 1?
 - What was the highest percentage obtained for Paper 2?
 - What was the median mark in Paper 1?
 - Write down the '5-Number Summary' for Paper 2.
 - Calculate the range and interquartile range for Paper 1 and 2 and use these measures of spread to comment on the spread of both papers.
 - Which measure of spread is a better representation of Mrs Ndhlovu's results? Give reasons for your answer.
- Find the 30th and the 70th percentile in each of the following data sets:
 - 5; 7; 8; 11; 14; 15; 16; 19; 21; 25
 - 1; 1; 3; 4; 5; 5; 5; 7; 8; 8; 10; 11; 13; 14; 16; 16; 18; 19; 20; 25
 - What is the BMI of a boy who weighs 50 kg and is 1,5 m tall?

Use $BMI = \frac{\text{weight in kg}}{(\text{length in m})^2}$
 - The chart on p. 186 shows the body mass index percentiles of boys of different ages. Use this chart to answer the questions that follow:
 - In which percentile would the boy in Question 4 be classified if he is 15?
 - What is the average BMI of an 18 year old boy?
 - Give the BMI range (the lowest to the highest) of an 18 year old boy that could be considered healthy. The healthy category is from the 5th percentile to the 85th percentile.
 - Give the BMI range of an 18 year old boy that could be considered at risk of being overweight. This category is from the 85th percentile to less than the 95th percentile.

BODY MASS INDEX PERCENTILES FOR BOYS (2 - 20 YEARS)



- ▶ 95th percentile: 95% of boys have a BMI of less than/equal to this value.
- ▶ 50th percentile (the average): boys with a BMI above the 50th percentile have an **above-average** BMI.
- ▶ 5th percentile: 5% of boys have a BMI below/equal to this specific value.



- 5.5 Give the BMI range of an 18 year old boy that could be considered as underweight. The underweight category is less than the 5th percentile.
- 5.6 Find the age and BMI of the boy labelled 'A' on the graph.
- 5.7 Find the age and BMI of the boy labelled 'B' on the graph.
- 5.8 How do you think the girls' body mass index percentile graph will differ from this graph?

Test Your Understanding



1. In February 2007, a large-scale community survey (CS) was conducted in all of the provinces. The main objective of the survey was to provide data for the municipalities. The survey results showed that the population of South Africa increased from 44,8 million in 2001 to 48,5 million in 2007.

The **percentage** of households by province that have access to **piped water** are represented in the table below:

	Eastern Cape	Free State	Gauteng	KwaZulu- Natal	Limpopo	Mpumalanga	Northern Cape	North West	Western Cape
Survey 2001	63,2	95,7	97,1	72,5	78,1	85,7	93,9	86,6	98,3
Community Survey (CS) 2007	70,4	97,5	97,9	79,4	83,6	91,3	94,8	89,9	98,9

- 1.1 Calculate the percentage increase in the population of South Africa from 2001 to 2007.
- 1.2 Which province had the greatest increase in access to piped water from the 2001 survey to CS 2007?
- 1.3 Determine the mean of the data for the CS 2007.
- 1.4 Determine the median of the data for the 2001 survey.
- 1.5 Determine the range and interquartile range for the 2001 survey.

2. Mr Daniel's Maths Literacy class obtained the following results for Paper 1 and 2:

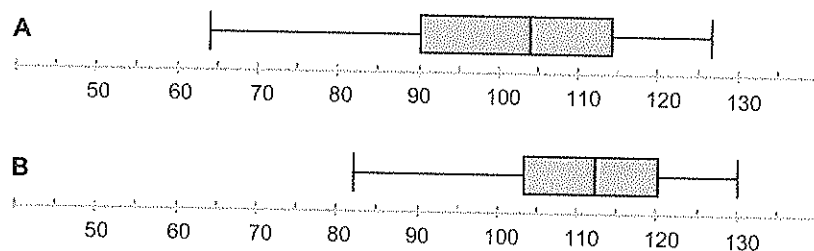
Maths Literacy Paper 1 results:

104, 97, 124, 130, 118, 118, 82, 110, 121, 108, 113, 117, 101, 119, 127, 103

Maths Literacy Paper 2 results:

120, 103, 87, 90, 117, 98, 105, 74, 126, 110, 112, 64, 119, 90, 100, 109

- 2.1 Determine the measures of central tendency for Mr Daniel's Maths Literacy Paper 2 results.
- 2.2 Which data set (Paper 1 or 2) is depicted in each of the box-and-whisker plots?



- 2.3 Write down the '5-Number Summary' for box-and-whisker plot **A**. (Estimate values off the plot).
- 2.4 Determine the range and interquartile range for box-and-whisker plot **B**. (Estimate values off the plot).
- 2.5 What conclusion can you make about the spread of each paper? How does the spread of the two examination papers compare?
3. Bathini High School and Vuka Secondary School entered some of their learners in a science competition. The scores (in percentages) for the first round of the competition are given below.

BATHINI HIGH SCHOOL

59 67 67 67 67 72 78 87 87 90 99

VUKA SECONDARY SCHOOL

90 67 67 89 50 78 54 67 95 90 98 57 49 78

- 3.1 The table below shows the median, mode, mean and range for both schools:

TABLE: Median, mode, mean and range

NAME OF SCHOOL	MEDIAN	MODE	MEAN	RANGE
Bathini High	72%	67%	76,4%	S
Vuka Secondary	P	Q	R	48

- 3.1.1 Determine the missing values P, Q, R and S.
- 3.1.2 Which school performed better in the competition? Explain your answer.
- 3.2 The table below shows the percentiles of scores for both schools:

TABLE: Scores for the two schools

NAME OF SCHOOL	25 th Percentile	60 th Percentile	75 th Percentile
Bathini High	67%	75,6%	87%
Vuka Secondary	57%	78%	90%

- 3.2.1 Use the percentage scores and this table to list the scores of the Vuka Secondary learners who scored at the 75th percentile or more.
- 3.2.2 Use the percentage scores and this table to determine how many Vuka Secondary learners obtained scores that were less than the 25th percentile of Bathini High?

UNIT

5

REPRESENTING DATA

glo

- Once the data has been summarised, it is often very useful to represent the data **visually**.
- The following representations of data can be drawn:
 - 1 Pie charts
 - 2 Single bar graphs and compound bar graphs (multiple and stacked)
 - 3 Histograms
 - 4 Line and broken line graphs
 - 5 Scatter plots
 - 6 Box-and-whisker plots



1 PIE CHARTS

- Pie charts are **circular** diagrams, where **each sector** of the circle ('slice' of the pie) **represents a data value**. Each sector can be expressed as a fraction, decimal or percentage.
- It is often used for representing **categorical data**.
- Method to determine the size of each sector:

$$\text{Size of Sector (in degrees)} = \text{fraction of the whole} \times 360^\circ$$



Why $\times 360^\circ$? Because the total number of degrees around a point (i.e. a revolution) is 360° !

You will not be asked to draw pie charts, but you must be able to **interpret and read values** from a pie chart and be able to **explain** how the **sizes of the different sectors** have been determined.



Worked Examples



The table below shows the number of beneficiaries of the Government's Social Grants for 2013/14:

	2013/14
State Old Age Grant	1 265
State Old Age Grant, over 75s	1 285
War Veterans Grant	1 285
Disability Grant	1 265
Foster Care Grant	800
Care Dependency Grant	1 265
Child Support Grant	295

1. Calculate the size of the sector representing the number of beneficiaries of the foster care grant.

Total number of beneficiaries

$$= 1\,265 + 1\,285 + 1\,285 + 1\,265 + 800 + 1\,265 + 295$$

$$= 7\,460$$

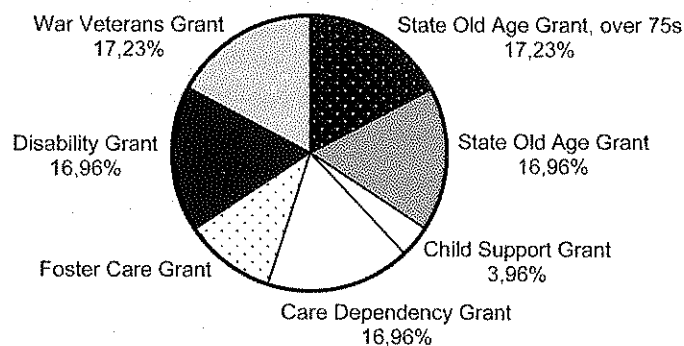
$\therefore$ Size of sector = fraction of whole $\times 360^\circ$

$$= \frac{\text{number of foster care beneficiaries}}{\text{total number of beneficiaries}} \times 360^\circ$$

$$= \frac{800}{7\,460} \times 360^\circ$$

$$= 38,61^\circ$$

2. The pie chart of the Social Grants Beneficiaries for 2013/14 is given below:



- 2.1 Which grant has the smallest number of beneficiaries?

Child Support Grant

- 2.2 Calculate how many children benefit from the child support grant, if there are a total of 7 460 beneficiaries for all the social grants.

$$\text{Number of children benefitting} = \frac{3,96}{100} \times 7\,460$$

$$= 295,42 \approx 295 \text{ children}$$

*Discrete data
 $\therefore$ round down!*



- 2.3 Determine the percentage allocated to the Foster Care Grant.

$$\text{Foster Care Grant} = 100 - 17,23 - 16,96 - 3,96 - 16,96 - 16,96 - 17,23$$

$$= 10,7\%$$

② SINGLE, MULTIPLE AND STACKED BAR GRAPHS

- A bar graph shows the **frequency** of each data value, by means of **bars**.
- It is used for **discrete categorical data**.
- Single bar graphs** represent **one data value per category**.
- Compound bar graphs** include multiple and stacked bar graphs.
- Multiple bar graphs:** **two or more data values per category** are compared and represented by **bars next to each other** (see Question 2 on p. 189).
- Stacked bar graphs:** **two or more data values per category** are compared and represented by **bars being stacked on top of each other**. Stacked bar graphs indicate the **cumulative totals** per category.
- The **spaces** between the bars indicate the **discrete nature** of the data.
- The bars are **equally spaced** and are of the same width.
- The **height** of each bar represents the **frequency** of each category.
- There is usually a space at both the start and end of the graph.
- Categories** are plotted on the **x-axis**; while the **frequency** is plotted on the **y-axis**.



*Bars are usually vertical
but can also be horizontal.*

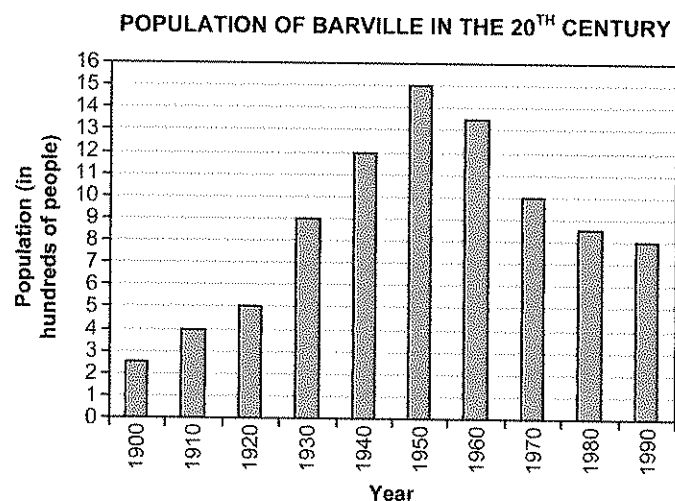
Worked Examples



1. The population of Barville is shown in the table below:

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990
Population (in hundreds of people)	2,5	4	5	9	12	15	13,5	10	8,5	8

- 1.1 Draw a bar graph of the population of Barville in the 20th Century:



- 1.2 What is the difference in the size of the population between 1900 and 1990?

$$\begin{aligned} \text{Difference} &= 800 - 250 \\ &= 550 \text{ people} \end{aligned}$$

NOTE! Numbers in 'hundreds of people'
 $\therefore 1 \text{ actually} = 100$



- 1.3 Describe the general trend in the population growth from 1900 to 1990.

The population of Barville steadily grew from 1900 to 1950.

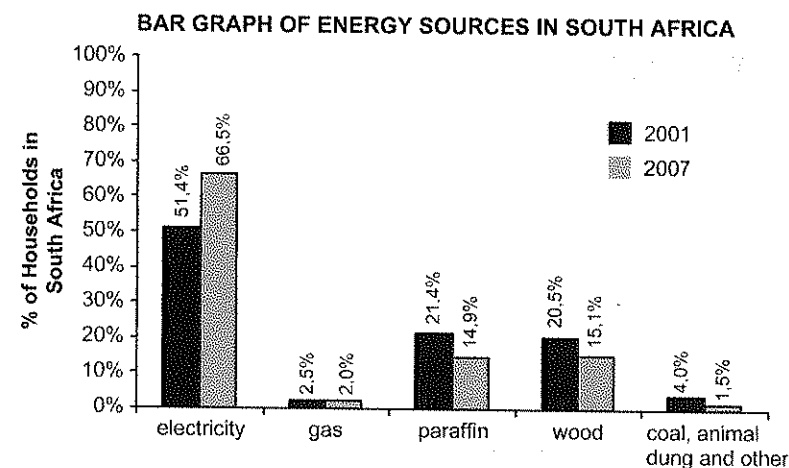
Thereafter the size of the population gradually decreased to 1990.

2. Statistics regarding the percentage of households in South Africa using different energy sources for cooking was collected in 2001 and 2007. Use the information collected below to answer the following questions:

% of Households in South Africa Using Different Energy Sources for Cooking					
Year	Electricity	Gas	Paraffin	Wood	Coal, Animal Dung and Other
2001	51,4	2,5	21,4	20,5	4,0
2007	66,5	2,0	14,9	15,1	1,5

Source: Statistics SA, Statistical Release PO301: Community Survey, 2007 (Revised Edition), p.51

- 2.1 Draw a multiple bar graph showing the different energy sources used by the % of households in South Africa, for 2001 and 2007.



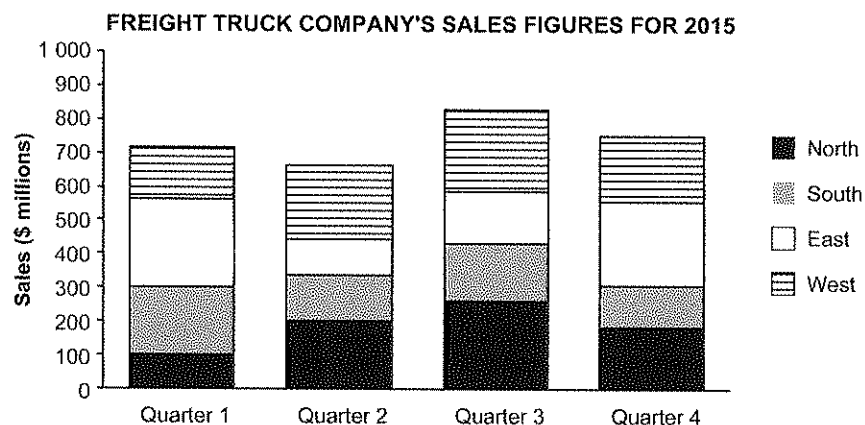
- 2.2 Which energy source showed a growth in usage from 2001 to 2007?

Electricity

- 2.3 Calculate the percentage decrease in the use of paraffin from 2001 to 2007.

$$\begin{aligned} \% \text{ Decrease} &= 21,4\% - 14,9\% \\ &= 6,5\% \end{aligned}$$

3. A freight truck company analysed and represented their sales figures per region for 2015, as shown below:



- 3.1 How many regions are represented?

4 - North, South, East and West

- 3.2 Which quarter showed the highest sales figures?

Quarter 3

- 3.3 Which region(s) reported sales of approximately \$100 million dollars in a Quarter?

North (Quarter 1: $100 - 0 = 100$)

East (Quarter 2: $450 - 350 = 100$)

- 3.4 List the approximate sales figures per region for Quarter 1.

North: \$100 mil - \$0 mil = \$100 mil

South: \$300 mil - \$100 mil = \$200 mil

East: \$550 mil - \$300 mil = \$250 mil

West: \$700 mil - \$550 mil = \$150 mil

- 3.5 What was the combined sales totals for the North and South regions for Quarter 3?

Approximately \$425 million

HISTOGRAMS

- A histogram shows the **frequency** of each data value, by means of **bars**.
- It is used for **continuous data**.
- The data is usually **grouped** into **class intervals** (e.g. height intervals, age groups).
- Intervals are represented by bars with **no spaces between them**, to indicate the continuous nature of the data.
- The **class intervals** are on the horizontal axis where each bar represents one class or interval.
- The vertical axis shows the **frequency** and the height of the bar represents the frequency of the class or interval.
- The bars are of the same width.

Worked Example

A civil engineer had to evaluate the roads of a small town and determine how many sections of roads (in metres) needed to be re-tarred. The table below gives the results of his findings:

Length (ℓ) of road to be re-tarred (in metres)	Frequency
$4\ 000 \leq \ell < 5\ 000$	2
$5\ 000 \leq \ell < 6\ 000$	6
$6\ 000 \leq \ell < 7\ 000$	2
$7\ 000 \leq \ell < 8\ 000$	9
$8\ 000 \leq \ell < 9\ 000$	6
$9\ 000 \leq \ell < 10\ 000$	5
$10\ 000 \leq \ell < 11\ 000$	2

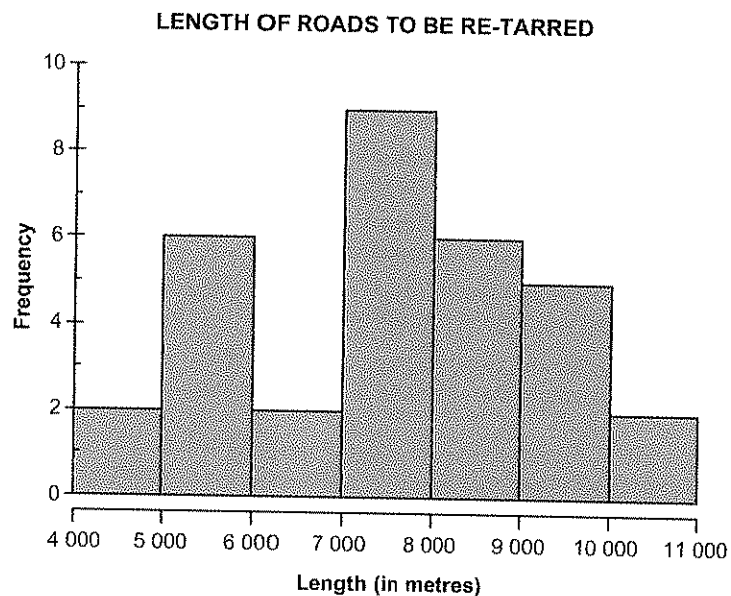
1. What type of data is being recorded here?

Numerical, continuous data

2. How many sections of road needs to be re-tarred?

Number of sections = $2 + 6 + 2 + 9 + 6 + 5 + 2 = 32$

3. Represent this data in a histogram.



4. What is the modal class interval for this set of data?

Modal class = $7\ 000 \leq l < 8\ 000$
 = (i.e. 9 sections of road fell into this length class interval)

Modal class: The class interval that has the highest frequency of data values.

④ LINE AND BROKEN LINE GRAPHS

- A line graph shows the trend between plotted points of continuous data.
 - Points are joined to show continuous nature of data.
- A broken line graph shows the trend between plotted points of discrete data.
 - Points are not joined to show discrete nature of data.
- These graphs are also effective in showing the relationship between two variables and multiple sets of data; and how these data sets change in relation to each other.



Refer to Module 2 for more on line graphs.

Worked Examples

NOTE! The question may ask for a line graph, but you are expected to know whether the graph will be a broken line or solid line graph.

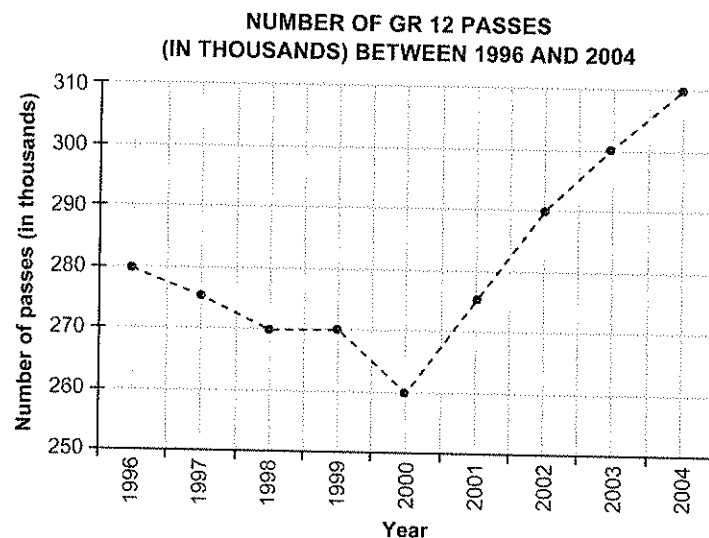
REMEMBER!

Discrete data $\Rightarrow$ broken line Continuous data $\Rightarrow$ solid line

1. The table below shows the number of learners that passed Grade 12 in South Africa between 1996 and 2004.

Year	Total number of passes
1996	280 000
1997	275 000
1998	270 000
1999	270 000
2000	260 000
2001	275 000
2002	290 000
2003	300 000
2004	310 000

- 1.1 Draw a line graph to illustrate the number of learners that have passed Grade 12 between 1996 and 2004.



NOTE!
 Time is continuous data but assessment of learners passing only occurs once a year.



1.2 Which year had the greatest number of Grade 12 passes?

2004

1.3 Which year had the least number of passes?

2000

1.4 Use this information to make a prediction regarding the number of Grade 12 passes in 2005.

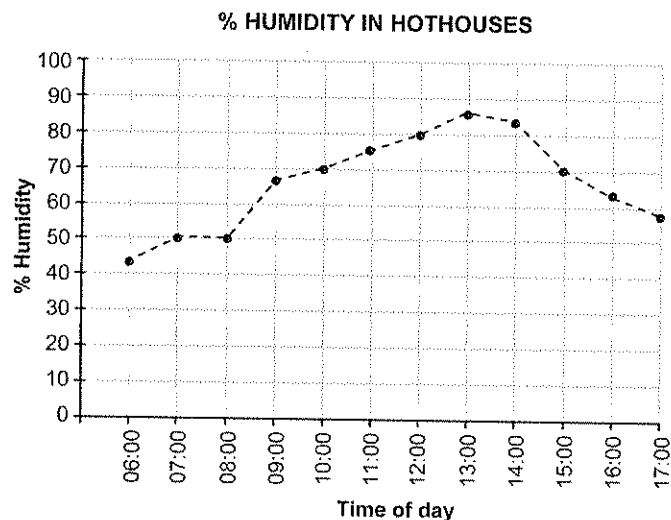
*The pass rate seems to be increasing at a steady rate from 2001 onwards.
One therefore estimates that the number of passes in 2005 would be 320 000.*

2. A nursery recorded the humidity levels in its hothouses on the hour for 12 hours a day. The recorded information is shown below.

Time of day	06:00	07:00	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00
% Humidity	42	50	50	67	70	75	80	85	82	70	62	58

2.1 Draw a line graph of the % humidity in the hothouses over 12 hours.

NOTE! Recordings were taken at specific times so the data is considered discrete - we don't know what happened between each recording
∴ broken line graph.



2.2 What was the maximum humidity reading?

85%

2.3 During what time of day did the humidity remain constant?

from 07:00 to 08:00

2.4 What was the difference in the % humidity between 12:00 and 07:00?

% Difference = $80\% - 50\%$
= 30%

6 SCATTER PLOTS

- A **scatter plot** is a graph whereby **one variable is plotted against another variable**; in order to show the relationship between the two variables.
- The scattered points may form a 'pattern':
 - If the points form an **increasing straight line** 'pattern', then we say that there is a **positive correlation**.
 - If the points form a **decreasing straight line** 'pattern', then we say that there is a **negative correlation**.
 - If the points are **scattered randomly** without any noticeable 'pattern', then we say that there is **no correlation**.
- If one were to try and draw a straight line to fit either the positive or negative correlation, then that line is known as the line of **best fit**.



You are **not** expected to know how to draw the line of best fit.

Worked Example



Appliance City records the sales of different appliances and the temperature on the day of the sale; in order to see if there is any correlation between these factors. The following information was recorded:

Table 1:

Temperature (°C)	18	20	32	22	35	37	27	28	29	25
Airconditioner sales	1	2	5	3	5	6	5	4	5	4

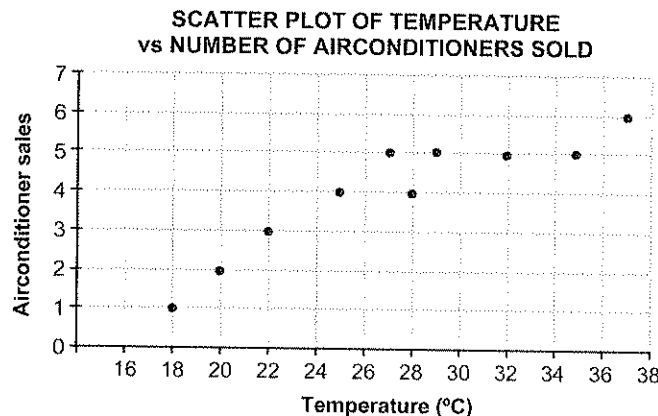
Table 2:

Temperature (°C)	20	23	21	17	18	15	16	19	22	12
Heater sales	2	1	1	7	6	7	6	4	0	9

Table 3:

Temperature (°C)	30	16	25	28	19	20	24	32	17	35
Vacuum cleaner sales	2	4	3	4	7	1	3	3	1	2

Use the data on p. 192 to draw three scatter plots, and comment on whether there is any correlation between the temperature and sales of the different appliances.



The points form an increasing straight line 'pattern' $\therefore$ a positive correlation.



NOTE: When plotting each factor against the other, they form co-ordinates.



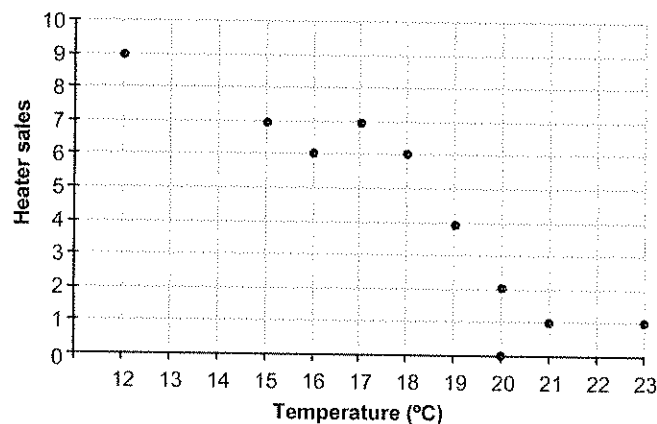
(Temperature ; No. of airconditioners sold)

(18 ; 1)

(x ; y)

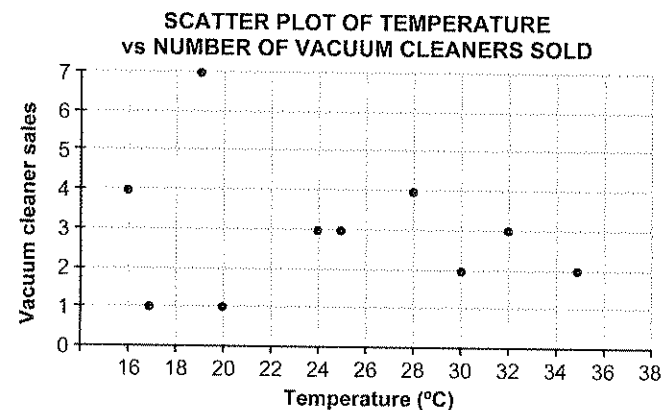
$\therefore$ There is a positive correlation between temperature and the number of airconditioners sold. As the temperature increases, so do the airconditioner sales.

SCATTER PLOT OF TEMPERATURE vs NUMBER OF HEATERS SOLD



The points form a decreasing straight line 'pattern' $\therefore$ a negative correlation.

There is a negative correlation between the temperature and number of heaters sold, because as the temperature increases, the number of heaters sold decreases.



There is no correlation between the temperature and number of vacuum cleaners sold, as the points are scattered randomly, without any pattern.

6 BOX-AND-WHISKER PLOTS

Also see Unit 4: Summarising Data - box-and-whisker plots on p. 184.



You are only expected to **interpret** box-and-whisker plots. You will **not** be expected to draw the plots.

Appropriate Representations of Data



- Some representations are more appropriate for particular types of data.
- In general, use:
 - **Pie charts** when you are trying to compare **parts of a whole**.
 - **Bar graphs** to compare the **frequency of discrete data**.
 - **Histograms** to compare the **frequency of continuous data**.
 - **Line and broken line graphs** to **track trends/changes over time**.
 - **Scatter plots** to show whether there is any **correlation between 2 variables**.
 - **Box-and-whisker plots** to show the **spread of data**.

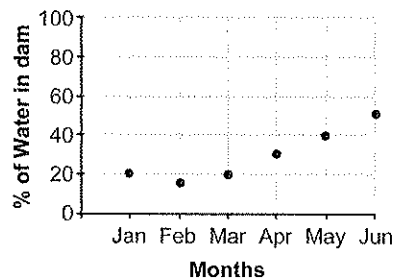
Factors that Affect the Impression created by a Graph



- The way in which a graph is drawn alters the impression of the data being represented.
- The following factors affect the impression created by a graph:
 - Scale of the axes**
 - the more spread out the axes, the larger the changes appear

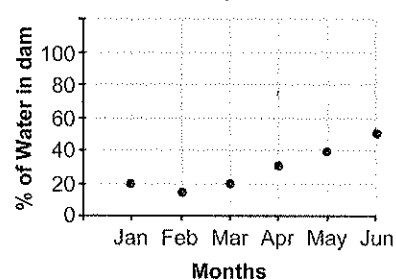
Dam Level Percentages Per Month

Graph A



Axes spread out
∴ changes more visible.

Graph B



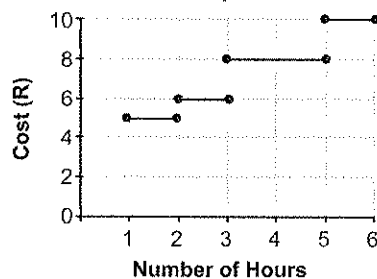
Axes condensed
∴ changes less visible.

Point at which the axes cross

- by excluding the section of axes where no points appear (i.e. breaking the axis), it 'zooms-in' on the relevant data points
- this alters the impression of the graph by highlighting small changes e.g.

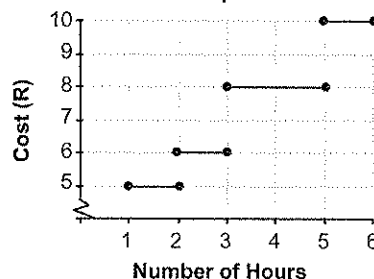
Cost of Parking per Hour

Graph A



Full axes creates a
general impression.

Graph B



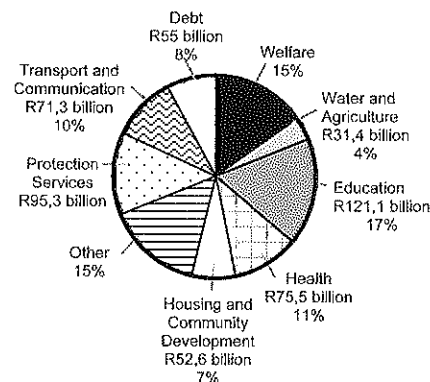
'Zooming-in' shows the
relevant portion of the axes
and highlights small changes.

Test Your Understanding

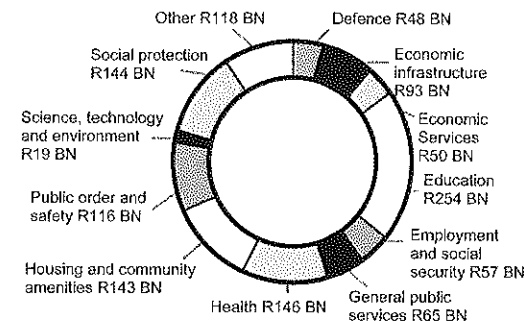


- Pie charts to show the South African budget allocations for 2008 and 2014:

2008 Budget Allocations



2014 Budget Allocations



- How much money was allocated to Transport and Communication in 2008?
- Which sector was allocated the smallest budget in the 2014 budget allocation?
- What percentage of the 2008 budget was allocated to Protection Services?
- Calculate the size of the 'Welfare' sector in the 2008 budget.
(Do not measure the size of the sector, as it is not drawn to scale).
- Calculate the size of the 'Social Protection' sector in the 2014 budget.
(Do not measure the size of the sector, as it is not drawn to scale).
- Calculate the % increase in the education budget between 2008 and 2014.

- Public transport is an essential part of the world in which we live. Most people in South Africa cannot afford their own cars, and so rely heavily on public transport or other forms of transport such as walking. The table below is a comparative study of the modes of transport for learners in South Africa, the United Kingdom and Australia.

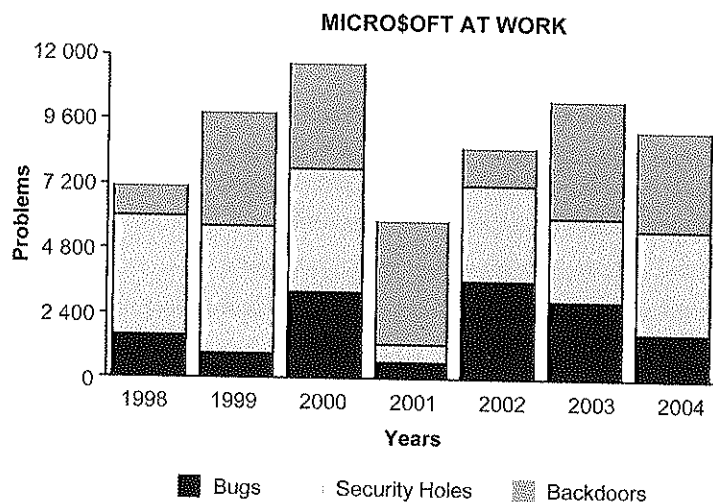
	Walk	Car	Bus	Bicycle	Train	Taxi	Other
South Africa	73,1%	12,9%	6,4%	1,2%	0,7%	5,3%	0,4%
United Kingdom	18,3%	52,3%	15,2%	0,7%	11,9%	0%	1,6%
Australia	37,9%	34,9%	22,6%	1,9%	1,7%	0%	1%

[Source: www.statsa.gov.za]

- What is the most popular mode of transport in South Africa?
- In 2008, there were 1,2 million learners in South African schools. Calculate the estimated number of learners who walked to schools in South Africa.

2.3 Draw a suitable bar graph to compare the modes of transport in South Africa and Australia.

3. Study the following graph and answer the questions below:



3.1 What type of graph is represented here?

3.2 In which year were the least number of total problems reported?

3.3 How many total problems were reported in 1998?

3.4 Estimate the number of bug problems that were reported in 2002.

3.5 In which year was the total number of bug and security hole problems approximately 1 200?

4. Each year South Africa generates income from exports (products sold to other countries). The income generated from these exports varies from year to year. Part of the income generated by exports comes from agricultural products. The table below shows the total income from exports, as well as the percentages of the total earned from agricultural products.

RELATIONSHIP BETWEEN SOUTH AFRICAN EXPORTS OF AGRICULTURAL AND OTHER PRODUCTS

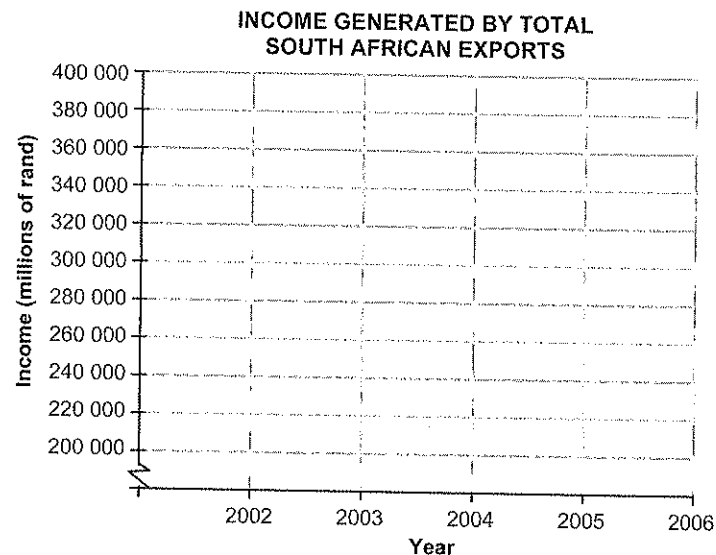
Year	Total income generated by South African exports (in millions of rand)	Income generated by agricultural exports (in millions of rand)	Percentage of the total income earned by agricultural products
2002	314 927	25 460	8,1
2003	273 127	22 670	8,3
2004	292 079	22 074	
2005	326 385	25 458	7,8
2006	393 047	26 978	6,9

[Source: South African Year Book, 2007]

4.1 Calculate the total income generated by agricultural exports from 2002 to the end of 2006.

4.2 What percentage of the total income earned by South African exports in 2004 was by agricultural products?

4.3 Draw a line graph of the total income generated by South African exports, using the system of axes below:



5. A botanist gathered the diameters of Loblolly Pine Trees in Duke Forest Tract, as shown below.

Diameter (d) of Tree (cm)	Frequency
$0 < d \leq 20$	1
$20 < d \leq 30$	7
$30 < d \leq 40$	40
$40 < d \leq 50$	67
$50 < d \leq 60$	75
$60 < d \leq 70$	48
$70 < d \leq 80$	10
$80 < d \leq 90$	2

5.1 How many class intervals are there?

5.2 How many trees were measured in Duke Forest Tract?

5.3 What is the modal class?

5.4 What type of data is being recorded?

5.5 Draw a histogram to represent this data.



6. It is said that often people with big hands have big feet and people with small hands have small feet. To investigate this, Liesl measured the length of 12 of her friends' hands. She also wrote down their shoe sizes.

Shoe size	6	8	5	11	4	7	10	9	6	3
Length of hand (cm)	16	21	15	25	12	19	25	22	13	10

6.1 Represent this data graphically using a scatter plot.

6.2 Is there any truth in this saying? Motivate your answer.



UNIT

6

INTERPRETING AND ANALYSING DATA

- After representing the data visually, it is important to **interpret** and **analyse** the data, by taking the following into account:
 - ① using percentages in a table or graph is useful for comparing relationships in size, but does not give any information regarding the actual sample or population size.
 - ② using actual sample or population values gives an indication of the size, but not of the relationship between data categories.
 - ③ the choice of scale of the axes and the point at which the axes cross will affect the impression created by the graph.



Also see 'Factors that affect the impression created by a graph' in Unit 5, p. 194.

- ④ graphs show **trends in data** more clearly than data values in a table.

- It is important to question the way in which data was collected, organised, summarised and represented in order to identify any errors, bias or misinterpretations. Therefore, the following questions should be asked:

- ① What was the size of the sample?
- ② Was the sample randomly chosen and representative?
- ③ What methods were used to collect the data and did the collector/recorder remain neutral and impartial?
- ④ Was the data collected fact or opinion?
- ⑤ How was the data organised and/or grouped?
- ⑥ Which measures of central tendency and spread were used?

- Be aware of the fact that data can be used and manipulated to favour an argument or circumstance.
- Interpretation and analysis of the data should happen at every stage of the statistical cycle.



Worked Examples

1. The table below shows how the government allocated its funds to certain expenditure items in 2012/13 and 2014/15:

ALLOCATIONS OF STATE EXPENDITURE AS A PERCENTAGE OF GOVERNMENT SPEND, 2012/13 AND 2014/15.

Expenditure	2012/13	2014/15
Education	5,29%	6,52%
Social Development	4,61%	4,82%
Health Care	3,3%	3,79%
Housing	3,08%	3,58%

- 1.1 What is the benefit of presenting percentages data in a table?

Percentages allow for comparisons between the different data categories as each data value is a percentage of the same whole.

- 1.2 What information is not revealed in this table?

The actual rand value budget allocation per data category is not revealed i.e. we don't know how much money was actually allocated to each expenditure item.

- 1.3 Why would it be better to represent this data visually, as opposed to data values in a table?

Visual representation of data in graphs shows trends more easily than data values in a table.

- 1.4 Name the most appropriate representation of this data and give a reason for your answer.

Double/multiple bar graph, in order to show the changes in percentages in the two different years.



Recap on the
'Appropriate Representations of Data' (see p. 193).

2. The following table shows the cost in cents of a litre of Lead Replacement Petrol (LRP).

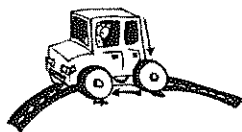
Petrol	Cost (LRP '95)
04 January 2012	1 031
01 February 2012	1 065
07 March 2012	1 093
04 April 2012	1 159
02 May 2012	1 187
06 June 2012	1 132
04 July 2012	1 047
01 August 2012	1 069
05 September 2012	1 162
03 October 2012	1 185

<http://www.aa.co.za/on-the-road/calculator-tool/fuel-pricing.html>

- 2.1 Draw a broken line graph to represent this data:

y-axis: Cost/l (c)

x-axis: Months

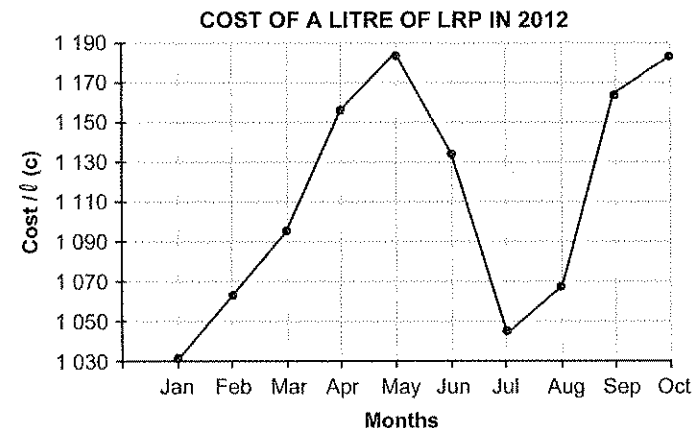


- 2.2 Describe any trends that you notice in the graph drawn.

The cost increased steadily from January 2012 to May 2012.

From May 2012 to July 2012 there was a sharp decrease in price.

From August 2012 to September 2012 the price rose very steeply.



- 2.3 What factors do you think affect these trends?

- *Weakness in the rand.*
- *Political unrest in parts of the world.*
- *Decrease in oil production due to extreme weather conditions e.g. hurricanes.*

- 2.4.1 In which month was the largest increase in the cost of petrol made?
What was the increase (in cents)?

September - 93 cents per litre increase

- 2.4.2 Explain how this increase is represented on your graph.

Steepest gradient

- 2.5 How does the line graph (Q2.1) highlight the changes to the cost of petrol?

The vertical axis starts at 1 030 cents (i.e. where the axes cross), to create a 'zooming-in' effect, changing the impression of the graph and highlighting small changes to the cost of petrol.



Also see 'Factors that affect the impression created by a graph' in Unit 5, p. 194.

- 2.6 Comment on a criticism that is evident in the data collection process.

The cost of petrol was recorded on different dates each month. In order for it to be more accurate and comparable, the data should have been collected on the same date each month (e.g. 1st of each month).

3. During the season the netball team scored the following number of goals in their matches.

11 13 14 15 16 17 18 19 21 58

- 3.1 The team goes through to the play-offs and the coach tries to determine what the team's chances are of winning. She decides to calculate their average match score by finding the mean. What impression does the value of the mean give in terms of their chances of winning the play-offs?

$$\text{Mean} = \frac{202}{10}$$

$$= 20,2$$

$$\approx 20 \text{ goals}$$

$$\text{Mean} = \frac{\text{sum of all values in data set}}{\text{total number of values in data set}}$$

REMEMBER! Round down for discrete data.



Since the mean of 20 goals is very high, it would give the impression that the team's chances of winning the play-off's is good.

- 3.2 Which measure of central tendency would be a better indicator of the team's chances of winning the play-offs? Give a reason for your answer.

Median, as it is not affected by outliers (i.e. 58).

- 3.3 Which data value would you choose to tell your opponents, in order to make them fearful of your team's netball ability?

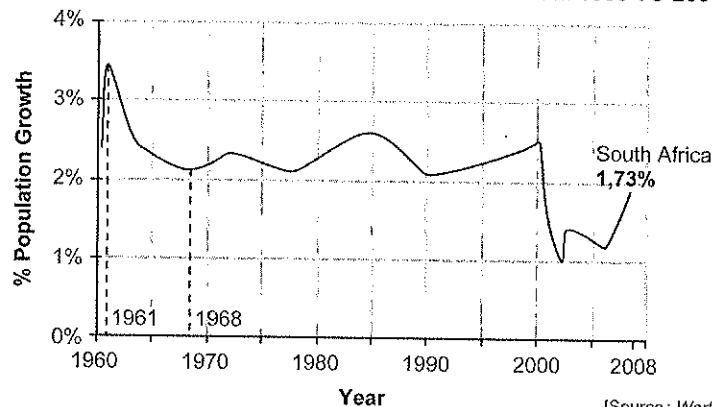
I would tell them that the team scored 58 goals.

Test Your Understanding



1. The following graph shows the population growth in South Africa from 1960 to 2008.

POPULATION GROWTH IN SOUTH AFRICA FROM 1960 TO 2008



[Source: World Bank, World Development Indicators - Last updated July 26, 2010]

- 1.1 Explain the population trend between 1961 and 1968, as shown on the graph.

Copyright © The Answer: Photocopying of this material is illegal

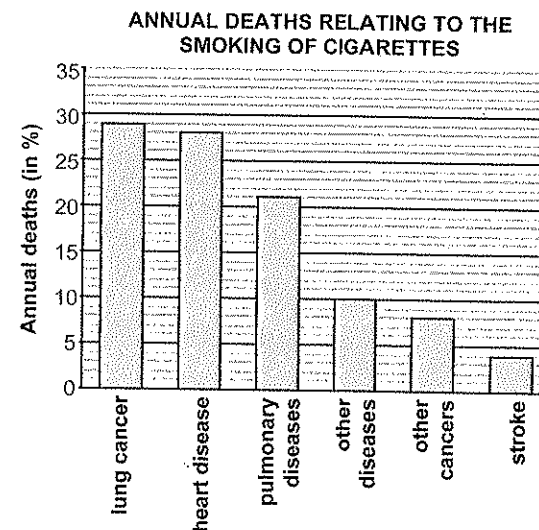
- 1.2 If the population was approximately 44 000 000 in 2008, calculate the total population figure for 2009 after a 1,73% increase.

- 1.3 Why do you think there was a decrease in the population growth rate from 2000 to 2002?

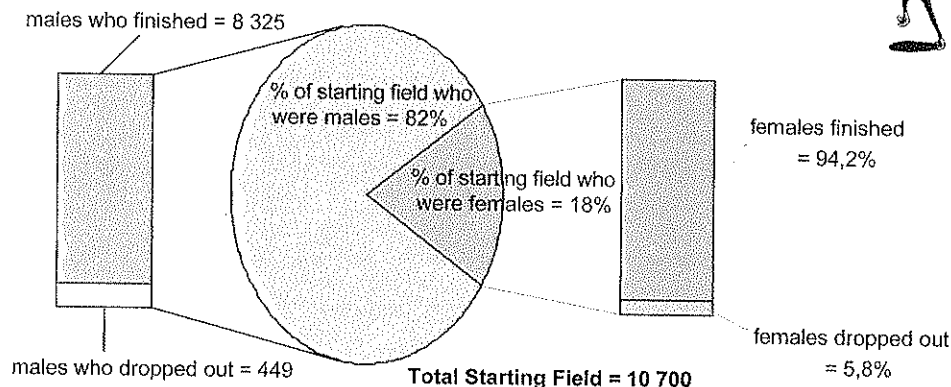
2. The manager at ABC Cigs used the graph alongside to try to convince Timothy to accept the position at the tobacco company. The manager stated:

'The graphs clearly show that there is an annual decline in the number of deaths relating to the smoking of cigarettes.'

Criticise the manager's statement with at least TWO justifications.



3. The graph below shows the percentage of males and females who started the Comrades Marathon and the number and/or percentage of males and females who finished the race.



- 3.1 How many men started this Comrades Marathon?
- 3.2 How many females started the Comrades Marathon?
- 3.3 How many of the females who started the marathon, finished it?
- 3.4 Did a higher percentage of male or female runners drop out of this Comrades Marathon? Explain your answer and show all working out.



7 PROBABILITY

UNIT

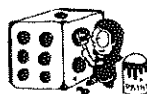
1

EXPRESSIONS OF PROBABILITY

Language of Probability

- Probability refers to the **chance** or **likelihood** of an event occurring.
- An **experiment** in probability refers to an activity which involves observing and recording results.
e.g. Rolling a die 15 times
- If you **roll** the die once, it is considered a **trial**.
- The **result** of a **trial** (i.e. 1; 2; 3; 4; 5 or 6) is an **outcome**.
- Rolling the die and obtaining an outcome, is called an **event**.

die = singular
dice = plural



Probability Scale

- Probability can be expressed in three different numerical ways:
 - percentage
 - common fraction
 - decimal
- The probability of an event occurring can be plotted on a probability scale.
 - Probability values range from zero to one, or from 0% to 100%. In other words, you **cannot** have a probability less than zero, nor greater than 1 (or 100%).

	Impossible		Equally likely		Certain
Percentage:	0%	25%	50%	75%	100%
Fraction:	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Decimal:	0.00	0.25	0.50	0.75	1.00

e.g. Tossing a coin and getting Heads AND Tails

Tossing a coin and getting Heads OR Tails

Tossing a coin and getting EITHER Heads or Tails

Impossible as a coin needs to land on only 1 side $\therefore$ can't get both!



There are only 2 possible outcomes $\therefore$ it is certain that you'll get either Heads or Tails!

Calculating Probability

- The universal symbol for calculating the probability of an event is **P(event)**.
- The formula used for calculating probability is:

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

e.g. What is the probability of selecting an 'Ace' out of a standard deck of cards?

NOTE: There are 52 cards in a deck.

$$P(\text{Ace}) = \frac{4 \text{ aces}}{52 \text{ cards in a deck}} = \frac{1}{13} (= 7,69\%)$$

Always remember to **simplify the fraction** as far as possible.



Worked Examples

1. Using the probability scale, describe and determine the probability of the following events:

- 1.1 A man giving birth
Impossible - 0%
- 1.2 A woman giving birth to either a baby boy or a baby girl.
Equally likely - 50%
- 1.3 A woman experiencing excruciating pain during natural childbirth.
Certain - 100%

2. In order to win the South African National Lottery, you need to select 6 winning numbers from 1 to 49. These numbers can be selected in any order.

What is the probability of selecting:

- 2.1 The number '7'?

$$P(7) = \frac{1}{49} (= 2,04\%)$$



$$P(\text{event}) = \frac{\text{no. of favourable outcomes}}{\text{total no. of possible outcomes}}$$

- 2.2 The number '50'?

$$P(50) = \frac{0}{49} (= 0\%)$$

$$= 0$$

- 2.3 A number greater than '45' first?

$$P(> 45) = \frac{4}{49} (= 8,16\%)$$

'A number > 45' means 46, 47, 48 and 49 ∴ 4 numbers!



Wednesday, 14 July 2010		
WINNING LOTTO NUMBERS		
06 - 21 - 27 - 32 - 37 - 49 + Bonus → 18		
Matches	No. of winners	Payouts per winner
Match 6 Numbers	1	R1 500 000
Match 5 Numbers + Bonus	1	R244 429
Match 5 Numbers	41	R13 749
Match 4 Numbers + Bonus	A	R1 996
Match 4 Numbers	2 470	R414
Match 3 Numbers + Bonus	4 542	R147
Match 3 Numbers	52 299	R42



Test Your Understanding

1. Arrange the following probability values in order from **most probable (likely)** to **most improbable (unlikely)**

$$\frac{9}{11} ; 90\% ; 0,87 ; \frac{7}{8} ; 0,25 ; 80\%$$



Most probable (likely) is closer on the probability scale to '**certain**'; while **most improbable (unlikely)** is close on the probability scale to '**impossible**'.

2. A bag has 2 bananas, 3 apples and 5 pears.

Find the following probabilities and express your answer as a fraction and as a %.

- 2.1 P(banana is chosen) 2.2 P(pear is chosen)
- 2.3 P(banana or apple is chosen) 2.4 P(no apples are chosen)

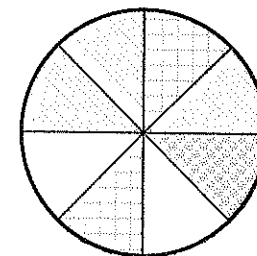
3. You are told to choose a number from 2 to 10 (including both 2 and 10).

What is the probability (as a fraction) that the number you choose will be:

- 3.1 the number 5? 3.2 an even number?

- 4.1 When spinning the spinner (illustrated alongside), which event(s) is:

- 4.1.1 most likely to occur?
- 4.1.2 equally likely to occur?
- 4.1.3 least likely to occur?



KEY

- Skip a turn
- Have another turn
- Go back 3 spaces
- Collect R500

- 4.2 What is the probability of the spinner coming to rest on:
- 4.2.1 'Go back 3 spaces'?
- 4.2.2 'Have another turn'?
- 4.2.3 'Give your opponent R500'?

5. What is the probability of selecting the following cards from a standard deck of cards?

- 5.1 A six of Spades
- 5.2 Any Heart
- 5.3 A number 2 of any suit
- 5.4 Not a King of Clubs

A standard deck of cards has 52 cards divided into 4 suits:

spades (black), hearts (red), clubs (black), diamonds (red).

Each suit has cards 2 to 10, Jack, Queen, King and Ace.



Probability is a prediction

- Probability refers to the chance or likelihood of an event recurring.
- We can use probability to predict future events or outcomes, based on the historical data/statistics of past events or outcomes over a long period of time. e.g. If a horse has won $\frac{4}{7}$ of its last races, the probability that this horse will win its next race is more probable than a horse who has only won $\frac{1}{7}$ of its last races.

Worked Example

A company has developed a new brand of toothpaste and tested the toothpaste on a group of people. The results of the test showed that 1 in every 5 people who used the toothpaste ended up with black teeth. Based on this data, how many people are predicted to have black teeth after using the new toothpaste, if a group of 200 people test it?

We can use this probability fraction to **estimate** the number of people whose teeth might turn black if the toothpaste were tested on 200 people in the following way:

$$\begin{aligned}
 &\text{Number of people whose teeth might turn black} \\
 &= P(\text{black teeth}) \times \text{number of people using the toothpaste} \\
 &= \frac{1}{5} \times 200 \\
 &= 40
 \end{aligned}$$



Theoretical vs Experimental Probability

- Theoretical probability** refers to the purely mathematical calculation of determining the probability of an event occurring out of the total number of possible events.

$$\text{Theoretical probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

e.g. When rolling a die, the theoretical probability of rolling a '6' is $\frac{1}{6}$.

die = singular
dice = plural

- Experimental probability** refers to the **actual results** obtained when calculating the **relative frequency** of an event, after repeated trials.
 - Frequency** of an event is the number of times an outcome has occurred. e.g. Roll a die 10 times, and if a '6' appears twice, the frequency of '6' is 2.
 - Relative frequency/Experimental probability** is the number of times the event occurred, relative to the number of trials of the event. e.g. Roll a die 10 times and if a '6' appears twice, the relative frequency of '6' is $\frac{2}{10}$.

$$\text{Experimental probability (relative frequency)} = \frac{\text{number of favourable outcomes}}{\text{total number of events}}$$

The more times an experiment is repeated, the closer the value for the experimental probability comes to the theoretical probability.

Worked Example

A pregnancy test states that it has a 98% accuracy in determining whether you are pregnant or not. A researcher decides to test the accuracy of the pregnancy test, by testing 50 pregnant women and seeing whether the test gave a positive result.

The results are shown in the frequency table below:

Pregnancy Test Results	
Outcome	Frequency
Pregnant	47
Not pregnant	3
Total	50

- What is the theoretical probability of being tested positive using the pregnancy test?

98%

2. What is the relative frequency / experimental probability of being tested positive using the pregnancy test? Give your answer as a fraction and %.

$$\frac{47}{50} = 94\%$$

3. Would you rely on the results of this pregnancy test? Explain your answer.

No, I wouldn't rely on this pregnancy test as 6% of the tested women (i.e. 3 women) were actually pregnant, when they tested negative for pregnancy.

Replacement vs Non-Replacement

- When trying to determine the total number of ways in which an event can occur and the probability of each outcome, it is important to consider the issue of replacement and non-replacement.

To explain the difference, consider the following example:

Benni works as a tester for a company who manufactures sweets. Benni's job is to randomly pick up sweets from a tray and check them for defects. On every tray there are 12 toffees and 8 chocolates.

Scenario 1: Replacement

Benni selects a sweet, inspects the sweet and then replaces it. He then picks up a second sweet and inspects it.

In this scenario, the probability that the first sweet that Benni will choose will be a toffee is:

$$\therefore P(1^{\text{st}} \text{ sweet toffee}) = \frac{12}{20}$$

Because he replaces this toffee, there are still 12 toffees left on the tray and a total of 20 sweets. This means that the probability that the second sweet that he chooses will be a toffee is

$$\therefore P(2^{\text{nd}} \text{ sweet toffee}) = \frac{12}{20}$$



Scenario 2: Non-Replacement

Benni selects a sweet and then eats the sweet. He then picks up a second sweet for inspection.



In this scenario, the probability that the first sweet that Benni will choose will be a toffee is:

$$\therefore P(1^{\text{st}} \text{ sweet toffee}) = \frac{12}{20}$$

But now Benni doesn't replace the sweet - he eats it. This means that there are now only 11 toffees left on the tray out of a remaining 19 sweets:

$$\therefore P(2^{\text{nd}} \text{ sweet toffee}) = \frac{11}{19}$$

- Whether an item is replaced or not replaced during an event will have an important impact on the probability of the outcome of that event. It is important, then, to establish whether replacement or non-replacement occurs before any probability calculations take place.



Worked Example

A magician asks you to pick a card out of a standard deck of cards.

1. What is the probability that you pick an Ace of Hearts?

$$P(\text{Ace of Hearts}) = \frac{1}{52}$$



2. What is the probability that your friend standing next to you picks ...

- 2.1 an Ace out of the deck, while you are still holding the Ace of Hearts card?

$$P(\text{Ace with non-replacement}) = \frac{3 \text{ aces}}{51 \text{ cards in deck}} = \frac{1}{17}$$



You are still holding the Ace of Hearts $\therefore 4 \text{ Aces} - 1 \text{ Ace} = 3 \text{ Aces}$ available in the deck. Also, since you are still holding a card $\therefore 52 \text{ cards} - 1 \text{ card} = 51 \text{ cards}$ available in the deck.

- 2.2 an Ace out of the deck, after you put the Ace of Hearts card back into the deck?

$$P(\text{Ace with replacement}) = \frac{4 \text{ aces}}{52 \text{ cards in deck}} = \frac{1}{13}$$

Test Your Understanding



1. A company is testing a new cream for people who suffer from dry skin. The table below shows the results of the test.

The Cream Helped	The Cream Made No Difference
37	13

- 1.1 What is the probability that a person selected randomly from the group would say that the cream helped their skin? Write your answer as a percentage.
- 1.2 Based on the test results, do you think the cream is successful in helping with dry skin? Explain your answer.
- 1.3 If 500 people were to buy this cream, how many of them could the manufacturers expect to be unhappy with the cream?
- 2.1 A chocolatier offers 5 varieties of chocolate, namely Nutty Delight; Hazelnut Praline; Caramel Cup; Mint Fresh and Strawberry Cream. The chocolatier records the number of sales of the different varieties of chocolates in a day, as shown below:

Monday Chocolate Sales	
Chocolate Variety	Frequency
Nutty Delight	42
Hazelnut Praline	27
Caramel Cup	16
Mint Fresh	55
Strawberry Cream	10

- 2.1.1 What is the theoretical probability of selling a Mint Fresh chocolate?
- 2.1.2 What is the frequency of Hazelnut Praline chocolate sales?
- 2.1.3 What is the relative frequency of the Strawberry Cream chocolate sales?
- 2.2 The chocolatier decides to put 5 Nutty Delights; 4 Hazelnut Pralines; 3 Caramel Cups; 2 Mint Fresh's; and 1 Strawberry Cream on a tester plate; that he offers to potential customers.
- 2.2.1 What is the probability of the first potential customer picking a Caramel Cup?
- 2.2.2 What is the probability that the second potential customer chooses a Nutty Delight?
- 2.2.3 What is the probability that the third potential customer takes a Caramel Cup?
3. In March 2008 there were approximately 9 168 000 motor vehicles on the road. Of these, approximately 375 000 were believed to be unroadworthy.
- 3.1 What is the probability that a driver that was pulled over, was driving a roadworthy car?
- 3.2 How many cars can the traffic officer expect to be unroadworthy, if he pulls over 30 cars in a day?

UNIT

3

REPRESENTATIONS FOR DETERMINING POSSIBLE OUTCOMES

Compound Events



- Situations involving the probability of **more than one event** are known as **compound events**.
These events can either take place at the same time or one after the other.
- It is important to note that the events in a **compound event take place independently** of each other (i.e. the results of the one event have no bearing on the results of the other event).
e.g. Rolling a die twice; Tossing a coin twice; Drawing a card from a deck of cards more than once, etc.
- In order to **represent compound events visually**, we make use of **tree diagrams** and **two-way tables** (also known as **contingency tables**).

Tree Diagrams



- Tree diagrams provide us with a way to **visually represent** all of the possible outcomes of **compound events**.
- A tree diagram is shaped like a tree, with the branches representing all the different possible outcomes.
- Remember that each **event** is **independent** of the other.

Worked Example

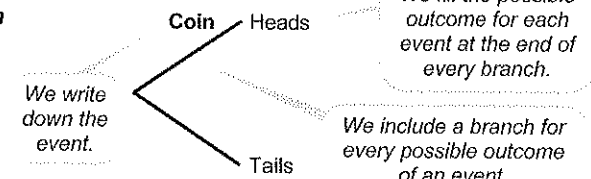


1. Draw a tree diagram to represent the possible outcomes of tossing a coin and rolling a die.

Event 1: Tossing a coin

Two possible outcomes:

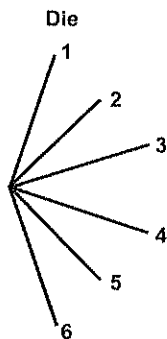
Heads or Tails



Event 2: Rolling a die

Six possible outcomes:

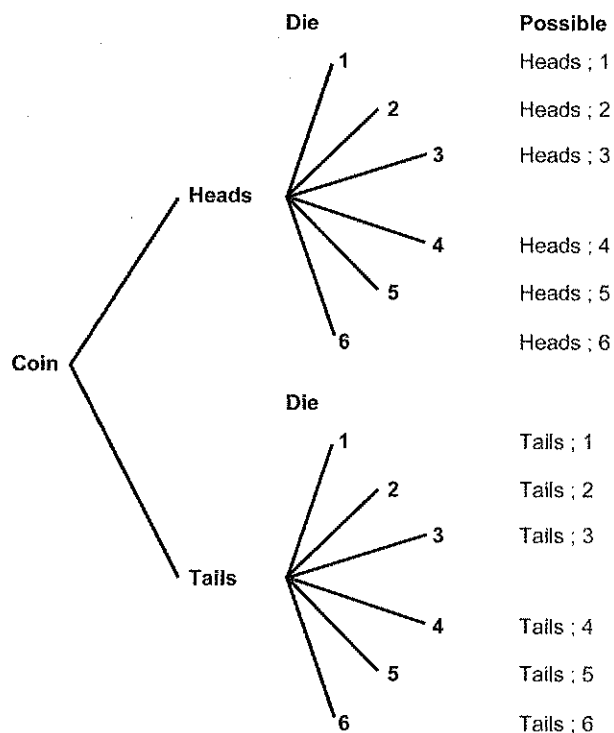
1; 2; 3; 4; 5; 6



NOTE! There are 6 branches to represent each possible outcome.



die = singular
dice = plural

Tree Diagram of a Coin Toss and Die Roll

It is important to list all the possible outcomes of the compound event by following the 'path' of each branch from the beginning of the tree to the very end.

2. Using your tree diagram from Question 1.1, answer the following questions:

2.1 How many possible outcomes are there?

12

2.2 What is the probability of getting Heads and the number 5?

$$\frac{1}{12} (= 8,33\%)$$

2.3 What is the probability of getting Tails?

$$\frac{6}{12} = \frac{1}{2} (= 50\%)$$

Six out of the 12 possible outcomes have tails; irrespective of the number that appears on the dice.



2.4 What is the probability of getting the number 4?

$$\frac{2}{12} = \frac{1}{6} (= 16,67\%)$$



Two out of the 12 possible outcomes have the number 4; irrespective of the outcome of the coin toss.

Two-way Tables

- Also known as **contingency tables**.
 - Two-way tables provide us with another way in which to **visually represent** all the possible outcomes of **compound events**.
 - A two-way table shows all the possible outcomes of the one event down a column; and the possible outcomes of the other event across a row.
- Therefore, the compound event outcomes or the frequencies of the events, are displayed in the 'middle' of the table.

Worked Examples



- 1.1 Draw up a two-way table to represent all the possible outcomes of throwing two dice.

DIE 1 \ DIE 2	1	2	3	4	5	6
1	1; 1	1; 2	1; 3	1; 4	1; 5	1; 6
2	2; 1	2; 2	2; 3	2; 4	2; 5	2; 6
3	3; 1	3; 2	3; 3	3; 4	3; 5	3; 6
4	4; 1	4; 2	4; 3	4; 4	4; 5	4; 6
5	5; 1	5; 2	5; 3	5; 4	5; 5	5; 6
6	6; 1	6; 2	6; 3	6; 4	6; 5	6; 6

Event 2
rolling
die 2

Event 1
rolling
die 1

Possible outcomes of compound event

- 1.2 Use the two-way table drawn in Question 1.1, to answer the following questions:

- 1.2.1 How many possible outcomes are there?
36

- 1.2.2 What is the probability of rolling a double six?
 $\frac{1}{36}$ (= 2,78%)



- 1.2.3 What is the probability of rolling first a 5, then a 1?
 $\frac{1}{36}$ (= 2,78%)

'5 and 1' in any order means a possible outcome of 5;1 or 1;5.

- 1.2.4 What is the probability of rolling a 5 and a 1 in any order?
 $\frac{2}{36} = \frac{1}{18}$ (= 5,56%)

The 3 could be rolled first or second $\therefore$ count all possible outcomes containing a 3.

- 1.2.5 What is the probability of rolling a 3?
 $\frac{11}{36}$ (= 30,56%)

2. A company is doing research into the link between smoking and cancer. Their results are summarised in the two-way table below:

Smoke	Cancer		Total
	Yes	No	
Yes	25	8	33
No	14	73	87
Total	39	81	120

This two-way table can be interpreted as follows:

'25' represents the number of people surveyed who **smoke** and who **do have or have had cancer**.

'8' represents the number of people surveyed who **smoke** and who **do not have or have never had cancer**.

'73' represents the number of people surveyed who **don't smoke** and who **do not have or have never had cancer**.

'14' represents the number of people surveyed who **do not smoke** but **do have or have had cancer**.

Smoke	Cancer		Total
	Yes	No	
Yes	25	8	33
No	14	73	87
Total	39	81	120

'33' = 25 + 8: This represents the total number of people who **smoke**.

'87' = 14 + 73: This represents the total number of people who **do not smoke**.



'39' = 25 + 14: This represents the total number of people who **have or have had cancer**.

'81' = 8 + 73: This represents the total number of people who **do not have or have never had cancer**.

'120' = 33 + 87
OR
'120' = 39 + 81
This represents the **total number of people surveyed** - both smokers and non-smokers; and those with and without cancer.

Use the two-way table above to answer these questions:

- 2.1 Determine the probability that a person chosen at random from the sample group will be a smoker who has or has had cancer:

$$P(\text{smoker with cancer}) = \frac{25}{120} = \frac{5}{24} (= 20,83\%)$$

- 2.2 Determine the probability of selecting a person who is a non-smoker and who does not have or has never had cancer:

$$P(\text{non-smoker without cancer}) = \frac{73}{120} (= 60,83\%)$$

- 2.3 Determine the probability that a person chosen from the group will be a smoker:

$$P(\text{smoker}) = \frac{33}{120} = \frac{11}{40} (= 27,5\%)$$

- 2.4 What does the information in the table tell us about smoking and cancer?

$$P(\text{smoker with cancer}) = 20,83\% \text{ (see Question 2.1)}$$

$$P(\text{non-smoker with cancer}) = \frac{14}{120} = 11,67\%$$

$\therefore$ It appears that there is a higher probability of getting cancer if you smoke.

ANSWERS

MODULE 1

NUMBERS AND CALCULATIONS WITH NUMBERS

UNIT 1

NUMBER FORMATS AND CONVENTIONS

1. Four million, eight hundred and twenty-seven thousand and thirty rand.
2. R950 000
- 3.1 R200 3.2 -R150
- 4.1 -5°C 4.2 510 4.3 -80 m

UNIT 2

OPERATIONS ON NUMBERS AND CALCULATOR SKILLS

1.
 - ▷ Eggs : R12,99 ≈ R13
 - ▷ Bread : R14,50 ≈ R15
 - ▷ Cheese : R27,85 ≈ R30
 - ▷ Tomato : R3,22 ≈ R3
 - ▷ Bacon : R19,89 ≈ R20



$$\therefore \text{Total estimated cost} \approx \text{R13} + \text{R15} + \text{R30} + \text{R3} + \text{R20} \approx \text{R81}$$

2. Estimated number of people ≈ 30% of 150
 $\approx 30\% \times 150$
 ≈ 45

$$\begin{aligned} 3.1 \quad & \frac{46 + 6 \times 4}{55 - 5 \times 7} \\ &= \frac{46 + 24}{55 - 35} \\ &= \frac{70}{20} \\ &= \frac{7}{2} (= 3,5) \end{aligned}$$

$$\begin{aligned} 3.3 \quad & 5 + \frac{2}{3} \text{ of } 150 \\ &= 5 + \frac{2}{3} \times 150 \\ &= 5 + 100 \\ &= 105 \end{aligned}$$

$$\begin{aligned} 3.5 \quad & 17\pi \times \frac{11}{100} \\ &= (17 \times 3,142) \times \frac{11}{100} \\ &= 53,414 \times \frac{11}{100} \\ &= 5,87554 \\ &= 5,88 \end{aligned}$$

rounded off to 2 decimal places



$$\begin{aligned} 3.2 \quad & 14\% \text{ of R63,15} \\ &= \frac{14}{100} \times \text{R63,15} \\ &= \text{R8,84} \end{aligned}$$

$$\begin{aligned} 3.4 \quad & 48,6^2 \div 10 + \sqrt{121} \\ &= 2\,361,96 \div 10 + 11 \\ &= 236,196 + 11 \\ &= 247,196 \\ &= 247,20 \end{aligned}$$

rounded off to 2 decimal places

UNIT 3

ROUNDING

$$1.1 \quad 235,02 \approx 235,0$$

$$1.2 \quad 12,9477 \approx 12,9$$

$$1.3 \quad 38,98 \approx 39,0$$

$$2.1 \quad 14,111 \approx 14,11$$

$$2.2 \quad 134,6983 \approx 134,70$$

$$2.3 \quad 7\,482,5104 \approx 7\,482,51$$

$$3.1 \quad 7,9999 \approx 8,000$$

$$3.2 \quad 15,9983 \approx 15,998$$

$$3.3 \quad 6,7053 \approx 6,705$$

$$4.1 \quad \text{R}335,73 \approx \text{R}335,75$$

$$4.2 \quad \text{R}109,98 \approx \text{R}110,00$$

$$4.3 \quad \text{R}34,56 \approx \text{R}34,55$$

$$5.1 \quad \text{R}199,45 \approx \text{R}199$$

$$5.2 \quad \text{R}39,98 \approx \text{R}40$$

$$5.3 \quad \text{R}2\,719,35 \approx \text{R}2\,719$$

$$6.1 \quad \text{R}4\,532 \approx \text{R}4\,530$$

$$6.2 \quad \text{R}2\,462,15 \approx \text{R}2\,460$$

$$6.3 \quad \text{R}3\,617,95 \approx \text{R}3\,620$$

$$7.1 \quad 894 \approx 890$$

$$7.2 \quad 788 \approx 790$$

$$7.3 \quad 2\,174 \approx 2\,170$$

$$8.1 \quad 38\,192 \approx 38\,200$$

$$8.2 \quad 7\,135 \approx 7\,100$$

$$8.3 \quad 448 \approx 400$$

$$9.1 \quad 1\,241 \approx 1\,000$$

$$9.2 \quad 94\,355 \approx 94\,000$$

$$9.3 \quad 6\,551 \approx 7\,000$$



$$10. \quad \text{R}65 \div \text{R}9,95$$

$$= 6,5327$$

$$\approx 6 \text{ bars}$$

You would normally round this number up to 7 but you don't have enough money for 7 bars $\therefore$ round down.

$$11. \quad 130 \div 3 = 43,33 \approx 43 \text{ books}$$



Only 43 books can fit on Roy's bookshelf. The length (130 cm) is not long enough for 44 books.

UNIT 4 RATIOS

1.1 $12 : 960$
 $1 : 80$ $\div$ HCF = 12

1.2 $140 : 24$
 $35 : 6$ $\div$ HCF = 4

1.3 $12,3 : 6,6$
 $123 : 66$
 $41 : 22$ $\div$ HCF = 3

1.4 $12 \text{ s} : 1 \text{ h}$
 $12 \text{ s} : 3 600 \text{ s}$
 $12 : 3 600$
 $1 : 300$ $\div$ 12
 $\dots 1 \text{ h} \times 60 = 60 \text{ min}$
 $\dots 60 \text{ min} \times 60 = 3 600 \text{ seconds}$

1.5 $15 \text{ cm} : 0,65 \text{ m}$
 $15 \text{ cm} : 65 \text{ cm}$
 $15 : 65$
 $3 : 13$ $\div$ 5
 $\dots 0,65 \text{ m} \times 100 = 65 \text{ cm}$

1.6 $\frac{3}{8} : \frac{2}{5}$
 $\frac{15}{40} : \frac{16}{40}$
 $15 : 16$
 $\dots \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$ and $\frac{2 \times 8}{5 \times 8} = \frac{16}{40}$
 $\therefore$ common denominator of 40.

2. Daniel's : Andrea's
R25 000 : R10 000

Daniel earns R25 000 because he earns R15 000 more than Andrea (earns R10 000).

$25 000 : 10 000$
 $5 : 2$ $\div$ HCF = 5 000

3. Sally : David
 $5 : 7$
 $? : 189 \text{ cm}$ $\times 27$

$\therefore$ Sally's height
 $= 5 \times 27$
 $= 135 \text{ cm}$

Sally : David
 $5 : 7$
 $? : 189 \text{ cm}$

$\therefore 1 \text{ Part} = 189 \text{ cm} \div 7$
 $= 27$

$\therefore$ Sally's height
 $= 5 \text{ parts} \times 27$
 $= 135 \text{ cm}$

4. R300 000 : R500 000
 $3 : 5$

Total number of parts = $3 + 5 = 8 \text{ parts}$

Shareholder A: $\frac{3}{8} \times \frac{1 200 000}{1} = \text{R}450 000$

Shareholder B: $\frac{5}{8} \times \frac{1 200 000}{1} = \text{R}750 000$

OR R300 000 : R500 000
 $3 : 5$

Total number of parts = $3 + 5 = 8 \text{ parts}$

$\therefore 1 \text{ Part} = \text{R}1 200 000 \div 8 = \text{R}150 000$

Shareholder A: $\text{R}150 000 \times 3 \text{ parts} = \text{R}450 000$

Shareholder B: $\text{R}150 000 \times 5 \text{ parts} = \text{R}750 000$

5.1 learners : teachers OR learners : teachers

$\times 32$ $25 : 1$
 $800 : ?$

$\therefore$ Number of teachers
 $= 1 \times 32$
 $= 32 \text{ teachers}$

$25 : 1$
 $800 : ?$

$\therefore 1 \text{ Part} = 800 \div 25$
 $= 32$

$\therefore$ Number of teachers
 $= 1 \times 32$
 $= 32 \text{ teachers}$

5.2 learners : teachers

$860 : 20$
 $43 : 1$ $\div$ HCF = 20

5.3

School A
learners : teachers
 $750 : 15$
 $50 : 1$

School B
learners : teachers
 $920 : 20$
 $46 : 1$

$\therefore$ School B has the smaller learner to teacher ratio.

UNIT 5 PROPORTION

1. mass : cost OR mass : cost
 $+ 2,4$ $480 \text{ kg} : \text{R}3 600$
 $200 \text{ kg} : ?$ $\div 2,4$

$\therefore \text{Cost} = \text{R}3 600 \div 2,4$
 $= \text{R}1 500$

OR

mass : cost
 $480 \text{ kg} : \text{R}3 600$
 $200 \text{ kg} : ?$

mass : cost
 $480 \text{ kg} : \text{R}3 600$
 $200 \text{ kg} : ?$

$\therefore \frac{480 \text{ g}}{\text{R}3 600} = \frac{200 \text{ kg}}{x}$

$480 \times x = 3 600 \times 200$

$480x = 720 000$

$x = \text{R}1 500$

$\therefore 1 \text{ kg costs} = \text{R}3 600 \div 480 = \text{R}7,50$

$\therefore 200 \text{ kg costs} = \text{R}7,50 \times 200 = \text{R}1 500$

2. boy scouts : time taken OR

$+ 1,5$ $12 : 3$
 $8 : ?$ $\times 1,5$

$\therefore$ Time taken
 $= 3 \times 1,5$
 $= 4,5 \text{ h}$

NOTE!
opposite
operations
 $\times$ and $\div$

Boy scouts	12	8
Time taken	3	?

$12 \times 3 = 8 \times ?$
 $= 36 = 36$

$\therefore \text{Time taken} = 36 \div 8$
 $= 4,5 \text{ h}$

3.1 Minimum total area = $1,6 \text{ m}^2 \times 52 \text{ learners}$
 $= 83,2 \text{ m}^2$

3.2 Maximum number of learners = $96 \text{ m}^2 \div 1,6 \text{ m}^2$
 $= 60 \text{ learners}$

4.1 10 hours 4.2 5 workers 4.3 7,5 hours

5.1 1 box = 2 cabbages + 12 carrots
 $\therefore$ Average mass of 1 box = $(2 \times 2 \text{ kg}) + (12 \times 0,12 \text{ kg})$
 $= 4 \text{ kg} + 1,44 \text{ kg}$
 $= 5,44 \text{ kg}$

5.2.1 number of boxes : number of carrots

$$\times 2 \left(\begin{array}{l} 1 : 12 \\ 2 : A \end{array} \right) \times 2$$

$$\therefore \text{Number of carrots (A)} = 12 \times 2 = 24 \text{ carrots}$$

OR

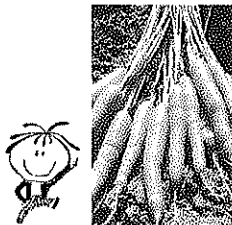
number of boxes : number of carrots

$$\begin{array}{l} 1 : 12 \\ 2 : A \end{array}$$

$$\therefore \frac{1}{12} = \frac{2}{A}$$

$$1 \times A = 12 \times 2$$

$$A = 24 \text{ carrots}$$



5.2.2 number of boxes : number of cabbages

$$\times 12 \left(\begin{array}{l} 1 : 2 \\ B : 24 \end{array} \right) \times 12$$

$$\therefore \text{Number of boxes (B)} = 1 \times 12 = 12 \text{ boxes}$$

OR

number of boxes : number of cabbages

$$\begin{array}{l} 1 : 2 \\ B : 24 \end{array}$$

$$\therefore \frac{1}{2} = \frac{B}{24}$$

$$2 \times B = 1 \times 24$$

$$2B = 24$$

$$B = 12 \text{ boxes}$$



5.3 cabbages : carrots OR cabbages : carrots

$$\times 6 \left(\begin{array}{l} 2 : 12 \\ 12 : ? \end{array} \right) \times 6$$

$$\therefore \text{Number of carrots}$$

$$= 12 \times 6$$

$$= 72$$

$$\therefore \frac{2}{12} = \frac{12}{x}$$

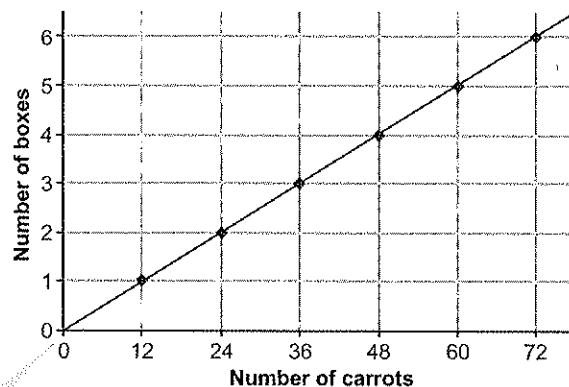
$$2 \times x = 12 \times 12$$

$$2x = 144$$

$$x = 72 \text{ carrots}$$

5.4

NUMBER OF CARROTS PER BOX



NOTE! Direct proportion graphs always start at the origin.



UNIT

6

RATES

$$1.1 \text{ Number of kilolitres} = 342 \ell \div 1\,000 = 0,342 \text{ k}\ell$$

$$\text{Cost} = R11,72 \times 0,342 \text{ k}\ell = R4,00$$

$$1.2 \text{ Number of kilolitres} = R164,80 \div R11,72 = 14 \text{ k}\ell$$

$$2. \quad 500 \text{ km on } 35 \ell$$

$$\therefore 100 \text{ km on } 35 \div 5 = 7 \ell$$

$$\therefore 7 \ell \text{ per } 100 \text{ km}$$

$$3. \quad \text{Number of hours} = 33 \text{ minutes} \div 60 = 0,55 \text{ h}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{25 \text{ km}}{0,55 \text{ h}}$$

$$= 45,45 \text{ km/h}$$



4. time : weight

$$\begin{array}{l} + 500 \left(\begin{array}{l} 30 \text{ min} : 500 \text{ g} \\ 0,06 : 1 \text{ g} \end{array} \right) + 500 \\ \times 1\,200 \left(\begin{array}{l} ? : 1\,200 \text{ g} \end{array} \right) \times 1\,200 \end{array}$$

$\therefore$ Time

$$= (30 \text{ min} \div 500) \times 1\,200 \text{ g}$$

$$= 72 \text{ min}$$

NOTE!

Change units to the same, smallest unit, i.e. grams

$$\therefore 1,2 \text{ kg} \div 1\,000 = 1\,200 \text{ g}$$

$$5. \quad \text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{160 \text{ km}}{120 \text{ km/h}}$$

$$= 1,33 \text{ h}$$

$$= 1 \text{ h and } 20 \text{ min}$$

Convert to minutes: multiply by 60

$$\therefore 0,33 \times 60 = 20 \text{ minutes}$$

$$6. \quad \text{Smooth Delight: } R89,99 \div 175 \text{ ml} = R0,514/\text{ml}$$

$$\text{Creamy Soft: } R92,99 \div 180 \text{ ml} = R0,516/\text{ml}$$

$$\text{Silky Vanilla: } R70,99 \div 150 \text{ ml} = R0,473/\text{ml}$$

$\therefore$ Creamy Soft is the most expensive face cream

UNIT

7

PERCENTAGES

$$1. \quad \text{Person A} = R2,50 + 0,75\% \text{ of } R760$$

$$= R2,50 + \frac{0,75}{100} \times R760$$

$$= R2,50 + R5,70$$

$$= R8,20$$

$$\text{Person B} = R1,50 + 0,85\% \text{ of } R760$$

$$= R1,50 + \frac{0,85}{100} \times R760$$

$$= R1,50 + R6,46$$

$$= R7,96$$

$\therefore$ Person A pays higher bank fees.



$$2. \text{ Increase} = \frac{16}{100} \times 1\,255 \text{ kg} = 200,8 \text{ kg}$$

$$\therefore \text{ Total} = 1\,255 + 200,8 \text{ kg} \\ = 1\,455,8 \text{ kg}$$

OR

$$\text{Total} = 1\,255 \text{ kg} \times (100\% + 16\%) \\ = 1\,255 \times 116\% \\ = 1\,255 \times \frac{116}{100} \\ = 1\,455,8 \text{ kg}$$

$$3. \text{ Discount} = 75\% \text{ of R350}$$

$$= \frac{75}{100} \times \text{R350} \\ = \text{R262,50}$$

$$\therefore \text{ Sale price} = \text{R350} - \text{R262,50} \\ = \text{R87,50}$$

OR

$$\text{Sale Price} = \text{R350} \times (100\% - 75\%) \\ = \text{R350} \times 25\% \\ = \text{R350} \times \frac{25}{100} \\ = \text{R87,50}$$



$$4. \text{ \% Increase} = \frac{6\,100\,000 - 5\,600\,000}{5\,600\,000} \times 100 \\ = \frac{500\,000}{5\,600\,000} \times 100 \\ = 8,9\%$$

$$5. \text{ \% Decrease} = \frac{314\,927 \text{ million} - 273\,127 \text{ million}}{314\,927 \text{ million}} \times 100 \\ = \frac{41\,800 \text{ million}}{314\,927 \text{ million}} \times 100 \\ = 13,27\%$$

$$6. \text{ Sale Price : Original Price} \\ + 80 \left(\begin{array}{c} 80\% : 100\% \\ 1 : \end{array} \right) + 80 \\ \times 1\,800 \left(\begin{array}{c} R1\,800 : R? \end{array} \right) \times 1\,800$$

$$\therefore \text{ Original Price} = (100 + 80) \times 1\,800 \\ = \text{R2}\,250$$

OR

$$\text{Original Price} = \text{R1}\,800 \div (100\% - 20\%) \\ = \text{R1}\,800 \div 80\% \\ = \text{R2}\,250$$

$$7. \text{ Decrease} = \frac{4}{100} \times \text{R6}\,999 \\ = \text{R279,96}$$

$$\therefore \text{ Value} = \text{R6}\,999 - \text{R279,96} \\ = \text{R6}\,719,04$$

OR

$$\text{Value} = \text{R6}\,999 \times (100\% - 4\%) \\ = \text{R6}\,999 \times 96\% \\ = \text{R6}\,999 \times \frac{96}{100} \\ = \text{R6}\,719,04$$

$$8. \text{ \% Profit} = \frac{\text{R2}\,400 - \text{R1}\,150}{\text{R1}\,150} \times 100 \\ = \frac{\text{R1}\,250}{\text{R1}\,150} \times 100 \\ = 108,7\%$$



$$9. \text{ Saturday's charge:}$$

$$\text{Increase} = \frac{50}{100} \times \text{R35,90} \\ = \text{R17,95}$$

$$\therefore \text{ Total} = \text{R35,90} + \text{R17,95} \\ = \text{R53,85}$$

OR

$$\text{Total} = \text{R35,90} \times (100\% + 50\%) \\ = \text{R35,90} \times 150\% \\ = \text{R35,90} \times \frac{150}{100} \\ = \text{R53,85}$$

$$\text{Friday's labour} = \text{R35,90} \times 6 \text{ h} = \text{R215,40}$$

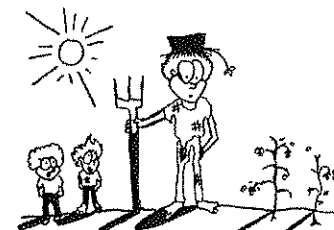
$$\text{Saturday's labour} = \text{R53,85} \times 6 \text{ h} = \text{R323,10}$$

$$\therefore \text{ Total labour cost} = \text{R215,40} + \text{R323,10} \\ = \text{R538,50}$$

$$\therefore \text{ His invoice of R638,50 was incorrect.}$$

$$10. \text{ Area used for farming} = \frac{80}{100} \times 1\,219\,090 \text{ km}^2 \\ = 975\,272 \text{ km}^2$$

$$\therefore \text{ Area suitable for crops} = \frac{11}{100} \times 975\,272 \text{ km}^2 \\ = 107\,279,92 \text{ km}^2$$



MODULE 2

PATTERNS, RELATIONSHIPS AND REPRESENTATIONS

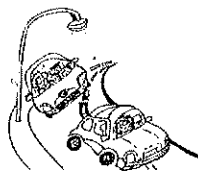
UNIT 1

1

MAKING SENSE OF GRAPHS THAT TELL A STORY

- 1.1 Independent variable: Time
Dependent variable: Distance
- 1.2 The flat (horizontal) part of the graph represents the time when the North family stopped travelling (as the distance remains the same at 480 km); possibly due to the family taking a break.
- 1.3 The family travelled the fastest between 6 - 8,65 hours. The slope of the straight line graph is the steepest during 6 - 8,65 hours; and since the slope represents $\frac{\text{distance}}{\text{time}}$, the speed of the family was greatest then.

$$\begin{aligned} \text{1.4 Speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{905 - 480 \text{ km}}{2,65 \text{ h}} \\ &= \frac{425 \text{ km}}{2,65 \text{ h}} \\ &= 160,38 \text{ km/h} \end{aligned}$$



∴ The North family was speeding.

- 2.1 Time 2.2 2 km
- 2.3 1h15 mins = 60 + 15 = 75 minutes
- 2.4 No, because Bongani's distance constantly increased as the time went on. **OR**
No, there is no flat/horizontal straight line on the graph.

2.5 First 15 min: Speed = $\frac{\text{distance}}{\text{time}}$

$$\begin{aligned} &= \frac{2 \text{ km}}{15 \text{ min}} \\ &= \frac{2 \text{ km}}{0,25 \text{ h}} \\ &= 8 \text{ km/h} \end{aligned}$$



$$\frac{15}{60 \text{ min}} = 0,25 \text{ h}$$

Rest of race (07:15 - 09:00):

$$\begin{aligned} \text{Speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{19,1 \text{ km}}{1\text{h}45 \text{ min}} \\ &= \frac{19,1 \text{ km}}{1,75 \text{ h}} \\ &= 10,91 \text{ km/h} \end{aligned}$$

$$\begin{aligned} 21,1 - 2 \text{ km} &= 19,1 \text{ km} \\ 09:00 - 07:15 &= 1 \text{ hr } 45 \text{ min} \\ \frac{60 + 45}{60 \text{ mins}} &= \frac{105}{60 \text{ mins}} \\ &= 1,75 \text{ hours} \end{aligned}$$

Bongani ran at a slower pace for the first 15 min of the race, after which he increased his run rate to 10,91 km/h. A possible reason for this could be that he first wanted to see the pace of the other runners in the race or that he was just warming his muscles for the first 15 minutes.

- 3.1 Richard's hand is 1 metre from the ground.
- 3.2 4 seconds; 17 metres
- 3.3 8 seconds
- 3.4 He caught the ball. The height at 8 seconds was 1 metre.



UNIT 2

2

PATTERNS AND RELATIONSHIPS



Constant/Fixed Relationship

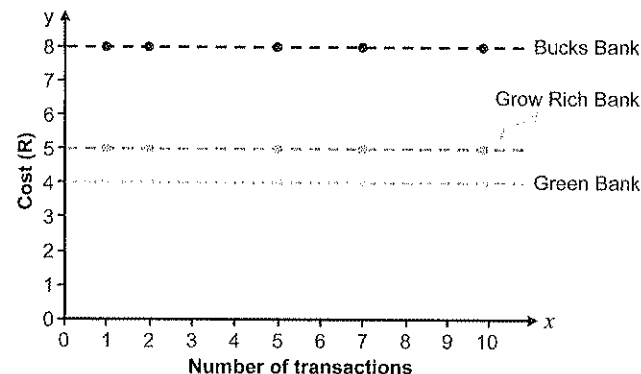
- 1.1 A = R5
B = R8
C = R4
D = 8 **OR** 9 transactions
- 1.2 Costs at Grow Rich Bank = R5
Costs at Bucks Bank = R8
Costs at Green Bank = R4



General formula for a constant/fixed relationship:
Dependent variable = constant/fixed value

1.3

BANK CHARGES AT DIFFERENT BANKS



- 2.1 Constant/fixed relationship
- 2.2 Cost at Hacker's Golf Range = R75

2.3 R100

2.4 60 golf balls



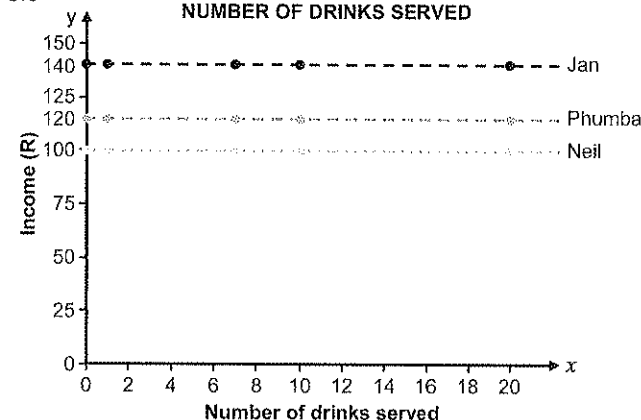
Since the barmen are paid per shift, their income will be the same, regardless of the number of drinks they serve, i.e. Phumba will earn R120 if he serves 0 or 100 drinks!

3.1 Neil's income = R100

Number of drinks served	0	1	5	10	20
Phumba's income (R)	120	120	120	120	120
Neil's income (R)	100	100	100	100	100
Jan's income (R)	140	140	140	140	140

3.3

BARMEN'S INCOME BASED ON NUMBER OF DRINKS SERVED



3.4 R120

3.5 R140



Linear Relationship in Direct Proportion



1.1 Number of cupcakes	0	10	20	30	40	50	60	70
Income (R)	0	50	100	150	200	250	300	350

1.2 Ashley's income = R5 × number of cupcakes sold

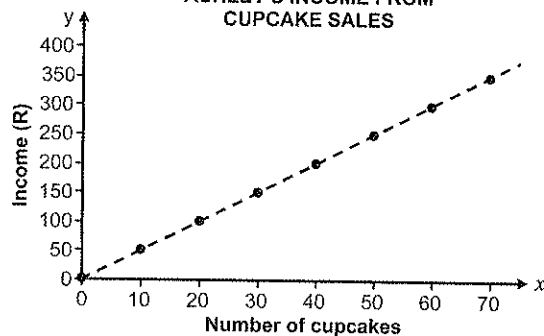


General formula for a linear relationship in direct proportion:

Dependent Variable = Constant Ratio × Independent Variable

1.3 Ashley's income = R5 × 120 = R600

1.4 **ASHLEY'S INCOME FROM CUPCAKE SALES**



2.1 Distance covered = 500 m × number of minutes

2.2 A = 1 500 m

B = 2 500 m

C = 6 minutes

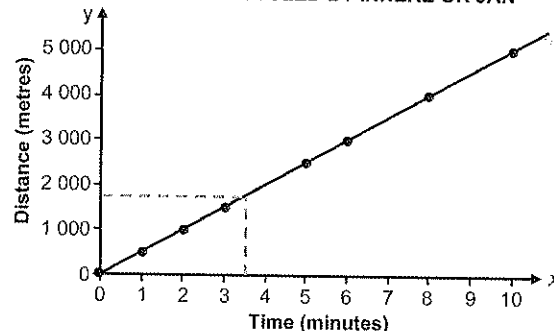
D = 8 minutes



Finding the missing values can either be done by inspection (i.e. following the pattern) or by substituting into your formula.

2.3

DISTANCE CYCLED BY INNEKE OR JAN



The straight line is a solid line as the data is continuous (i.e. time = independent variable is continuous).

2.4 1 750 metres

3.1 A linear relationship in direct proportion

It is an increasing straight line graph that starts at the origin.



3.2 Number of grams = 28,35 × number of ounces



Constant ratio is illustrated at 1 ounce = 28,35 g

3.3 113,4 g

3.4 6 ounces



Linear Relationship not in Direct Proportion



1.1 Number of hours	0	1	2	3	4	8	10
Total cost (R)	500	600	700	800	900	1 300	1 500

1.2 Total costs = (R100 × number of hours) + R500



General formula for a linear relationship not in direct proportion:

Dependent Variable = (Constant Difference × Independent Variable) + Starting Value

1.3 Total costs = (R100 × 9) + R500
= R1 400

Substitute into the formula and solve for the unknown.

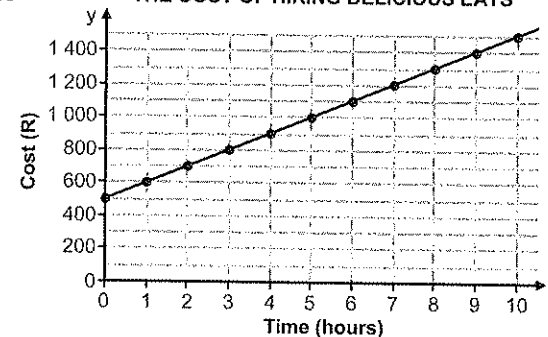
1.4 R1 200 = (R100 × number of hours) + R500

R700 = R100 × number of hours

∴ Number of hours = R700 ÷ R100
= 7 hours

1.5

THE COST OF HIRING DELICIOUS EATS



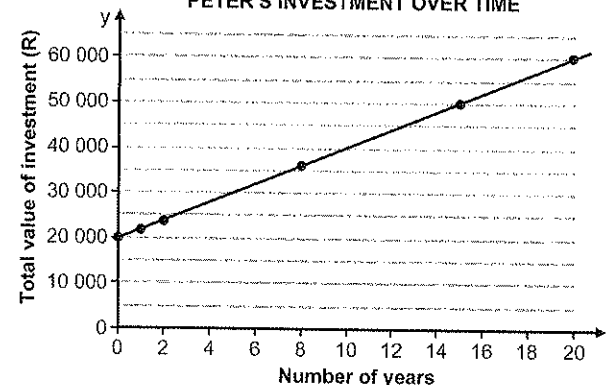
2.1 A = 20 000 ; B = R24 000 ; C = 15 years

2.2 Total value of investment

= (R2 000 × number of years) + R20 000

2.3

PETER'S INVESTMENT OVER TIME



3.1 Linear relationship that is not in direct proportion

This is an increasing straight line graph that starts at a point on the y-axis, but not at the origin.



3.2 105°F

3.3 20°C

Copyright © The Answer: Photocopying of this material is illegal

3.4 Temperature in Fahrenheit
 $= (1,8 \times \text{Temperature in Celsius}) + 30$

The general formula for a linear relationship that is not in direct proportion is:

Dependent Variable = (Constant Difference $\times$ Independent Variable + Starting Value



Indirect/Inverse Proportion Relationship



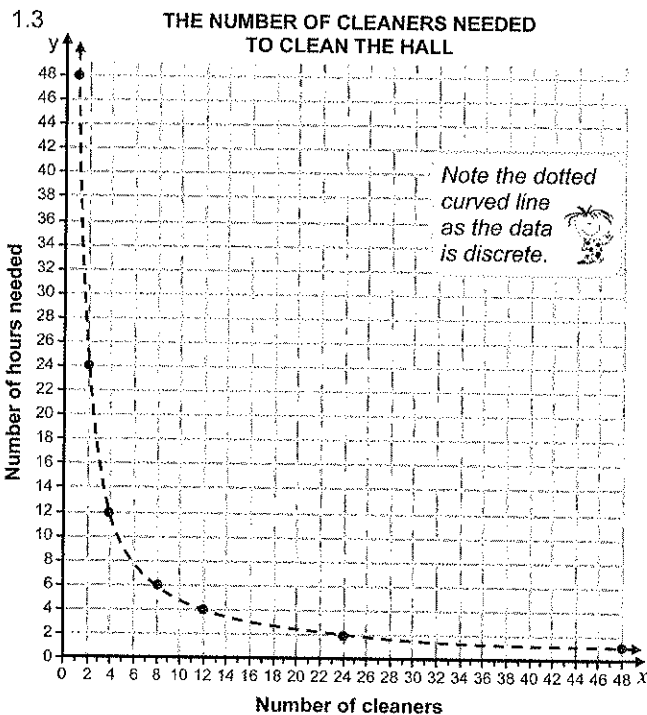
1.1	Number of cleaners	1	2	4	8	12	24	48
	Number of hours	48	24	12	6	4	2	1

1.2 Number of cleaners $\times$ Number of hours = 48
 $\therefore$ Number of cleaners = $48 \div \text{Number of hours}$



The general formula for an indirect/inverse proportion graph is:

Independent Variable $\times$ Dependent Variable = Constant Product



1.4 16 cleaners

1.5 Approximately 2,5 hours

1.6 Indirect/inverse proportion relationship

2.1 travel time $\times$ speed = 520

To find the constant product, multiply the independent variable by the dependent variable, i.e. $100 \times 5,2 = 520$.



2.2 $A \times 65 = 520$

$A = 8$ hours

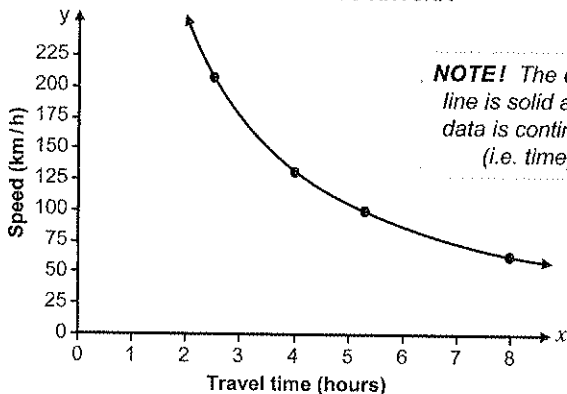
$4 \times B = 520$

$B = 130$ km/h

Substitute into your formula to solve for the missing values



2.3 THE TRAVEL TIME TO KNYSNA



NOTE! The curved line is solid as the data is continuous (i.e. time).



2.4 Speed = $\frac{\text{distance}}{\text{time}}$

$100 = \frac{\text{distance}}{5,2}$

$\therefore 100 \times 5,2 = \text{Distance}$

$\therefore \text{Distance} = 520$ km



3.1 5 people

3.2 R125

3.3 number of people per car $\times$ cost per person = R250



Constant Ratio Relationship



1.1 Year 1 = starting value
 = R5,20

Year 2 = $(7,1\% \times \text{year 1}) + \text{year 1}$
 $= \left(\frac{7,1}{100} \times 5,20 \right) + 5,20$
 = R5,57

Year 3 = $(7,1\% \times \text{year 2}) + \text{year 2}$
 $= \left(\frac{7,1}{100} \times 5,57 \right) + 5,57$
 = R5,97

Calculate 7,1% of the previous year and add to the previous value.



Year 4 = $(7,1\% \times \text{year 3}) + \text{year 3}$
 $= \left(\frac{7,1}{100} \times 5,97 \right) + 5,97$
 = R6,39

Year 5 = $(7,1\% \times \text{year 4}) + \text{year 4}$
 $= \left(\frac{7,1}{100} \times 6,39 \right) + 6,39$
 = R6,84



Year 6 = $(7,1\% \times \text{year 5}) + \text{year 5}$
 $= \left(\frac{7,1}{100} \times 6,84 \right) + 6,84$
 = R7,33

Year	1	2	3	4	5	6
Price of butter (R)	5,20	5,57	5,97	6,39	6,84	7,33

1.2 Constant Ratio = $\frac{5,57}{5,20} = 1,071$

Constant Ratio = $\frac{\text{Term 2}}{\text{Term 1}}$
 = $\frac{\text{Term 3}}{\text{Term 2}}$

Constant Ratio = $\frac{5,97}{5,57} = 1,071$

1.3 Year 7 = year 6 $\times$ constant ratio
 = $7,33 \times 1,071$
 = R7,85

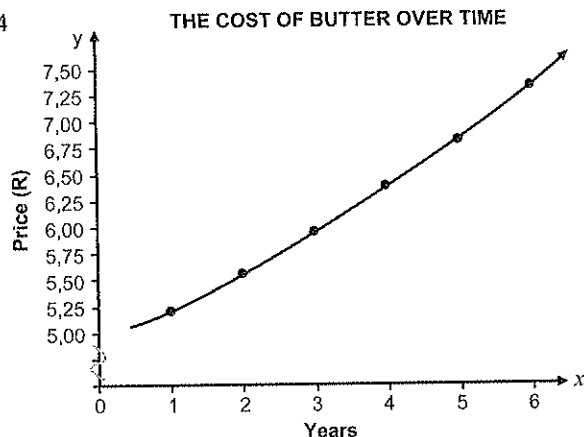


Year 8 = year 7 $\times$ constant ratio
 = $7,85 \times 1,071$
 = R8,41

Year 9 = year 8 $\times$ constant ratio
 = $8,41 \times 1,071$
 = R9,01

$\therefore$ The price of butter in 9 years' time will be R9,01

1.4



2.1 R500

2.2 $R3\ 000 - R500 = R2\ 500$

Investment after 30 years = R3 000;
 Initial deposit = R500
 $\therefore$ Growth = $R3\ 000 - R500$



2.3 ≈ 35 years

2.4 After 40 years the value of the investment is approximately R5 100,00

Original investment = R500,00

Value of investment after 40 years $\approx$ R5 100,00

Value of growth = $R5\ 100,00 - R500,00 = R4\ 600,00$

$$\begin{aligned} \% \text{ growth} &= \frac{\text{growth}}{\text{original investment}} \times 100\% \\ &= \frac{R4\ 600,00}{R500,00} \times 100\% \\ &\approx 920\% \end{aligned}$$

Angela's investment would have grown by 920% after 40 years.



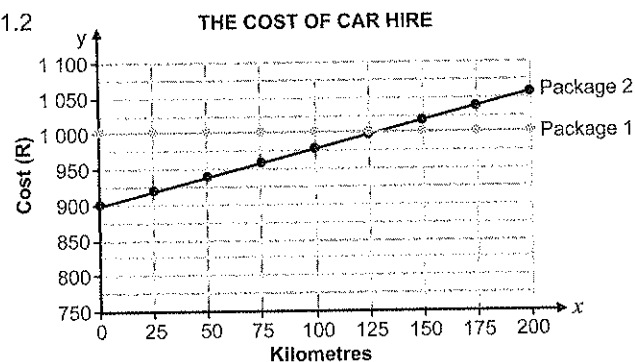
Combination of the different types of relationships



1.1

Number of kilometres	Cost of Package 1	Cost of Package 2
0	1 000	900
25	1 000	920
50	1 000	940
75	1 000	960
100	1 000	980
125	1 000	1 000
150	1 000	1 020
175	1 000	1 040
200	1 000	1 060

1.2



1.3 Package 2 has a constant difference of R20 for every 25 km.

$$\begin{aligned} \therefore 225 \text{ km} &= \text{cost of } 200 \text{ km} + R20 \\ &= 1\ 060 + 20 \\ &= R1\ 080 \end{aligned}$$

$$\begin{aligned} \therefore 250 \text{ km} &= \text{cost of } 225 \text{ km} + R20 \\ &= 1\ 080 + 20 \\ &= R1\ 100 \end{aligned}$$

1.4 125 km

Where the graphs intersect, the cost of both packages is the same.

2.1.1 A - linear relationship in direct proportion
 B - linear relationship not in direct proportion

2.1.2 A - Daniel's income B - Gemma's income

2.2 Gemma earns more money.

2.3 Point P represents when both Daniel and Gemma earn the same amount of money.

2.4 P(7 hours; R60)

REMEMBER! The question asked for coordinates - so first the x and then the y values!



2.5 Daniel: $w = R8,50 \times t$
 Gemma: $w = (R4,50 \times t) + R30$

2.6 Daniel: $w = R8,50 \times 15$
 = R127,50

Gemma: $w = (R4,50 \times 15) + R30$
 = R97,50

$\therefore$ Daniel earned more money by:
 $R127,50 - R97,50 = R30$

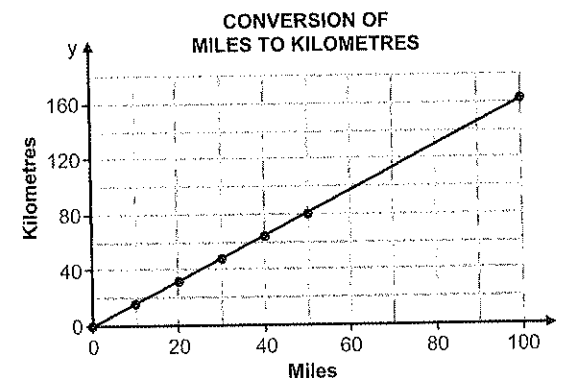


Test Your Understanding



Miles	0	10	20	30	40	50	100
Kilometres	0	16,1	32,2	48,3	64,4	80,5	161

1.2



1.3 Linear relationship in direct proportion.

Copyright © The Answer: Photocopying of this material is illegal

1.4 Number of kilometres = $1,61 \times$ number of miles

The general formula for a linear relationship that is in direct proportion is:

Dependent Variable =
(Constant Ratio $\times$ Independent Variable)



1.5 Number of kilometres
= $1,61 \times 205$
= 330,05

2.1 $A = 330 \div 2$
= 165 g

$B = 330 \div 55$
= 6 servings

This is an inverse/indirect proportion relationship with a constant product of 330.



2.2 Number of servings per day

2.3 Discrete data, as the independent variable (number of servings per day) is discrete (i.e. you can't have $2\frac{3}{4}$ servings!).

2.4 An inverse/indirect proportion relationship

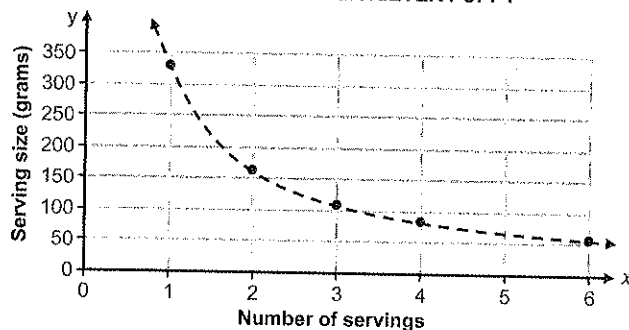
2.5 Number of servings per day $\times$ serving size = 330

General formula for an indirect/inverse proportion relationship:

Independent Variable $\times$ Dependent Variable = Constant Product



2.6 THE NUMBER OF SERVINGS VS SERVING SIZE FOR A GOLDEN RETRIEVER PUPPY

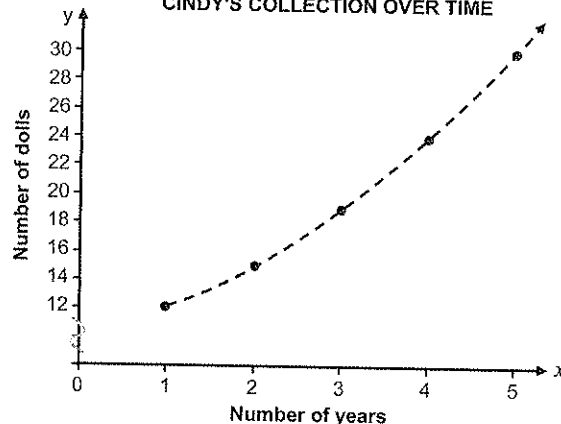


3.1	Year	1	2	3	4	5
	Number of dolls	12	15	19	24	30

3.2 Constant ratio relationship

3.3

THE NUMBER OF BARBIE DOLLS IN CINDY'S COLLECTION OVER TIME



4.1.1 A - indirect/inverse proportion relationship
B - constant/fixed relationship

4.1.2 A - bakkie ; B - minivan

4.1.3 A: Cost per person $\times$ Number of people = R2 400
B: Cost per person = R1 200

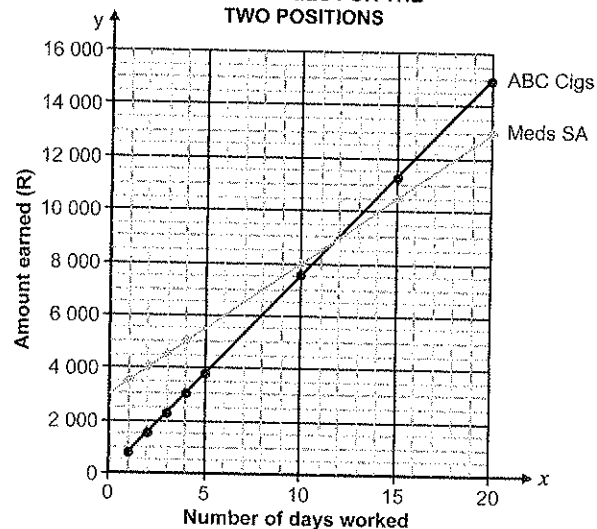
4.2.1 R1 200 per person 4.2.2 R800 per person

4.3 (2 people; R1 200) 4.4 3 people

5.1 Salary = R750 $\times$ number of days worked

5.2

THE SALARIES FOR THE TWO POSITIONS



5.3.1 Reading where the two graphs cross each other = 12 days

5.3.2 18 days at Med SA = R12 000
R12 000 at ABC Cigs = 16 days

6.1 $A = 160 \times R120 = R19\ 200$

$B = (12\ 000 - 3\ 000) \div 90 = 100$ tickets

$C = 150 \times R150 = R22\ 500$

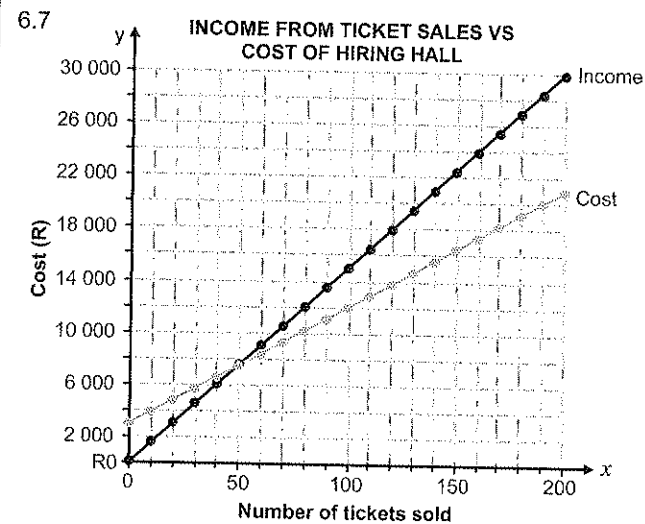
6.2 100 tickets

6.3	Cost of Impala Conference Centre	:	Cost of Community Hall
	24 000	:	21 000
	8	:	7

6.4 Cost (c)

6.5 Discrete data

6.6 Linear relationship in direct proportion, because there is a constant difference of R150 and the graph starts at the origin (0; 0).



6.8.1 50 tickets

6.8.2 120 tickets

6.8.3 R20 000



MODULE 3

FINANCE

UNIT 1

FINANCIAL DOCUMENTS

- 1.1 Parklands Junction
- 1.2 VAT-inclusive items are indicated with an 'A'
- 1.3 A plastic bag
- 1.4 Mrs Vosloo paid with her ABSA debit card
- 1.5 Weight = $R46,12 \div R78,99/\text{kg} = 0,58 \text{ kg}$ or 580 g
- 1.6 VAT-exclusive price = $R224,39 - R17,86 = R206,53$
- 1.7 $R224,39 \approx R224,35$



REMEMBER! South Africa doesn't have 1c and 2c pieces
∴ we round down.

- 2.1 R1 172,33
- 2.2 Customer paid extra = $R3\,700 - R1\,172,33 = R2\,527,67$
- 2.3 Charge per square metre = $\frac{R1\,646,76}{920\text{m}^2} = R1,79/\text{m}^2$
- 2.4 Account is in credit, as the total due is a negative value (i.e. -R881,00)
- 3.1 Invoice number: 10100005
Invoice date: 19 October 2010
- 3.2 $R3\,480 \div R580 = 6$
- 3.3 $R6\,580 \div 4 = R1\,645$
- 3.4 Total cost = $R7\,100 \times 9 = R63\,900$



- 3.5 Subtotal = $R3\,480 + R6\,580 + R12\,300 + R63\,900 = R86\,260$
∴ VAT = $\frac{14}{100} \times R86\,260 = R12\,076,40$
- 3.6 Cost of shipping = 5,5% of subtotal + R1 520
= $\frac{5,5}{100} \times R86\,260 + R1\,520 = R4\,744,30 + R1\,520 = R6\,264,30$
- 3.7 Amount due = Subtotal + VAT + Shipping
= $R86\,260 + R12\,076,40 + R6\,264,30 = R104\,600,70$
- 4.1 Alexandra Schoeman
- 4.2 176 hours
- 4.3 Hourly rate = $R100\,000 \div 176 \text{ hours} = R568,18$
- 4.4 R10 410
- 4.5 UIF = $\frac{1}{100} \times R100\,000 = R1\,000$
∴ Under-paid by $R1\,000 - R124 = R876$
- 4.6 Net pay = $R100\,000 - R10\,410 \text{ (tax)} - R1\,000 \text{ (UIF)} = R88\,590$
Net pay =
Gross income - Deductions
5. Fixed cost of a R365 000 vehicle = R104 374 per annum
Fuel costs = $22\,000 \text{ km} \times 134,7 \text{ cents} = 2\,963\,400 \text{ cents} = R29\,634$
Maintenance costs = $22\,000 \text{ km} \times 67,7 \text{ cents} = 1\,489\,400 \text{ cents} = R14\,894$
∴ Total annual vehicle costs = $R104\,374 + R29\,634 + R14\,894 = R148\,902$
∴ Monthly travel allowance = $R148\,902 \div 12 = R12\,408,50$



- 6.1 Total loan length = $78 + 282 = 360 \text{ months} = 30 \text{ years}$

- 6.2 Total monthly payment = $R761,78 + R82,35 + R206,13 = R1\,050,26$



- 6.3 Property tax and insurance = $R967,91 - R761,78 = R206,13$

- 6.4 Total paid = $(R1\,050,26 \times 78) + (R967,91 \times 282) = R81\,920,28 + R272\,950,62 = R354\,870,90$

- 6.5 Interest = total paid back - loan amount = $R354\,870,90 - R150\,000 = R204\,870,90$
∴ % = $\frac{R204\,870,90}{R150\,000} \times 100 = 136,58\%$

UNIT 2

TARIFF SYSTEMS

- 1.1 053
- 1.2 Pietermaritzburg
- 1.3 R0,007
- 1.4 Ballito
- 1.5 $12 \text{ minutes} \times 60 = 720 \text{ seconds}$
∴ $12 \text{ minutes } 39 \text{ seconds} = 720 + 39 = 759 \text{ seconds}$
(021) number falls in category B (national call)
The call is made during callmore time (19:00 - 07:00)
∴ Cost of call = $759 \times R0,006 = R4,55$

- 1.6 The call was made at 18:53, it was 11 min 35 sec long and it was a local call (i.e. (033) number)

The call falls in standard time as well as callmore time. Calculate each part separately and then add together.

Standard time: 7 minutes = 420 seconds

$$\therefore \text{Cost of call during standard time} = 420 \times R0,007 = R2,94$$

Callmore time:

$$\begin{aligned} 11 \text{ min } 35 \text{ sec} - 7 \text{ min} &= 4 \text{ min } 35 \text{ sec} \\ 4 \text{ min } 35 \text{ sec} &= (4 \times 60) + 35 \\ &= 275 \text{ seconds} \end{aligned}$$

$$\therefore \text{Cost of call during callmore time} = 275 \times R0,003 = R0,83$$

$$\therefore \text{Total cost of call} = R2,94 + R0,83 = R3,77$$

- 2.1.1 28 kℓ **REMEMBER!** When you add up the kℓ of water used, you must get the total of the question, i.e. $6 + 7 + 15 = 28$ kℓ

- 2.1.2 Charge

$$= (R0,00 \times 6 \text{ kℓ}) + (R3,49 \times 7 \text{ kℓ}) + (R8,56 \times 15 \text{ kℓ})$$

$$= R0,00 + R24,43 + R128,40$$

$$= R152,83$$

- 2.1.3 VAT = $\frac{14}{100} \times R152,83 = R21,40$

$$\therefore \text{Grand Total Due} = R152,83 + R21,40 = R174,23$$

- 2.2 6 to 13 kℓ bracket:
 % increase in price from 2013/2014 to 2014/2015

$$= \frac{(R3,49 - R3,23)}{R3,23} \times 100$$

$$= \frac{R0,26}{R3,23} \times 100$$

$$= 8,05\%$$

 > 13 kℓ bracket:
 % increase in price from 2013/2014 to 2014/2015

$$= \frac{(R8,56 - R7,93)}{R7,93} \times 100$$

$$= \frac{R0,63}{R7,93} \times 100$$

$$= 7,94\%$$

$\therefore$ The 6 - 13 kℓ tariff bracket increased the most.

- 2.3 > The horizontal portion of the graph (from 0 kℓ to 6 kℓ) represents the free allocation of water. This portion is horizontal because there is no cost.
 > The second portion of the graph represents the cost of using between 6 kℓ and 13 kℓ of water, where the water tariff is R3,49/kℓ.
 > The third and steepest part of the graph represents the cost of using more than 13 kℓ of water. This portion of the graph is steeper than the other two portions since the cost of water in this bracket is more expensive than the other two brackets.

- 3.1 Single trip from Bellville to Cape Town:

> Bus = R20,40

> Taxi = R9,50 + R5,80
 = R15,30

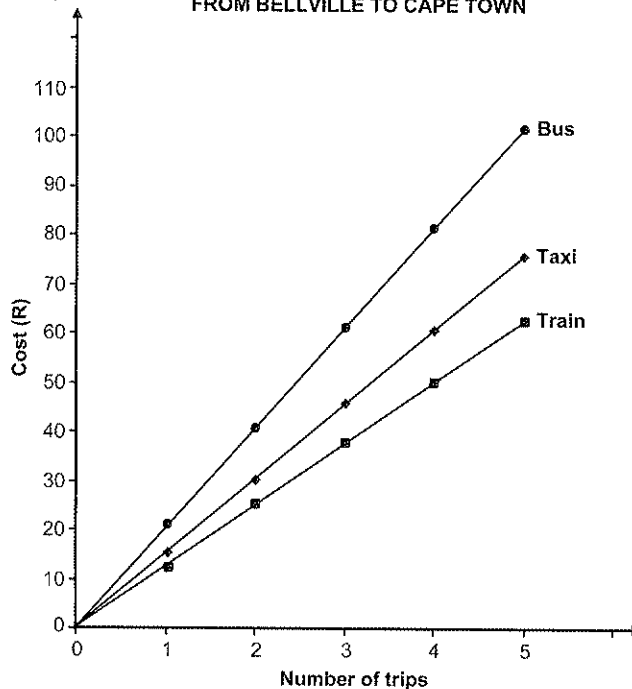
> Train = R25 ÷ 2
 = R12,50

$\therefore$ Train is cheapest.

2 taxis needed so add costs.

Train fare is a return trip (i.e. there and back) $\therefore$ divide by 2.

3.2
 TRANSPORT COSTS
 FROM BELLVILLE TO CAPE TOWN



4. Consumption for February
 = 377 957 - 377 449
 = 508 kWh

NOTE!
 377 449 is the current reading in January.

$$\begin{aligned} \text{Cost of electricity} &= 508 \text{ kWh} \times 31,5 \text{ c/kWh} \\ &= 16\,002 \text{ c} \\ &= R160,02 \end{aligned}$$

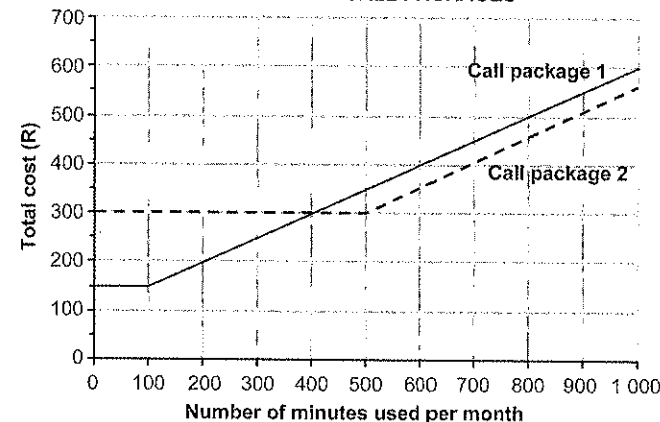
$$\text{Cost including VAT} = R160,02 \times \frac{114}{100} = R182,42$$

$$\begin{aligned} \text{Total amount due} &= 182,42 + 67,03 + 23,03 \\ &= R272,48 \end{aligned}$$

- 5.1 8:45 - 08:01 = 44 minutes
 This is less than 1 hour so the parking is free.
 17:30 - 12:15 = 5 hours 15 minutes
 $\therefore$ She pays R8 for parking.
- 5.2 13:25 - 08:45 = 4 hours 40 minutes
 $\therefore$ She should be charged the '3 to 5 hour' rate, which is R6.
 The charge of R8 is incorrect. She was overcharged by R2.
- 6.1.1 Total cost (in rand)
 = R300 + 0,50 × (number of minutes more than 500)
- 6.1.2 Total cost (in rand) = R300 + 0,50 × (510 - 500)
 = 300 + 0,50 × 10
 = 300 + 5
 = R305

- 6.2.1

LANDLINE CALL PACKAGES



6.2.2 400 minutes 6.2.3 Call package 2.

6.2.4 Call package 1.

7. Total monthly cost
 = service charge + network charge + energy charge
 = (R228,95 + R109,85)
 + (2 700 kWh × R0,5210/kWh)
 = R338,80 + R1 406,70
 = R1 745,50

REMEMBER! The energy charge is in cents,
 so divide by 100 to get it to Rands,
 i.e. 52,10c = R0,5210
 Don't round off until your final answer!

UNIT 3

INCOME, EXPENDITURE, PROFIT/LOSS, I & E STATEMENTS AND BUDGETS

1.1 A = Fixed Expenses B = Variable Expenses

1.2 20% of Salary expense = $\frac{20}{100} \times R45\ 000$
 = R9 000

∴ Advertising expense (C) = R45 000 - R9 000
 = R36 000

1.3 Stock Purchases (D)
 = R592 700 - R4 432,00 - R2 400,55 - R1 800,45 -
 R11 867,00
 = R572 200

1.4 Subtotal of Fixed Expenses (A)
 = R20 200 + R36 000(C) + R45 000 + R2 800
 = R104 000

Total expenses

= fixed expenses (A) + variable expenses (B)
 = R104 000 + R592 700
 = R696 700

∴ Profit = income - expenses
 = R855 200 - R696 700
 = R158 500

∴ Yes, they will make a profit.

$$\begin{aligned} 1.5 \text{ Profit Margin} &= \frac{\text{profit}}{\text{income}} \times 100 \\ &= \frac{R158\ 500}{R855\ 200} \times 100 \\ &= 18,53\% \end{aligned}$$

2.1 Possible running costs:

rental for business premises; telephone; electricity;
 water; maintenance costs; taxi services etc.

2.2 Total expenses = R325 400 + R100 000 + R50 000
 = R475 400

$$\begin{aligned} \% &= \frac{\text{salaries}}{\text{total expenses}} \times 100 \\ &= \frac{R100\ 000}{R475\ 400} \times 100 \\ &= 21,03\% \end{aligned}$$



2.3 Profit = income - expenses
 = R682 500 - R475 400
 = R207 100

∴ Yes, Fatima did make a profit.

2.4 February increase = $\frac{13,5}{100} \times R682\ 500 = R92\ 137,50$

∴ February's income = R682 500 + R92 137,50
 = R774 637,50

2.5 Profit margin for January = $\frac{\text{profit}}{\text{income}} \times 100$
 $= \frac{R207\ 100}{R682\ 500} \times 100$
 = 30,34%

Profit for February = income - expenses
 = R774 637,50 - R490 200
 = R284 437,50

∴ Profit margin for February = $\frac{\text{profit}}{\text{income}} \times 100$
 $= \frac{R284\ 437,50}{R774\ 637,50} \times 100$
 = 36,72%

∴ February had the higher profit margin.

3.1.1 R5 000 million

3.1.2 5,3%

3.1.3 2005/2006

3.1.4 Total income tax = 132 475 + 95 201 + 1 890
 = R229 566 million

3.1.5 R23 194 200 000

3.1.6 The value is negative because the amount
 decreased from 2005/2006 to 2006/2007.

3.1.7 Transfer duty decrease = 6 210 - 8 700
 = -2 490

$$\begin{aligned} &\frac{-2\ 490}{8\ 700} \times 100 \\ &= -28,6\% \end{aligned}$$



FIRST SUBTRACT the two quantities
 to calculate the increase/decrease.
 Then divide by the original amount
 and multiply by 100 to calculate 100%.

3.1.8 Total Customs and Excise
 = 18 600 + 15 799 + 20 700 + 1 231 - 14 145
 = R42 185 million

3.2 Profit/Loss = income - expenses
 = 446 356,2 - 495 247,7
 = -R48 891,5 million

∴ The government made a loss.

4.1 Amount budgeted on education in 2012/2013:

Total expenditure = R834,3 billion

% spent on education = 17%

∴ 17% of 834,3 billion = $834\ 300\ 000\ 000 \times \frac{17}{100}$
 = 141 831 000 000
 = R141,831 billion

Amount budgeted on education in 2013/2014:

Total expenditure = R900,9 billion

% spent on education

$$= 100 - (\text{sum of other \%s})$$

$$= 100 - (10 + 17 + 10 + 11 + 15 + 4 + 15)$$

$$= 100 - 82$$

$$= 18\%$$

$$\therefore 18\% \text{ of } 900,9 \text{ billion}$$

= amount budgeted on education

$$= 900\,900\,000\,000 \times \frac{18}{100}$$

$$= 162\,162\,000\,000$$

$$= R162,162 \text{ billion}$$

$\therefore$ Difference between budget for education in 2013/2014 and 2012/2013

$$= 162\,162\,000\,000 - 141\,831\,000\,000$$

$$= R20\,331\,000\,000$$

$$\approx R20,331 \text{ billion}$$

$\therefore$ The difference is more than R20 000 000 000 (i.e. R331 000 000)

4.2 $\triangleright$ There is a shortage of teachers in schools. More money should be spent on education so that more teachers can be employed. This will increase the literacy and matric pass rate in schools.

$\triangleright$ More money should be spent on education so that more textbooks and other learning aids can be given to schools. This will also increase the literacy and matric pass rate in schools.

5. BUDGET FOR MT TRADING'S MARKETING DAY

Income		Expenses	
Rental income (50 m ² $\times$ R80/m ²)	R4 000	Hall rental	R500
Sponsor donation ($\frac{15}{100} \times R4\,000$)	R600	Jumping castles (R275 $\times$ 2)	R550
		Security guards	R1 100
		Local band (R280 $\times$ 3 [30 min])	R840
Total	R4 600	Total	R2 990

$$\begin{aligned}\therefore \text{Profit} &= \text{income} - \text{expenses} \\ &= R4\,600 - R2\,990 \\ &= R1\,610\end{aligned}$$

$\therefore$ MT Trading is expected to make a profit of R1 610.

UNIT

4

COST PRICE AND SELLING PRICE

1.1 Profit = 30% of cost price

$$= \frac{30}{100} \times R1\,200$$

$$= R360$$

$\therefore$ Selling Price = cost price + profit

$$= R1\,200 + R360$$

$$= R1\,560$$

1.2 Cost Price = selling price - profit

$$= R2\,400 - R890$$

$$= R1\,510$$



2.1 Profit per set = R2 999,00 - R999,00

$$= R2\,000,00$$

Profit from 14 sets = R2 000,00/set $\times$ 14 sets

$$= R28\,000,00$$

2.2 Expenses on 14 sets = R999,00 $\times$ 14

$$= R13\,986,00$$

Income from one set = R2 999,00

$$\begin{aligned}\text{No. of sets needed to break even} &= \frac{R13\,986,00}{R2\,999,00} \\ &\approx 4,66 \text{ sets}\end{aligned}$$

Therefore 5 sets need to be sold in order to cover the costs, i.e. to break even.

2.3 Income from sale of discounted set

$$= R2\,999,00 - (R2\,999,00 \times 15\%)$$

$$= R2\,999,00 - R449,85$$

$$= R2\,549,15$$

$$\text{Profit} = R2\,549,15 - R999,00 = R1\,550,15$$

$$\text{Profit Margin} = \frac{\text{profit}}{\text{income}} \times 100\%$$

$$= \frac{R1\,550,15}{R2\,549,15} \times 100\%$$

$$\approx 60,81\%$$

Sbusiso would still maintain his 60% profit margin even if he sold the sets at a discounted price (but only just ☺).

3.1 Profit on key ring = R7,00 - R4,80 = R2,20

$$\% \text{ Profit on key ring} = \frac{R2,20}{R4,80} \times 100 = 45,83\%$$

Profit on coffee mug = R10,00 - R7,00 = R3,00

$$\% \text{ Profit on coffee mug} = \frac{R3,00}{R7,00} \times 100 = 42,86\%$$

$\therefore$ Key ring has the highest % profit.

3.2.1 Number of key rings sold

$$= (R175 + R385 + R259 + R231 + R259 + R259 + R322) \div R7$$

$$= R1\,890 \div R7$$

$$= 270$$



$$\begin{aligned}\text{Number of key rings} \\ &= \frac{\text{total income}}{\text{selling price}}\end{aligned}$$

Number of coffee mugs sold

$$= (R250 + R350 + R370 + R380 + R270 + R350 + R370) \div R10$$

$$= R2\,340 \div R10$$

$$= 234$$

$\therefore$ More key rings were sold by:
270 - 234 = 36 items.

3.2.2 Total profit on key rings = 270 $\times$ R2,20 = R594

$$\begin{aligned}\text{Profit} &= \text{selling price} - \text{cost price} \\ &\text{(see Answer 3.1)}\end{aligned}$$

$\therefore$ Profit margin on key rings for the week

$$= \frac{\text{profit for week}}{\text{income for week}} \times 100$$

$$= \frac{R594}{R1\,890} \times 100$$

$$= 31,43\%$$

Total profit on coffee mugs = 234 $\times$ R3,00 = R702

$\therefore$ Profit margin on coffee mugs for the week

$$= \frac{\text{profit for week}}{\text{income for week}} \times 100$$

$$= \frac{R702}{R2\,340} \times 100$$

$$= 30\%$$



$\therefore$ The key rings had the biggest profit margin for the week.

BREAK-EVEN ANALYSIS

1.1.1 Fixed costs = R3 450 + R4 000 = R7 450

1.1.2 Variable costs = 20 + 50 + 1 000 + 780 = R1 850

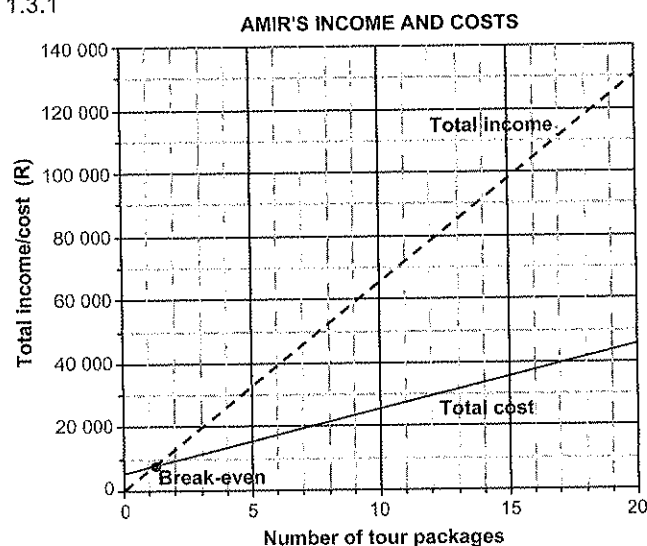
1.1.3 Total Expenditure
= variable costs + fixed costs
= R1 850 × number of tour packages + R7 450

- 1.1.4 a) R7 450 c) R16 700
b) R9 300 d) R44 450

1.2.1 Total income = R6 500 × number of tour packages

- 1.2.2 a) 0 c) R32 500
b) R6 500 d) R130 000

1.3.1



1.3.2 Break-even point on the graph is
≈ 1,6 tour packages

∴ Amir must sell at least 2 packages to break-even



REMEMBER! Discrete data must be rounded up in order to at least break even.

1.3.3 Profit = income - expenses
= R40 000 - R20 000
= R20 000



2.1.1 ≈ R205

2.1.2 Rent, salaries/wages, loan repayments, vehicle repayments, insurance, etc.

2.2 Reading from the graph:
2 chairs cost R100,00
→ 1 chair costs R100,00 ÷ 2 = R50,00

2.3 According to the graph, the break-even point (the point where the graphs intersect) is 6½ chairs. This means that Happiness must sell more than 6 chairs – i.e. 7 chairs – in order to break even.

2.4 Income from 12 chairs ≈ R540,00

2.5 Profit from selling 15 chairs = income - expenses
≈ R680,00 - R400,00
≈ R280,00

2.6 The Income graph would be affected and would become steeper because his income per chair would be more. The Expenses graph would not be affected as the only thing that is increasing is the selling price (and hence the income).

3.1 Fresh Vegetables = R70,00
Packets = R12,00 ÷ 14 days
= R0,86 or 86c per day

Transport = R22,00
Permit = R280 ÷ 365 days
= R0,77 or 77c per day

Total daily expenses
= R70,00 + R0,86 + R22,00 + R0,77
= R93,63

3.2 Weekly income
= R15,00/packet × 12 packets/day × 7 days
= R1 260,00

3.3 To break even Joyce must cover all of her expenses.
(R93,63 per day)

So: No. of packets that must be sold to break even
= R93,63 ÷ R15,00/packet
= 6,242.

This means that Joyce must sell just more than 6 packets per day to break even. Therefore, she must sell 7 packets.

3.4 Profit = income - expenses
∴ Income = expenses + profit

To make R200 profit:
Income = expenses + R200,00
= R93,63 + R200,00
= R293,63



So, to make R200 profit Joyce must sell them at:
R293,63 for 15 packets ÷ 15 packets
≈ R19,58
≈ R20,00 per packet

4.1



$Time = \frac{distance}{speed} = \frac{600 \text{ km}}{80 \text{ km/h}}$
= 7,5 h
= 7 hours 30 minutes



Number of hours	0	1	2	3	4	7,5
Distance from Bloemfontein (km)	0	80	160	240	320	600

Distance = speed × time
= 80 km/h × 1 h
= 80 km

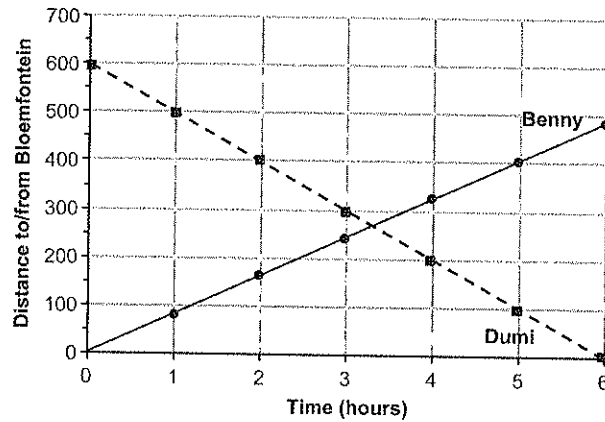
Distance = speed × time
= 80 km/h × 3 h
= 240 km

4.2

Number of hours	0	1	2	3	4	6
Distance to Bloemfontein (km)	600	500	400	300	200	0

4.3

DISTANCE TRAVELLED OVER TIME



4.4 Approximately 3,25 h = 3 hours 15 minutes

Benny and Dumi pass each other where the two lines intersect because that is when they are the same distance from Bloemfontein.



4.5 3,25 h = 3 h and 15 min ($0,25 \times 60 = 15$ minutes)
They both left at 6:00
 $\therefore$ 6:00 + 3 h and 15 min = 9.15 am

UNIT

6

INTEREST

1. Bank A: Yr 1 interest = $\frac{6}{100} \times R250\,000$
= R15 000

Yr 1 total = R250 000 + R15 000
= R265 000

Yr 2 interest = $\frac{6}{100} \times R265\,000$
= R15 900

Yr 2 total = R265 000 + R15 900
= R280 900

Bank B: Interest = $\left(\frac{5,8}{100} \times R250\,000\right) \times 3$ years
= R14 500 $\times$ 3
= R43 500

$\therefore$ Total after 3 years = R250 000 + R43 500
= R293 500

$\therefore$ The owner should invest with Bank B, as his return on his investment will be greater.

2.1 Interest = $\left(\frac{10,5}{100} \times R7\,400\right) \times \frac{3}{12}$ years
= R777 $\times \frac{1}{4}$ year
= R194,25

$\frac{3}{12}$ is a fraction of a year; since the interest rate is always given as per annum/year.

2.2 Total = R7 400 + R194,25
= R7 594,25

3. Yr 1 interest = $\frac{11}{100} \times R10\,000 = R1\,100$

Yr 1 total = R10 000 + R1 100 = R11 100

Yr 2 interest = $\frac{11}{100} \times R11\,100 = R1\,221$

Yr 2 total = R11 100 + R1 221 = R12 321

Yr 3 interest = $\frac{11}{100} \times R12\,321 = R1\,355,31$

Yr 3 total = R12 321 + R1 355,31 = R13 676,31

$\therefore$ Profit = R13 676,31 - R10 000 = R3 676,31

Profit in this context = Investment Total - Initial Investment/Principal Amount

4.1 Deposit = $\frac{15}{100} \times R3\,999 = R599,85$

Outstanding balance = R3 999 - R599,85
= R3 399,15

Interest charged = $\left(\frac{18}{100} \times R3\,399,15\right) \times 3$ yrs
= R611,85 $\times$ 3
= R1 835,54

Total repayment amount = R3 399,15 + R1 835,55
= R5 234,70

$\therefore$ Monthly instalments = R5 234,70 $\div$ (3 yrs $\times$ 12)
= R5 234,70 $\div$ 36 months
= R145,41

$\therefore$ Total monthly instalment
= R145,41 + R30 (insurance)
= R175,41

4.2 Total paid
= deposit + total monthly instalments for 36 months
= R599,85 + (R175,41 $\times$ 36)
= R599,85 + R6 314,76
= R6 914,61

4.3 Extra cost = HP total paid - cash price
= R6 914,61 - R3 999
= R2 915,61

5. Interest for 4 yrs = R17 160 - R13 000 = R4 160
Interest per year = R4 160 $\div$ 4 years = R1 040

$\therefore$ Interest rate = $\frac{R1\,040}{R13\,000} \times 100$
= 8% p.a.

REMEMBER! Interest rate = % increase

6.1 Graph A: straight line graph
Graph B: exponential graph

6.2 Graph A: simple interest
Graph B: compound interest



6.3 R12 000

6.4 Year 1

6.5 Plan B because her investment growth will be more due to compounded interest.

6.6 Interest after 1 Year = R13 200 - R12 000
= R1 200

$\therefore$ Interest Rate = $\frac{R1\,200}{R12\,000} \times 100 = 10\%$ p.a.



Interest rate = % Increase
 $\therefore \frac{\text{increase}}{\text{initial value}} \times 100$

$$1.1.1 \text{ Total stokvel payout} = R750 \times 20 \text{ members} \\ = R15\,000$$

$\therefore$ No, Pieter won't be able to afford the tractor.

$$1.1.2 \text{ Loan amount} = R21\,000 - R15\,000 = R6\,000$$

$$\% = \frac{R6\,000}{R21\,000} \times 100 \\ = 28,57\%$$

$$1.2.1 \text{ Monthly payout} \\ = (R800\,000 \div 12 \text{ months}) \div 12 \text{ years} \\ = R5\,555,55$$

1.2.2 No, as Pieter needs R6 000 in addition to his stokvel payout and the monthly retirement annuity payout is only R5 555,55. Therefore, Pieter still needs R444,44.

$$2.1.1 \text{ R1 902,96}$$

$$2.1.2 \text{ Total ATM withdrawals} = R1\,000,00 + R880,00 \\ = R1\,880,00$$

$$2.1.3 \text{ Missing account balance on 21 Feb} \\ = R2\,227,96 - R5,59 \\ = R2\,222,37$$

$$2.1.4 \text{ Car insurance} = R4\,452,96 - R3\,800,96 = R652,00$$

$$2.2.1 \text{ Transaction fee} \\ = R2,50 + 0,95\% \text{ of the amount deposited} \\ = R2,50 + (0,95\% \times R325,00) \\ = R2,50 + (0,0095 \times R325,00) \\ = R2,50 + R3,0875 \\ = R5,59$$

rounded off to 2 decimal places

$$2.2.2 \text{ Transaction fee} \\ = R0,90 \text{ for every R100 (or part thereof) deposited} \\ \text{Amount deposited} = R325,00 \\ \text{Transaction fee} \\ = [(R0,90 \times 3) \text{ for the first R300,00}] + (R0,90 \text{ for the last R25,00}) \\ = R3,60$$

$$2.2.3 \text{ Solly would have saved } R5,59 - R3,60 = R1,99 \\ \text{by depositing the money at the ATM.}$$

$$2.3.1 \text{ R8,70}$$

$$2.3.2 \text{ R7,20}$$

2.3.3 The equation for Graph 1 is:

$$\text{Fee} = R2,50 + 0,95\% \text{ of deposit} \\ = R2,50 + 0,0095 \times \text{deposit}$$

This is a graph that will increase by a value of 0,0095 for every R1,00 deposited; or by 0,95 for every R100 deposited. So, there is a constant increase or a direct proportion between the independent and dependent variables.

2.3.4 If you deposit no money then there is no transaction fee. If the graph started at R2,50 on the vertical axis this would imply that for a deposit of R0,00 the fee would be R2,50.

2.3.5 The fee for depositing money at an the ATM is R0,90 for every R100,00 or part thereof. This means that for anything less than or equal to R100,00 the fee is R0,90. For any deposit of an amount between R100,01 and R200,00 the fee is another R0,90, bringing the total fee to R1,80.

The stepped graph shows how the fee jumps for every additional R100,00 deposited. Each step is straight to show that if you deposit R50,00 or R80,00 or R100 the fee is the same because of the 'or part thereof' clause.

2.3.6 No, it is always better to deposit money at an ATM. i.e. Graph 2 is always below Graph 1, so no matter what amount you are depositing, it will always be cheaper to deposit it at an ATM than at the branch.



A16

2.4

ARO BANK – ACCOUNT FEES

Type of Transaction	Fee Formula	Fee on Solly's Transactions in February
ATM Withdrawal	R2,50 for the first R100 plus R1,10 for every additional R100 (or part thereof).	ATM withdrawal 1 = R1 000 Fee = R2,50 + (R1,10 × 9) = R2,50 + R9,90 = R12,40 ATM withdrawal 2 = R880,00 Fee = R2,50 + (R1,10 × 8) = R2,50 + R8,80 = R11,30
Debit Order	R5,00 per debit order	2 debit orders $\therefore$ Fee = 2 × R5,00 = R10,00
Stop Order	R5,00 per stop order	1 stop order $\therefore$ Fee = R5,00
Debit Card Purchase	R2,30 per purchase	1 × debit card purchase $\therefore$ Fee = R2,30
Cash deposit at Branch	R2,80 + 0,90% of the amount deposited	1 × cash deposit at branch Fee = R2,80 + 0,90% × deposit = R2,80 + 0,009 × R325,00 = R2,80 + R2,925 = R5,73
Monthly service fee	R6,70	1 × monthly fee $\therefore$ Fee = R6,70

rounded off to 2 decimal places

$$\therefore \text{Total ARO Bank transaction fees} \\ = R12,40 + R11,30 + R10,00 + R5,00 + R2,30 + R5,73 + R6,70 \\ = R53,43$$

$$\text{Total BRO Bank transaction fees} = R45,29$$

$\therefore$ Solly would be better off staying with BRO bank.

$$3.1 \text{ R478,00}$$

$$3.2 \text{ R800,00}$$

$$3.3 \text{ Interest} = R119,50 - R100 = R19,50$$

$$\therefore \text{Interest Rate} = \frac{R19,50}{R100} \times 100 = 19,5\%$$



REMEMBER!

Interest rate = % increase

$$\therefore \text{Interest rate} = \frac{\text{interest after 1 month}}{\text{loan amount}} \times 100$$

Copyright © The Answer: Photocopying of this material is illegal

4. **BNF:** Monthly repayment
 $= (R975\ 000 \div 1\ 000) \times 13,37$
 $= R13\ 035,75$
- ASBA:** Monthly repayment
 $= (R975\ 000 \div 1\ 000) \times 14,34$
 $= R13\ 981,50$
- $\therefore$ Callan would get the lowest monthly repayments with BNF bank.

- 5.1.1 R1 000,00
- 5.1.2 Interest rate $= (R90,00 \div R1\ 000,00) \times 100\%$
 $= 9\%$
- 5.1.3 Closing Balance after Payment
 $= \text{balance before payment} - \text{payment}$
 $= R607,63 - R225,00$
 $= R382,63$
- 5.1.4 The outstanding balance in December is R209,35. So, to pay off the loan a payment of R209,35 and not R225,00 is necessary.
- 5.1.5 Real cost of the loan $= (R225,00 \times 5) + R209,35$
 $= R1\ 125,00 + R209,35$
 $= R1\ 334,35$

5.2.1 Interest rate $= (R71,79 \div R717,85) \times 100 = 10\%$

5.2.2

End of month	Interest	Balance before payment	Payment	Closing balance after payment
				R1 000
July	R90,00	R1 090,00	R225,00	R865,00
Aug	R77,85	R942,85	R225,00	R717,85
Sept	R71,79	R789,64	R225,00	R564,64
Oct	R56,46	R621,10	R225,00	R396,10
Nov	R39,61	R435,71	R225,00	R210,71
Dec	R21,07	R231,78	R225,00	R6,78
Jan	R0,68	R7,46	R7,46	R0,00

- 5.2.3 Real cost of the loan $= (R225,00 \times 6) + R7,46$
 $= R1\ 350,00 + R7,46$
 $= R1\ 357,46$

- 5.2.4 A 1% increase in the interest rate will increase the total amount paid back for this small loan by R1 357,46 - R1 334,35 = R23,11 and will increase the amount of time taken to pay back the loan from 6 months to 7 months.

UNIT



INFLATION

- 1.1 Inflation rate from 2013 - 2014:

$$= \frac{R321\ 993 - R307\ 230}{R307\ 230} \times 100$$

$$= \frac{R14\ 763}{R307\ 230} \times 100$$

$$= 4,81\%$$

$$\text{Inflation rate} = \frac{\text{Price difference}}{\text{Original price}} \times 100$$

- Inflation rate from 2014 - 2015:

$$= \frac{R335\ 730 - R321\ 993}{R321\ 993} \times 100$$

$$= \frac{R13\ 733}{R321\ 993} \times 100$$

$$= 4,27\%$$

- $\therefore$ 2013 - 2014 showed the highest inflation rate.

1.2 Increase $= \frac{5,4}{100} \times R335\ 730 = R18\ 129,42$

$$\therefore \text{Price in 2016} = R335\ 730 + R18\ 129,42$$

$$= R353\ 859,42$$

2. Year 1 Increase $= \frac{6,8}{100} \times R30\ 000 = R2\ 040$

$$\text{Year 1 Cost} = R30\ 000 + R2\ 040 = R32\ 040$$

$$\text{Year 2 Increase} = \frac{6,8}{100} \times R32\ 040 = R2\ 178,72$$

$$\text{Year 2 Cost} = R32\ 040 + R2\ 178,72 = R34\ 218,72$$

$$\text{Year 3 Increase} = \frac{6,8}{100} \times R34\ 218,72 = R2\ 326,87$$

$$\text{Year 3 Cost} = R34\ 218,72 + R2\ 326,87 = R36\ 545,59$$

- $\therefore$ No, Sibusiso's fixed deposit of R35 000 will not be able to cover the costs in 3 years' time.

3. Decrease $= \frac{2,3}{100} \times R1\ 050$
 $= R24,15$

$$\text{Bill} = R1\ 050 - R24,15$$

$$= R1\ 025,85$$

REMEMBER!

A negative inflation rate results in a decrease in the price.

4.1.1 Price increase $= R27,46 - R23,68 = R3,78$

4.1.2 Price decrease $= R28,23 - R24,92 = R3,31$

4.1.3 % Price increase $= \frac{R212,65 - R162,23}{R162,23} \times 100$
 $= \frac{R50,42}{R162,23} \times 100$
 $= 31,1\%$

4.1.4 Food basket Jan 2014 Total $= R782,02$
 Food basket Feb 2014 Total $= R833,50$

$$\therefore \text{Inflation} = \frac{R833,50 - R782,02}{R782,02} \times 100$$

$$= \frac{R51,48}{R782,02} \times 100$$

$$= 6,58\%$$

4.1.5 Price of Flour in Feb 2014 $= R72,24$

$$\text{Increase} = \frac{1,6}{100} \times R72,24 = R1,16$$

$$\text{Price} = R72,24 + R1,16 = R73,40$$

- 4.2.1 It means that from March 2012 to March 2013, the prices increased on average by 6,1%.

- 4.2.2 Lowest inflation rate in April 2012 at 3,3%
 Highest inflation rate in March 2014 at 10,6%

4.2.3 % Price increase $= 7,9\% - 5,4\% = 2,5\%$



$$1. \text{ VAT} = \frac{14}{100} \times R820,99 \\ = R114,94$$

REMEMBER!
VAT is 14%
of the item price.



2. **Method 1** OR **Method 2**

$$\text{VAT} = \frac{14}{100} \times R12\,700 \\ = R1\,778$$

$$\therefore \text{VAT-incl.} \\ = R12\,700 + R1\,778 \\ = R14\,478$$

$$\text{VAT-incl.} \\ = R12\,700 \times \frac{114}{100} \\ = R14\,478$$

Multiply the VAT-excl.
price by
(100% + VAT of 14%).

Calculate the VAT
and add it to the
VAT-excl. price.



$$3. \text{ VAT-excl.} = R9\,899 \div \frac{114}{100} \\ = R8\,683,33$$

VAT excl. price
= VAT incl. $\div$ 114%

$$4. \text{ Case of candles VAT-excl.} = R6,59 \times 120 \\ = R790,80$$

$$\text{VAT on case of candles} = \frac{14}{100} \times R790,80 \\ = R110,71$$

$$\text{VAT-incl. price on case of candles} \\ = R790,80 + R110,71 \\ = R901,51$$

$$\text{Discount on case} = \frac{5}{100} \times R901,51 = R45,08$$

$$\therefore \text{Total VAT-incl. price after discount} \\ = R901,51 - R45,08 \\ = R856,43$$

$$5.1 \quad R11,4 \text{ billion}$$

$$5.2 \quad R6,9 \text{ billion}$$

$$5.3 \quad R8,8 \text{ billion}$$

$$5.4 \quad \% = \frac{R2,6 \text{ billion}}{R9,5 \text{ billion}} \times 100 = 27,37\%$$

$$5.5 \quad 2009 - 2010 \text{ and } 2012 - 2013$$

6.1 Mr Riet earns R7 986,50 so he falls into the R0 - R8 000 bracket.

Cost for medical aid:

Main member (Mr Riet) = R1 152

Wife = R816

2 children = $R424 \times 2 = R848$

$$\therefore \text{Total medical aid cost} = 1\,152 + 816 + 848 \\ = R2\,816$$

$$\therefore \frac{1}{3} \text{ of } R2\,816 = R938,67$$

6.2

MONTHLY DEDUCTIONS

CALCULATIONS

Union membership fee

R35,00

Pension fund

$$7\,986,50 \times 7,5\%$$

= 7,5% of gross salary

$$= 7\,986,50 \times \frac{7,5}{100}$$

$$= \mathbf{R598,99}$$

PAYE

= (Gross salary - R4 750)

$\times 18\%$

$$(7\,986,50 - 4\,750) \times \frac{18}{100}$$

$$= 3\,236,50 \times \frac{18}{100}$$

$$= \mathbf{R582,57}$$

UIF = 1% of gross salary

$$\frac{1}{100} \times 7\,986,50 = \mathbf{R79,87}$$

Medical aid contribution

R938,67

$$\text{TOTAL} = 35 + 598,99 + 582,57 + 79,87 \\ + 938,67$$

$$= \mathbf{R2\,235,10}$$

$$6.3 \quad \text{Net monthly salary} = R7\,986,50 - R2\,235,10 \\ = R5\,751,40$$

$$\therefore \text{Net annual salary} = R5\,751,40 \times 12 \\ = R69\,016,80$$

$$\text{Net salary} = \text{gross salary} - \text{total deductions}$$



7.1.1 Tax bracket 2

$$7.1.2 \quad R17\,100,00 \text{ per month} = 17\,100 \times 12 \text{ months} \\ = R205\,200,00 \text{ per year} \\ = \text{Tax bracket 2}$$

7.1.3 Primary rebate of R13 500,00

7.1.4 65 years and older

$$7.2.1 \quad \text{Annual income} = R15\,090 \times 12 \\ = R181\,080$$

$$\therefore \text{Annual tax} = \frac{18}{100} \times R181\,080 \text{ (tax bracket 1)} \\ = R32\,594,40$$

$$\therefore \text{Actual tax} = R32\,594,40 - R13\,500 \\ = R19\,094,40$$

$$\therefore \text{Monthly tax} = R19\,094,40 \div 12 \\ = R1\,591,20$$

$$7.2.2 \quad \text{Annual income with 13}^{\text{th}} \text{ cheque} \\ = R181\,080,00 + R15\,090,00 \\ = R196\,170,00 \text{ (tax bracket 2)}$$

Yearly tax

$$= R33\,840,00 + 25\% \times (R196\,170,00 - R188\,000,00) \\ = R33\,840,00 + 25\% \times R8\,170,00 \\ = R33\,840,00 + R2\,042,50 \\ = R35\,882,50$$

Actual tax payable per year

$$= R35\,882,50 - R13\,500,00 \text{ (primary rebate)} \\ = R22\,382,50$$

So, receiving a 13th cheque will mean that Solly will pay R22 382,50 - R19 094,40 = R3 288,10 more tax during the year.

$$7.2.3 \quad \text{Old salary} = R15\,090,00 \text{ per month}$$

$$\text{New salary} = R15\,090,00 + 8\% \times R15\,090,00 \\ = R15\,090,00 + R1\,207,20 \\ = R16\,297,20$$

$$\text{New annual salary} = R16\,297,20 \times 12 \\ = R195\,566,40 \text{ (tax bracket 2)}$$

Yearly tax
 $= R33\ 840,00 + 26\% \times (R195\ 566,40 - R188\ 000,00)$
 $= R33\ 840,00 + 26\% \times R7\ 566,40$
 $= R33\ 840,00 + R1\ 967,26$
 $= R35\ 807,26$

Actual tax payable per year
 $= R35\ 807,26 - R13\ 500,00$ (primary rebate)
 $= R22\ 307,26$
 $= R1\ 858,94$ per month

So, increase in salary of R1 207,20 (8%) brings about an increase in tax of R1 858,94 - R1 591,20 (monthly tax on original salary) = R267,74.

So, the amount of the increase in salary that Solly will actually take home is R939,46.

8.1 You only start paying tax when you earn more than the tax threshold.

8.2 Monthly tax in 2014/2015 $\approx$ R4 500,00
 Monthly tax in 2012/2013 $\approx$ R5 250,00
 $\therefore$ Decrease in tax = R5 250 - R4 500,00
 $\approx$ R750,00

8.3.1 Since 2012/2013, the amount of tax that taxpayers have to pay each month has decreased. The greatest decrease was from 2012/2013 to 2013/2014, with a smaller decrease from 2013/2014 to 2014/2015.

8.3.2 Inflation generally increases from year to year. This means that every year things become more expensive. So, if the amount of tax that taxpayers pay each year had to increase, then their buying power would decrease significantly from one year to the next. Tax rates are reduced to try to ensure that people do not lose their buying power, or that the loss in buying power is not too great.

UNIT 10 EXCHANGE RATE

1. Rands = Algerian dinar $\div$ rate
 $= 110,35 \div 9,48$
 $= R11,64$

Unit currency
 $= \text{multiple currency} \div \text{rate}$



- 2.1 $\triangleright$ United Kingdom weekly salary = £55
 Salary in rands = £55 $\times$ 14,66 = R806,30
 $\triangleright$ USA weekly salary = \$140
 Salary in rands = \$140 $\times$ 7,42 = R1 038,80
 $\triangleright$ Netherlands weekly salary = €340 $\div$ 4 = €85
 Salary in rands = €85 $\times$ 9,86 = R838,10
 $\therefore$ USA will pay the best salary per week.
- 2.2 British pound, because R14,66 is equivalent to £1.
- 2.3 It would be best to import goods from the USA, as it would be cheaper than the other places due to the dollar being weaker than the pound or euro.
- 3.1 Botswana pula = rands $\times$ rate
 $= R20 \times 0,95$
 $=$ BWP 19
- 3.2 $\triangleright$ Accommodation = 360 286 $\times$ 9 nights
 $=$ 3 242 574 ZMK
 $\triangleright$ Dinner = 85 134 $\times$ (2 people $\times$ 4 meals)
 $= 85\ 134 \times 8$ meals
 $=$ 681 072 ZMK
 $\therefore$ Total Costs = 3 242 574 + 681 072
 $=$ 3 923 646 ZMK
 $\therefore$ Total Payable = cost - deposit
 $= 3\ 923\ 646 - 1\ 021\ 605$
 $=$ 2 902 041 ZMK
 $\therefore$ Total Payable in rand = Zambian kwacha $\div$ rate
 $= 2\ 902\ 041 \div 681,07$
 $=$ R4 261,00

3.3 Botswana pula

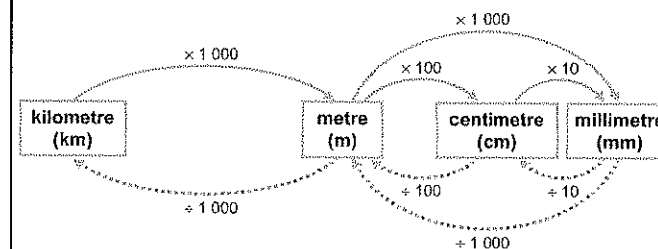
- 4.1 $\triangleright$ Over-locker sewing machine $\approx$ £500
 $\triangleright$ Needles $\approx$ £10 $\times$ 5 $\approx$ £50
 $\triangleright$ Thread $\approx$ £5 $\times$ 50 $\approx$ £250
 $\therefore$ Total Cost in pounds $\approx$ £500 + £50 + £250 $\approx$ £800
 $\therefore$ Total Cost in rand $\approx$ pounds $\times$ rate
 $\approx$ £800 $\times$ 20
 $\approx$ R16 000

- 4.2 $\triangleright$ Over-locker sewing machine = £510,99
 $\triangleright$ Needles = £9,50 $\times$ 5 $\approx$ £47,50
 $\triangleright$ Thread = £4,85 $\times$ 50 $\approx$ £242,50
 $\therefore$ Total Cost in pounds
 $=$ £510,99 + £47,50 + £242,50
 $=$ £800,99
 $\therefore$ Total Cost in rand = £800,99 $\times$ 18,42
 $=$ R14 754,24
 $\therefore$ Amy will be able to afford the sewing machine and accessories.

MODULE 4 MEASUREMENT

UNIT 2 MEASURING LENGTH AND DISTANCE

EXERCISE 1



- 1.1 2 m = 2 $\times$ 1 000 = 2 000 mm
 1.2 3,18 km = 3,18 $\times$ 1 000 = 3 180 m
 1.3 80 m = 80 $\times$ 100 = 8 000 cm
 1.4 6,7 cm = 6,7 $\times$ 10 = 67 mm
 1.5 400 m = 400 $\div$ 1 000 = 0,4 km
 1.6 0,89 km = 0,89 $\times$ 1 000 = 890 m
 $\therefore$ 890 m = 890 $\times$ 100 = 89 000 cm



- 1.7 $63 \text{ km} = 63 \times 1\,000 = 63\,000 \text{ m}$
 $\therefore 63\,000 \text{ m} = 63\,000 \times 1\,000 = 63\,000\,000 \text{ mm}$
- 1.8 $5,29 \text{ cm} = 5,29 \div 100 = 0,0529 \text{ m}$
- 1.9 $8 \text{ mm} = 8 \div 1\,000 = 0,008 \text{ m}$
 $\therefore 0,008 \text{ m} = 0,008 \div 1\,000 = 0,000008 \text{ km}$
- 1.10 $9\,301 \text{ cm} = 9\,301 \div 100 = 93,01 \text{ m}$
 $\therefore 93,01 \text{ m} = 93,01 \div 1\,000 = 0,09301 \text{ km}$

2. $400 \text{ m} = 400 \div 1\,000 = 0,4 \text{ km}$
 $\therefore$ No. of times around track $= 21 \text{ km} \div 0,4 \text{ km}$
 $= 52,5$
 ≈ 53 times



REMEMBER! Round up, otherwise he wouldn't equal the half-marathon distance.

3. Window-sill length $= 720 \text{ mm} = 720 \div 10 = 72 \text{ cm}$
Length of tile $= 20 \text{ cm}$
 $\therefore$ No. of mosaic tiles needed $= 72 \text{ cm} \div 20$
 $= 3,6$
 ≈ 4 tiles
- Length of 4 tiles $= 4 \times 20 \text{ cm} = 80 \text{ cm}$
- $\therefore$ Cost of tiles $= 80 \text{ cm} \times R6,20 = R496$

Round up!

EXERCISE 2

- 1.1 $40 \text{ miles} = 40 \times 1,6093$
 $= 64,37 \text{ km}$
- OR $= 40 \div 0,6214$
 $= 64,37 \text{ km}$
- 1.2 $650 \text{ yards} = 650 \times 0,9144$
 $= 594,36 \text{ m}$
- OR $= 650 \div 1,0936$
 $= 594,37 \text{ m}$

- 1.3 $50 \text{ inches} = 50 \times 2,54$
 $= 127 \text{ cm}$
- OR $= 50 \div 0,3937$
 $= 127 \text{ cm}$

$$1 \text{ inch} = 2,54 \text{ cm}$$

$$1 \text{ cm} = 0,3937 \text{ inch}$$

- 1.4 $0,24 \text{ feet} = 0,24 \times 0,3048$
 $= 0,073152 \text{ m}$
- $\therefore 0,073152 \text{ m} = 0,073152 \times 1\,000$
 $= 73,15 \text{ mm}$

NOTE! Only round off your final answer to 2 decimal places.



- 1.5 $3,7 \text{ m} = 3,7 \times 1,0936$
 $= 4,05 \text{ yards}$
- OR $= 3,7 \div 0,9144$
 $= 4,05 \text{ yards}$

$$1 \text{ m} = 1,0936 \text{ yds}$$

$$1 \text{ yd} = 0,9144 \text{ m}$$

- 1.6 $900,4 \text{ km} = 900,4 \times 0,6214$
 $= 559,51 \text{ miles}$
- OR $= 900,4 \div 1,6093$
 $= 559,50 \text{ miles}$

$$1 \text{ km} = 0,6214 \text{ mile}$$

$$1 \text{ mile} = 1,6093 \text{ km}$$

- 1.7 $180 \text{ cm} = 180 \div 100 = 1,8 \text{ m}$
 $\therefore 1,8 \text{ m} = 1,8 \times 3,2808$
 $= 5,91 \text{ ft}$

$$1 \text{ m} = 3,2808 \text{ ft}$$

NOTE! First convert cm to m; in order to convert to feet (since there wasn't a direct conversion factor from cm to feet).



- 1.8 $22\,169 \text{ mm} = 22\,169 \div 10 = 2\,216,9 \text{ cm}$
 $\therefore 2\,216,9 \text{ cm} = 2\,216,9 \times 0,3937$
 $= 872,79 \text{ inches}$
- $1 \text{ cm} = 0,3937 \text{ inch}$

2. Lindy: $2,4 \text{ feet} = 2,4 \times 0,3048 = 0,73 \text{ m}$
Lester: $0,85 \text{ m}$
- $\therefore$ Lester jumps the highest.

$$1 \text{ ft} = 0,3048 \text{ m}$$

3. $3\,250 \text{ m} = 3\,250 \div 1\,000 = 3,25 \text{ km}$
 $\therefore 3,25 \text{ km} = 3,25 \times 0,6214$
 $= 2,02 \text{ miles}$
- $\therefore$ Cost $= 2,02 \text{ miles} \times \$2,50/\text{mile} = \$5,05$

$$1 \text{ km} = 0,6214 \text{ mile}$$

Test Your Understanding



1. Vineyard race: $65 \text{ miles} = 65 \div 0,6214$
 $= 104,60 \text{ km}$

Ocean to Ocean race: $120\,000 \text{ yards}$
 $= 120\,000 \times 0,9144$
 $= 109\,728 \text{ m}$

$\therefore 109\,728 \text{ m}$
 $= 109\,728 \div 1\,000$
 $= 109,73 \text{ km}$

Karoo Fun race: $888\,500 \text{ feet}$
 $= 888\,500 \div 3,2808$
 $= 270\,818,09 \text{ m}$
 $\therefore 270\,818,09 \text{ m}$
 $= 270\,818 \div 1000$
 $= 270,82 \text{ km}$



Charity Fun sprint: $5\,632\,144 \text{ inches}$
 $= 5\,632\,144 \times 2,54$
 $= 14\,305\,645,76 \text{ cm}$
 $\therefore 14\,305\,645,76 \text{ cm}$
 $= 14\,305\,645,76 \div 100\,000$
 $= 143,06 \text{ km}$

$\therefore$ Total distance covered
 $= 104,60 + 109,73 + 270,82 + 143,06$
 $= 628,21 \text{ km}$

$\therefore$ Yes, Jacques will win the club cycling cup.

2. Dimension of pavers:

$$2,1872 \text{ yards} = 2,1872 \times 0,9144 = 2 \text{ m}$$

$\therefore$ Paver measures $2 \text{ m} \times 2 \text{ m}$.

Width of paving:

$$6,562 \text{ ft} = 6,562 \div 3,2808 = 2 \text{ m}$$

Length of paving:

$$32,81 \text{ ft} = 32,81 \div 3,2808 = 10 \text{ m}$$

Breadth of paving:

$$26,248 \text{ ft} = 26,248 \div 3,2808 = 8 \text{ m}$$

$\therefore$ No. of pavers across length

$$= 10 \text{ m} \div 2 \text{ m} = 5 \text{ pavers}$$

$\therefore$ No. of pavers across breadth

$$= \frac{8 \text{ m} - 2 \text{ m} - 2 \text{ m}}{2 \text{ m}} = \frac{4 \text{ m}}{2 \text{ m}} = 2 \text{ pavers}$$

$\therefore$ Total number of pavers = $(5 \times 2) + (2 \times 2)$

$$= 10 + 4$$

$$= 14$$

$\therefore$ Cost of pavers = $\$42,99 \times 14$

$$= \$601,86$$

$$\text{Import duties} = \frac{15}{100} \times \$601,86$$

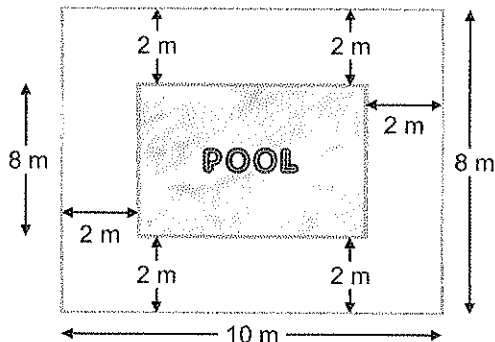
$$= \$90,28$$

$\therefore$ Total cost of pavers in Dollars

$$= \$601,86 + \$90,28 = \$692,14$$

$\therefore$ Total cost of pavers in Rand

$$= \$692,14 \times 9,68 = \text{R}6\,699,92$$



- 3.1 > Cape Town
- $\rightarrow$
- Berlin: 5 980 miles

Berlin $\rightarrow$ New York: 3 960 miles

$\therefore$ Cape Town $\rightarrow$ Berlin $\rightarrow$ New York:

$$= 5\,980 + 3\,960$$

$$= 9\,940 \text{ miles}$$

- > Cape Town
- $\rightarrow$
- London: 6 010 miles

London $\rightarrow$ New York: 3 460 miles

$\therefore$ Cape Town $\rightarrow$ London $\rightarrow$ New York:

$$= 6\,010 + 3\,460$$

$$= 9\,470 \text{ miles}$$

- >
- $\therefore$
- Cape Town
- $\rightarrow$
- London
- $\rightarrow$
- New York is the shortest route.

- 3.2 Distance of Cape Town
- $\rightarrow$
- London
- $\rightarrow$
- New York
-
- = 9 470 miles

$$\therefore 9\,470 \text{ miles} = 9\,470 \times 1,6093$$

$$= 15\,240,07 \text{ km}$$

$$\approx 15\,240 \text{ km}$$

- 3.3 Cape Town
- $\rightarrow$
- London = 6 010 miles

$$\therefore 6\,010 \text{ miles} = 6\,010 \times 1,6093$$

$$= 9\,671,89 \text{ km}$$

Flying time = 11 hours 45 minutes

$$= 11 \text{ hours} + \frac{45}{60} \text{ hours}$$

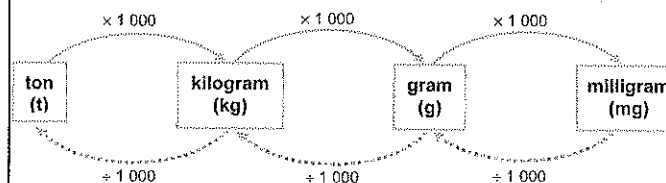
$$= 11,75 \text{ hours}$$

$$\therefore \text{Average speed} = 9\,671,89 \text{ km} \div 11,75 \text{ hours} = 823,14 \text{ km/h}$$

UNIT 3

MEASURING MASS (WEIGHT)

EXERCISE 1



$$1.1 \quad 3 \text{ mg} = 3 \div 1\,000 = 0,003 \text{ g}$$

$$1.2 \quad 75 \text{ g} = 75 \div 1\,000 = 0,075 \text{ kg}$$

$$1.3 \quad 64,7 \text{ t} = 64,7 \times 1\,000 = 64\,700 \text{ kg}$$

$$1.4 \quad 28 \text{ g} = 28 \times 1\,000 = 28\,000 \text{ mg}$$

$$1.5 \quad 0,03 \text{ kg} = 0,03 \times 1\,000 = 30 \text{ g}$$

$$1.6 \quad 84 \text{ kg} = 84 \div 1\,000 = 0,084 \text{ t}$$

$$1.7 \quad 0,0002 \text{ t} = 0,0002 \times 1\,000 = 0,2 \text{ kg}$$

$$\therefore 0,2 \text{ kg} = 0,2 \times 1\,000 = 200 \text{ g}$$

$$1.8 \quad 1,492 \text{ mg} = 1,492 \div 1\,000 = 0,001492 \text{ g}$$

$$\therefore 0,001492 \text{ g} = 0,001492 \div 1\,000$$

$$= 0,000001492 \text{ kg}$$

$$1.9 \quad 89,3 \text{ kg} = 89,3 \times 1\,000 = 89\,300 \text{ g}$$

$$\therefore 89\,300 \text{ g} = 89\,300 \times 1\,000 = 89\,300\,000 \text{ mg}$$

$$1.10 \quad 49\,110\,763 \text{ mg} = 49\,110\,763 \div 1\,000$$

$$= 49\,110,763 \text{ g}$$

$$49\,110,763 \text{ g} = 49\,110,763 \div 1\,000$$

$$= 49,110763 \text{ kg}$$

$$49,110763 \text{ kg} = 49,110763 \div 1\,000$$

$$= 0,049110763 \text{ t}$$

2. > Sweet Chews:
- $\text{R}13,99 \div 280 \text{ g} = \text{R}0,0499/\text{g}$

- > Happy Tastes:
- $\text{R}16,50 \div 320 \text{ g} = \text{R}0,0515/\text{g}$

$\therefore$ Sweet Chews is the better buy, as it is cheaper per gram.

$$3. \quad \text{BMI} = \frac{\text{mass (kg)}}{[\text{height (m)}]^2}$$

$$= \frac{58 \text{ kg}}{(1,24)^2}$$

$$= 37,72$$

$\therefore$ Sandra's weight is classified as obese.

4. > Weight of 10-year old boy in 50
- th
- percentile: 32 kg

- > Weight of 13-year old boy in 90
- th
- percentile: 60 kg

$$\therefore \text{Difference} = 60 - 32 = 28 \text{ kg}$$



EXERCISE 2

$$1.1 \quad 4,9 \text{ kg} = 4,9 \times 2,2046 \\ = 10,80 \text{ lb}$$

$$1 \text{ kg} = 2,2046 \text{ lb}$$

$$\text{OR} \quad = 4,9 \div 0,4535 \\ = 10,80 \text{ lb}$$

$$1 \text{ lb} = 0,4535 \text{ kg}$$

$$1.2 \quad 630 \text{ g} = 630 \times 0,0352 \\ = 22,18 \text{ oz}$$

$$1 \text{ g} = 0,0352 \text{ oz}$$

$$\text{OR} \quad = 630 \div 28,3495 \\ = 22,22 \text{ oz}$$

$$1 \text{ oz} = 28,3495 \text{ g}$$

There is a slight discrepancy in the answers due to the fact that the conversion factor/rate is a rounded off number!



$$1.3 \quad 21 \text{ oz} = 21 \times 28,3495 \\ = 595,34 \text{ g}$$

$$1 \text{ oz} = 28,3495 \text{ g}$$

$$\text{OR} \quad = 21 \div 0,0352 \\ = 596,59 \text{ g}$$

$$1 \text{ g} = 0,0352 \text{ oz}$$

$$1.4 \quad 0,03 \text{ lb} = 0,03 \times 0,4535 \\ = 0,01 \text{ kg}$$

$$1 \text{ lb} = 0,4535 \text{ kg}$$

$$\text{OR} \quad = 0,03 \div 2,2046 \\ = 0,01 \text{ kg}$$

$$1 \text{ kg} = 2,2046 \text{ lb}$$

$$1.5 \quad 8 \text{ oz} = 8 \times 28,3495 \\ = 226,796 \text{ g}$$

$$1 \text{ oz} = 28,3495 \text{ g}$$

$$\therefore 226,80 \text{ g} = 226,80 \div 1\,000 \\ = 0,23 \text{ kg}$$

NOTE! Only round off your final answer to 2 decimal places.



$$1.6 \quad 2 \text{ lb} = 2 \times 0,4535 \\ = 0,907 \text{ kg}$$

$$1 \text{ lb} = 0,4535 \text{ kg}$$

$$\therefore 0,907 \text{ kg} = 0,907 \times 1\,000 \\ = 907 \text{ g}$$

$$1.7 \quad 99,32 \text{ kg} = 99,32 \times 1\,000 \\ = 99\,320 \text{ g}$$

$$1 \text{ g} = 0,0352 \text{ oz}$$

$$\therefore 99\,320 \text{ g} = 99\,320 \times 0,0352 \\ = 3\,496,06 \text{ oz}$$

$$1.8 \quad 818,7 \text{ g} = 818,7 \div 1\,000 \\ = 0,8187 \text{ kg}$$

$$1 \text{ kg} = 2,2046 \text{ lb}$$

$$\therefore 0,8187 \text{ kg} = 0,8187 \times 2,2046 \\ = 1,80 \text{ lb}$$

$$2. \quad 6\,820 \text{ lb} = 6\,820 \times 0,4535 \\ = 3\,092,87 \text{ kg}$$

$$1 \text{ lb} = 0,4535 \text{ kg}$$

$$\therefore 3\,092,87 \text{ kg} = 3\,092,87 \div 1\,000 \\ = 3,09 \text{ t}$$

$\therefore$ Yes, the crane will be able to lift the container.

$$3. \quad 0,25 \text{ lb} = 0,25 \times 0,4535 \\ = 0,113\,375 \text{ kg}$$

$$1 \text{ lb} = 0,4535 \text{ kg}$$

$$\therefore 0,113\,375 \text{ kg} = 0,113\,375 \times 1\,000 \\ = 113,375 \text{ g}$$

$$113,375 \text{ g} = 113,375 \times 0,0352 \\ = 3,9908 \text{ oz}$$

$$1 \text{ g} = 0,0352 \text{ oz}$$

$$\therefore \text{Cost of gold} \\ = 3,9908 \text{ oz} \times \text{R}13\,070,58/\text{oz} \\ = \text{R}52\,162,07$$

Test Your Understanding



$$1. \quad \text{A} \quad 125\,000 \text{ mg} = 125\,000 \div 1\,000 = 125 \text{ g}$$

$$\therefore \text{Cost per gram} = \text{R}8,95 \div 125 \text{ g} = \text{R}0,0716/\text{g}$$

$$\text{B} \quad 500 \text{ g}$$

$$\therefore \text{Cost per gram} = \text{R}32,20 \div 500 \text{ g} = \text{R}0,0644/\text{g}$$

$$\text{C} \quad 0,75 \text{ kg} = 0,75 \times 1\,000 = 750 \text{ g}$$

$$\therefore \text{Cost per gram} = \text{R}38,99 \div 750 \text{ g} = \text{R}0,05199/\text{g}$$

$\therefore$ C - 0,75 kg (750 g) baby powder is most economical.

$$2. \quad \triangleright \text{Dozen and a half muffins} = 12 + 6 = 18 \text{ muffins}$$

$$\triangleright \text{Pound of dates per muffin} = 3,6 \text{ lb} \div 18 \\ = 0,2 \text{ lb/muffin}$$

$$\text{Mass of dates per muffin} = 0,2 \text{ lb} \div 2,2046 \\ = 0,0907... \text{ kg}$$

$$\therefore 0,0907 \text{ kg} \times 1\,000 = 90,72 \text{ g}$$

$$\therefore \text{Cost of dates per muffin} = 90,72 \text{ g} \times 4 \text{ cents} \\ = 362,88 \text{ cents}$$

$$\therefore 362,88 \text{ cents} \div 100 = \text{R}3,63$$

$$\triangleright \text{Ounces of coconut per muffin} = 5,4 \text{ oz} \div 18 \\ = 0,3 \text{ oz}$$

$$\text{Mass of coconut per muffin} = 0,3 \text{ oz} \times 28,3495 \\ = 8,50 \text{ g}$$

$$\therefore \text{Cost of coconut per muffin} = 8,50 \text{ g} \times 25 \text{ cents} \\ = 212,5 \text{ cents}$$

$$\therefore 212,5 \text{ cents} \div 100 = \text{R}2,13$$

$$\therefore \text{Total cost of dates and coconut} = \text{R}3,63 + \text{R}2,13 \\ = \text{R}5,76 \text{ per muffin}$$

$$3. \quad \triangleright \text{Paul's height} = 165 \text{ cm} \div 100 = 1,65 \text{ m}$$

$$\text{Paul's BMI} = \frac{76 \text{ kg}}{(1,65)^2} = 27,92$$

$\therefore$ Paul's weight is classified as 'overweight'.

$$\triangleright \text{Pauline's mass} = 99 \text{ lb} \times 0,4536 = 44,91 \text{ kg}$$

$$\text{Pauline's height} = 60 \text{ in} \times 0,0254 = 1,52 \text{ m}$$

$$\text{Pauline's BMI} = \frac{44,91 \text{ kg}}{(1,52)^2} = 19,44$$

$\therefore$ Pauline's weight is classified as 'normal'.

$\therefore$ Paul needs to choose a healthier lifestyle.

4.1 The girl has an 'average' (median) weight-for-age ratio. i.e. 50% of all other girls between 2 and 20 years have a similar weight-for-age ratio.

$$4.2 \approx 41,5 \text{ kg}$$

$$4.3 \approx 163 \text{ cm} (1,63 \text{ m})$$



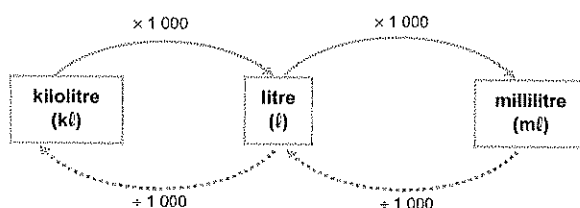
- 4.4 Above average. The girl's weight-for-age ratio lies above the 50th percentile and almost on the 90th percentile. This is far above average.
- 4.5 Below average. The girl's height-for-age ratio lies below the 50th percentile and almost on the 10th percentile. This is far below average.
- 4.6 25th percentile and 50th percentile
- 4.7 75th percentile and 90th percentile
- 4.8 More than 78 kg. A 16 year-old girl who weighs more than 78 kg has a weight-for-age ratio that lies above the 95th percentile curve. This means that she belongs to a group of only 5 out of every 100 children who share this high weight-for-age ratio.
- 4.9 Less than 46 kg. A 19 year old girl who weighs less than 46 kg has a weight-for-age ratio that lies below the 5th percentile curve. This means that she belongs to a group of only 5 out of every 100 children who share this low weight-for-age ratio.
- 4.10 Yes. This girl's height-for-age ratio puts her above the 95th percentile. This means that she belongs to a group of only 5 out of every 100 children who share this high height-for-age ratio.
- 4.11 From about age 9 or 10 years to about age 13 years. On the graph this is represented by the portion of the curve that is the steepest.

UNIT

4

MEASURING VOLUME

EXERCISE 1



- 1.1 $60 \text{ mℓ} = 60 \div 1\,000 = 0,06 \text{ ℓ}$
- 1.2 $4 \text{ ℓ} = 4 \div 1\,000 = 0,004 \text{ kℓ}$
- 1.3 $2 \text{ kℓ} = 2 \times 1\,000 = 2\,000 \text{ ℓ}$
- 1.4 $0,32 \text{ ℓ} = 0,32 \times 1\,000 = 320 \text{ mℓ}$
- 1.5 $2,4 \text{ kℓ} = 2,4 \times 1\,000 = 2\,400 \text{ ℓ}$
 $\therefore 2\,400 \text{ ℓ} = 2\,400 \times 1\,000 = 2\,400\,000 \text{ mℓ}$



- 1.6 $63\,000 \text{ mℓ} = 63\,000 \div 1\,000 = 63 \text{ ℓ}$
 $\therefore 63 \text{ ℓ} = 63 \div 1\,000 = 0,063 \text{ kℓ}$
- 2.1 $1,75 \text{ ℓ} = 1,75 \times 1\,000 = 1\,750 \text{ mℓ}$
 $\therefore \text{No. of cups} = 1\,750 \text{ mℓ} \div 250 = 7 \text{ cups}$
- 2.2 $\text{No. of teaspoons} = 45 \text{ mℓ} \div 5 = 9 \text{ teaspoons}$
- 2.3 $0,06 \text{ ℓ} = 0,06 \times 1\,000 = 60 \text{ mℓ}$
 $\therefore \text{No. of tablespoons} = 60 \text{ mℓ} \div 15 = 4 \text{ tablespoons}$

3. $\triangleright \text{Distance} = 70 \text{ km/h} \times 1,25 \text{ hours}$
 $= 87,5 \text{ km}$
- $1 \text{ h } 15 \text{ min}$
 $= 1 \text{ h } + \frac{15}{60} \text{ h}$
 $= 1 + 0,25$
 $= 1,25 \text{ h}$
- $\triangleright \text{No. of litres of fuel} = 87,5 \text{ km} \div 0,8 \text{ km}$
 $= 109,375 \text{ ℓ}$
- $\therefore \text{Cost of fuel} = 109,375 \text{ ℓ} \times \text{R}11,59$
 $= \text{R}1\,267,66$



EXERCISE 2

- 1.1 $60 \text{ mℓ} = 60 \times 0,0338 \text{ US fl. oz}$
 $= 2,03 \text{ fl. oz}$
- OR $= 60 \div 29,5735$
 $= 2,03 \text{ fl. oz}$
- $1 \text{ mℓ} = 0,0338 \text{ US fl.}$
 $1 \text{ US fl.} = 29,5735 \text{ mℓ}$

- 1.2 $4 \text{ UK pints} = 4 \times 0,5682$
 $= 2,27 \text{ ℓ}$
- OR $= 4 \div 1,7598$
 $= 2,27 \text{ ℓ}$
- $1 \text{ UK pint} = 0,5682 \text{ ℓ}$
 $1 \text{ ℓ} = 1,7598 \text{ UK pints}$

- 1.3 27 US fl. oz
 $= 27 \times 29,5735$
 $= 798,4845 \text{ mℓ}$
- $\therefore 798,4845 \text{ mℓ} = 798,4845 \div 1\,000 = 0,8 \text{ ℓ}$

NOTE! Only round off your final answer to 2 decimal places.



- 1.4 $0,8 \text{ US qt} = 0,8 \times 0,9464$
 $= 0,75712 \text{ ℓ}$
- $\therefore 0,75712 \text{ ℓ} = 0,75712 \times 1\,000 = 757,12 \text{ mℓ}$
- $1 \text{ US qt} = 0,9464 \text{ ℓ}$

- 1.5 $479 \text{ ℓ} = 479 \times 0,2199$
 $= 105,33 \text{ UK gal}$
- OR $= 479 \div 4,5461$
 $= 105,37 \text{ UK gal}$
- $1 \text{ ℓ} = 0,2199 \text{ UK gal}$
 $1 \text{ UK gal} = 4,5461 \text{ ℓ}$

The discrepancy in answers is due to the fact that the conversion factor is a rounded off number!



- 1.6 $1011,21 \text{ mℓ} = 1011,21 \div 1\,000$
 $= 1,01121 \text{ ℓ}$
- $\therefore 1,01121 \text{ ℓ} = 1,01121 \text{ ℓ} \times 1,7598 = 1,78 \text{ UK pints}$
- $1 \text{ ℓ} = 1,7598 \text{ UK pt}$

2. $\triangleright \text{Silver Stream: } 2\,750 \text{ UK gal}$
 $= 2\,750 \times 4,5461$
 $= 12\,501,78 \text{ ℓ}$
- $\triangleright \text{Placid Waters: } 13\,207,5 \text{ US qt}$
 $= 13\,207,5 \times 0,9464$
 $= 12\,499,58 \text{ ℓ}$
- $\therefore \text{Silver Stream has the biggest capacity.}$
- $1 \text{ UK gallon} = 4,5461 \text{ ℓ}$
 $1 \text{ US qt} = 0,9464 \text{ ℓ}$

3. $\triangleright 1,5849 \text{ US qt} = 1,5849 \times 0,9464$

$= 1,5 \text{ l}$

$\therefore \text{Two tubs} = 1,5 \text{ l} \times 2 = 3 \text{ l}$

$\triangleright 0,43995 \text{ UK pt} = 0,43995 \times 0,5682$

$= 0,25 \text{ l}$

$\therefore \text{Eight tubs} = 0,25 \text{ l} \times 8 = 2 \text{ l}$

$\therefore$ Mrs Hawthorne will get more ice-cream if she buys the two 1,5849 US qt tubs of gelato.

1 US qt
 $= 0,9464 \text{ l}$

1 UK pint
 $= 0,5682 \text{ l}$

EXERCISE 3

1. $150 \text{ g} = 150 \text{ ml}$

$\therefore \text{No. of ice-cube blocks}$

$= 150 \text{ ml} \div 25 \text{ ml}$

$= 6 \text{ blocks}$

$1 \text{ g} = 1 \text{ ml}$

2. $1,5 \text{ hours} = 1,5 \times 60 \text{ minutes}$

$= 90 \text{ minutes}$

$\therefore \text{Amount of water}$

$= 0,002 \text{ m}^3 \times 90 \text{ min}$

$= 0,18 \text{ m}^3$

$= 0,18 \times 1\,000$

$= 180 \text{ l}$

$1 \text{ m}^3 = 1\,000 \text{ l}$

3. $1,5 \text{ kg} = 1,5 \times 1\,000 = 1\,500 \text{ g}$

$\therefore \text{Amount of } E. coli$

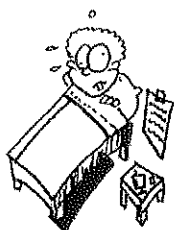
$= (1\,500 \text{ g} \div 100 \text{ g}) \times 8,5 \text{ cm}^3$

$= 15 \times 8,5 \text{ cm}^3$

$= 127,5 \text{ cm}^3$

$\therefore 127,5 \text{ cm}^3 = 127,5 \text{ g of } E. coli$

$1 \text{ cm}^3 = 1 \text{ g}$



Test Your Understanding



1. $\triangleright 135\,000 \text{ ml of water: } 135\,000 \div 1\,000 = 135 \text{ l}$

$\therefore 135 \text{ l} = 135 \div 1\,000 = 0,135 \text{ kl}$

$\therefore \text{Cost} = 0,135 \text{ kl} \times R12,90 = R1,74$

$\triangleright 0,065 \text{ l of car shampoo: } 0,065 \times 1\,000 = 65 \text{ ml}$

$\therefore \text{Cost} = 65 \text{ ml} \times R0,40 = R26,00$

$\therefore \text{Total cost} = R1,74 + R26,00 = R27,74$

2. $\triangleright \text{No. of dosages in 24-hour period} = 24 \text{ h} \div 4$
 $= 6 \text{ doses}$

$\triangleright 1 \text{ dose} = 2 \text{ teaspoons} = 2 \times 5 \text{ ml} = 10 \text{ ml}$

$\triangleright \text{Volume of 6 doses} = 6 \times 10 \text{ ml} = 60 \text{ ml}$

$\therefore 60 \text{ ml} = 60 \div 29,5753 = 2,03 \text{ US fl. ounces}$

3. $\triangleright 33 \text{ m}^3 = 33 \times 1\,000 = 33\,000 \text{ l}$

$\triangleright \text{Filled capacity} = \frac{85}{100} \times 33\,000 = 28\,050 \text{ l}$

$\therefore 28\,050 \text{ l} = 28\,050 \div 4,5461 = 6\,170,12 \text{ UK gallons}$

4. $\triangleright 2 \text{ cups flour} = 2 \times 250 \text{ ml} = 500 \text{ ml}$

$\triangleright 2 \text{ tsp vanilla essence} = 2 \times 5 \text{ ml} = 10 \text{ ml}$

$\triangleright 1 \text{ cup sugar} = 250 \text{ ml}$

$\triangleright 150 \text{ g water} = 150 \text{ ml}$

$\triangleright 4 \text{ tbs oil} = 4 \times 15 \text{ ml} = 60 \text{ ml}$

$\triangleright 180 \text{ g cm}^3 \text{ milk} = 180 \text{ ml}$

$\therefore \text{Total volume} = 500 + 10 + 250 + 150 + 60 + 180$
 $= 1\,150 \text{ ml}$

$\therefore \text{Total weight} = 1\,150 \text{ ml} = 1\,150 \text{ g}$

$\therefore 1\,150 \text{ g} = 1\,150 \div 1\,000 = 1,15 \text{ kg}$



In these calculations, we simplify conversions between liquids and solids by using the conversion factor for pure water:

$1 \text{ ml} = 1 \text{ cm}^3 = 1 \text{ g}$

to estimate the weight of a liquid.

UNIT 5

MEASURING TEMPERATURE



Test Your Understanding

1. $\triangleright \text{Beijing: } ^\circ\text{C} = (46^\circ\text{F} - 32^\circ) \div 1,8 = 7,78^\circ\text{C}$

$\triangleright \text{Dar es Salaam: } ^\circ\text{C} = (91^\circ\text{F} - 32^\circ) \div 1,8 = 32,78^\circ\text{C}$

$\triangleright \text{Minsk: } ^\circ\text{C} = (21^\circ\text{F} - 32^\circ) \div 1,8 = -6,11^\circ\text{C}$

1.1 Dar es Salaam

We need hot weather to dry clothes on the line!



1.2 Minsk

2. $\triangleright ^\circ\text{C} = (100,76^\circ\text{F} - 32^\circ) \div 1,8 = 38,2^\circ\text{C}$

$\triangleright \text{After 1 hour:}$

$\triangleright \text{Increase} = \frac{5}{100} \times 38,2^\circ\text{C} = 1,91^\circ\text{C}$

$\triangleright \text{Temperature} = 38,2 + 1,91 = 40,11^\circ\text{C}$

$\therefore$ After 1 hour Emily will need to go to the hospital.

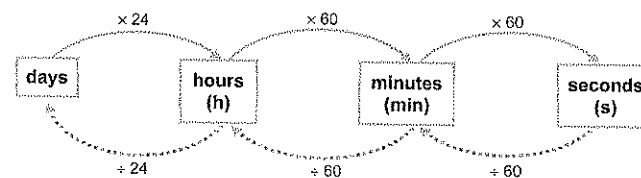
3. $^\circ\text{F} = (1,8 \times 16^\circ\text{C}) + 32^\circ = 60,8^\circ\text{F}$

Yes, I should plant carrots.

UNIT 6

MEASURING TIME

EXERCISE 1



1.1 $4 \text{ days} = 4 \times 24 = 96 \text{ hours}$

$\therefore 96 \text{ hours} = 96 \times 60 = 5\,760 \text{ minutes}$

- 1.2 $666 \text{ seconds} = 666 \div 60 = 11,1 \text{ minutes}$
 $\therefore 11,1 \text{ minutes} = 11,1 \div 60 = 0,185 \text{ hours}$
- 1.3 $121 \text{ days} = 121 \times 24 = 2\,904 \text{ hours}$
 $\therefore 2\,904 \text{ hours} = 2\,904 \times 60 = 174\,240 \text{ minutes}$
 $\therefore 174\,240 \text{ min} = 174\,240 \times 60 = 10\,454\,400 \text{ s}$
- 1.4 $432 \text{ minutes} = 432 \div 60 = 7,2 \text{ hours}$
 $\therefore 7,2 \text{ hours} = 7 \text{ hours} + 0,2 \text{ hours}$
 $= 7 \text{ hours} + 0,2 \text{ hours} \times 60 \text{ minutes}$
 $= 7 \text{ hours} + 12 \text{ minutes}$
 $\therefore 432 \text{ minutes} = 7 \text{ hours}; 12 \text{ minutes}$
- 1.5 $1\,000 \text{ s} = 1\,000 \div 60 = 16,66 \text{ min} \dots 16,6 \text{ min}$
 $\therefore 16,66\dots = 16 \text{ min} + 0,6 \dots \text{ s}$
 $= 16 \text{ min} + 0,6 \dots \text{ s} \times 60 \text{ min}$
 $= 16 \text{ min} + 40 \text{ s}$
 $\therefore 1\,000 \text{ s} = 16 \text{ minutes}; 40 \text{ seconds}$
- 1.6 $557\,799 \text{ s} = 557\,799 \div 60 = 9\,296,65 \text{ min}$
 $\therefore 9\,296,65 \text{ min} = 9\,296,65 \div 60 \text{ s}$
 $= 154,9441667 \text{ h}$
 $\therefore 154,9441667 \text{ h} = 154,9441667 \div 24$
 $= 6,456006944 \text{ days}$
 $\therefore 6,456006944 \text{ days}$
 $= 6 \text{ days} + 0,456006944 \text{ days}$
 $= 6 \text{ days} + 0,456006944 \text{ days} \times 24 \text{ h}$
 $= 6 \text{ days} + 10,94416667 \text{ h}$
 $\therefore 10,94416667 \text{ h}$
 $= 10 \text{ h} + 0,94416667 \text{ h}$
 $= 10 \text{ h} + 0,94416667 \times 60 \text{ min}$
 $= 10 \text{ h} + 56,65 \text{ min}$
 $\therefore 56,65 \text{ min}$
 $= 56 \text{ min} + 0,65 \text{ min} \times 60 \text{ s}$
 $= 56 \text{ min} + 39 \text{ s}$
 $\therefore 557\,799 \text{ s}$
 $= 6 \text{ days}; 10 \text{ h}; 56 \text{ min} \text{ and } 39 \text{ s}$



2. $04:25$; $4:30 \text{ am}$; $10:30 \text{ am}$; $16:05$; $4:15 \text{ pm}$;
 $22:15$; $10:45 \text{ pm}$
3. 18 October 1968 to 30 August 1991
 18 October 1990 - 18 October 1968 = 22 years
 18 August 1991 - 18 October 1990 = 10 months
 30 August 1991 - 18 August 1991 = 12 days
 The record was broken in 22 years, 10 months and 12 days $\approx 23 \text{ years}$
- 4.1 Runner A
- 4.2 Distance to Pinetown = 68,9 km
 Distance to Finish = 89,3 km
 $\therefore$ Distance from Pinetown to Finish
 $= 89,3 \text{ km} - 68,9 \text{ km}$
 $= 20,4 \text{ km}$
- 4.3 Running time at Lion Park = 1 h 8 min 7 s
 Running time at Half Way = 3 h 13 min 20 s
 $\therefore 3 \text{ h} ; 13 \text{ min} ; 20 \text{ s}$
 $- 1 \text{ h} ; 8 \text{ min} ; 7 \text{ s}$
 $2 \text{ h} ; 5 \text{ min} ; 13 \text{ s}$
- 4.4 Running time at Camperdown = 1 h 50 min 39 s
 Running time Mayville = 6 h 2 min 45 s
 $\therefore 6 \text{ h} ; 2 \text{ min} ; 45 \text{ s}$
 $- 1 \text{ h} ; 50 \text{ min} ; 39 \text{ s}$
 $4 \text{ h} ; 12 \text{ min} ; 6 \text{ s}$
- 4.5 Between Mayville and the Finish, i.e. Runner B came through Mayville in a quicker time than Runner A, but Runner A then finished the race before runner B. So Runner A passed Runner B somewhere between Mayville and the Finish.



EXERCISE 2

1. Ave. Speed = $\frac{12\,311 \text{ km}}{15 \text{ h} ; 48 \text{ min}}$ $\frac{12\,311 \text{ km}}{15,8 \text{ h}}$ $\frac{12\,311 \text{ km}}{15,8 \text{ h}}$
 $= 779,18 \text{ km/h}$ $15 \text{ h} ; 48 \text{ min}$
 $= 15 \text{ h} + \frac{48}{60} \text{ h}$
 $= 15 \text{ h} + 0,8 \text{ h}$
 $= 15,8 \text{ h}$
2. Neil: Speed = $\frac{280\,000 \text{ m}}{2 \text{ h} ; 10 \text{ min}}$ $\frac{280\,000 \text{ m}}{2,16 \text{ h}}$ $\frac{280\,000 \text{ m}}{2,16 \text{ h}}$
 $= 129,23 \text{ km/h}$ $2 \text{ h} ; 10 \text{ min}$
 $= 2 \text{ h} + \frac{10}{60} \text{ h}$
 $= 2 \text{ h} + 0,16 \text{ h}$
 $= 2,16 \text{ h}$
- Larry: Speed = $\frac{26\,000\,000}{120 \text{ minutes}}$ $\frac{26\,000\,000 \text{ cm}}{2 \text{ h}}$ $\frac{26\,000\,000 \text{ cm}}{2 \text{ h}}$
 $= 260 \text{ km}$ $= 260 \text{ km}$ $= 130 \text{ km/h}$
- NOTE!** Don't round off until your final answer.
- $\therefore$ Larry is travelling the fastest.

Convert both speeds into the same units i.e. km/h



$$120 \text{ min} = 120 \div 60 = 2 \text{ h}$$



3.1 1 hour

3.2 60 km

- 3.3 After Mr Goldman has travelled for 90 minutes it is 8.30 am ($7 \text{ am} + 1,5 \text{ hours} = 8,5 = 8.30 \text{ am}$)
 Distance travelled by Mr Goldman at 8.30 = 90 km
 Distance travelled by Mr Lebelo at 8.30 = 100 km
 $\therefore$ Distance between them = $100 \text{ km} - 90 \text{ km}$
 $= 10 \text{ km}$

Mr Lebelo is 10 km ahead of Mr Goldman at 8.30.

The point of intersection (i.e. where both straight lines cross each other) indicates the point at which both men have travelled the same distance.



$$\begin{aligned}
 3.4 \text{ Ave. Speed} &= \frac{120 \text{ km} - 60 \text{ km}}{08:45 - 08:00} \\
 &= \frac{60 \text{ km}}{45 \text{ min}} \\
 &= \frac{60 \text{ km}}{0,75 \text{ h}} \quad 45 \text{ min} = \frac{45}{60} = 0,75 \text{ h} \\
 &= 80 \text{ km/h}
 \end{aligned}$$



Test Your Understanding



$$1.1 \quad 3 \text{ h} + \frac{45}{60} \text{ h} = 3,75 \text{ hours}$$

$$1.2 \text{ Closing time of shop: } 17:30 + 00:30 = 18:00$$

$$\begin{aligned}
 1.3 \quad 05:30 &\rightarrow 18:00 \\
 &\quad 17 \quad 60 \\
 \therefore 18:00 \\
 &- \quad 05:30 \\
 &\quad 12:30 \quad \therefore 12 \text{ hours } 30 \text{ minutes}
 \end{aligned}$$

$$\begin{aligned}
 1.4.1 \quad \text{Speed} &= \frac{\text{distance}}{\text{time}} \\
 50 \text{ km/h} &= \frac{30 \text{ km}}{\text{time}} \\
 \therefore \text{Time} \times 50 \text{ km/h} &= 30 \text{ km} \\
 \therefore \text{Time} &= \frac{30 \text{ km}}{50 \text{ km/h}} \\
 &= 0,6 \text{ hours} \\
 &= 0,6 \text{ h} \times 60 \text{ min} \\
 &= 36 \text{ minutes}
 \end{aligned}$$

$$\begin{aligned}
 1.4.2 \quad 17:15 + 00:36 &= 17:51 \\
 17:51 - 17:30 &= 21 \text{ minutes late}
 \end{aligned}$$

1.4.3 The Van Zyl family needs to reach the gate in 15 minutes (since they leave at 17:15 and the gates close at 17:30)



$$\begin{aligned}
 \therefore \text{Speed} &= \frac{30 \text{ km}}{15 \text{ min}} \\
 &= \frac{30 \text{ km}}{0,25 \text{ h}} \quad 15 \text{ min} = \frac{15}{60} = 0,25 \text{ h} \\
 &= 120 \text{ km/h}
 \end{aligned}$$

$$\begin{aligned}
 2.1 \quad &\triangleright \text{Distance from start to Mayville} = 82,3 \text{ km} \\
 &\triangleright \text{Total running time} \\
 &\quad = 7 \text{ h } 33 \text{ min } 12 \text{ s} \\
 &\quad = (7 \times 60 \text{ min}) + 33 \text{ min} + \frac{12}{60} \text{ min} \\
 &\quad = 420 \text{ min} + 33 \text{ min} + 0,2 \text{ min} \\
 &\quad = 453,2 \text{ minutes}
 \end{aligned}$$

$$\begin{aligned}
 \triangleright \text{Average speed} &= \frac{\text{distance}}{\text{time}} \\
 &= \frac{82,3 \text{ km}}{453,2 \text{ min}} \\
 &= 0,182 \text{ km/min}
 \end{aligned}$$

$$\begin{aligned}
 \triangleright \text{Distance between Finish and Mayville} \\
 &= 89,3 \text{ km} - 82,3 \text{ km} \\
 &= 7 \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 \triangleright \text{Time taken from Mayville to Finish} \\
 &= \frac{\text{distance}}{\text{speed}} \\
 &= \frac{7 \text{ km}}{0,182 \text{ km/min}} \\
 &= 38,462 \text{ min} \\
 &= 38 \text{ min} + 0,462 \text{ min} \\
 &= 38 \text{ min} + (0,462 \text{ min} \times 60 \text{ s}) \\
 &= 38 \text{ min} + 27,72 \text{ s} \\
 &\approx 38 \text{ min} + 28 \text{ s}
 \end{aligned}$$



$$\begin{aligned}
 \triangleright \text{Approximate finishing time} \\
 &= 7 \text{ hrs } 33 \text{ min } 12 \text{ s} + 38 \text{ min } 28 \text{ s} \\
 &= 7 \text{ hrs } 71 \text{ min } 40 \text{ s} \\
 &= 8 \text{ hours } 11 \text{ minutes } 40 \text{ seconds}
 \end{aligned}$$

2.2 The athlete ran slower from Mayville to the finish than from Pinetown to Mayville. i.e. The portion of the graph from Mayville to the Finish is not as steep as the portion of the graph from Pinetown to Mayville.

3.1 12 February is a Thursday.

3.2 11 - 12 June

3.3 3 Public holidays: Human Rights day (21 March)
Freedom day (27 April)
Workers day (1 May)

NOTE! The following public holidays do not fall within the academic term:

- New Year's day (1 January) - University hasn't opened yet
- Good Friday (3 April) - Vacation time (30 March - 6 April)
- Easter (5 April) - Vacation time (30 March - 6 April)
- Family Day (6 April) - Vacation time (30 March - 6 April)
- Youth Day (16 June) - Vacation commences 13 June

3.4 7 April - 20 May: 30 academic days



Academic days = Mondays → Fridays
and excluding public holidays.

4.1 Bus departs Rocklands at 06:49 and arrives at the Convention centre at 07:13.

$$\begin{aligned}
 &\quad 06 \quad 60 + 13 = 73 \\
 \therefore 07:13 \\
 &- \quad 06:49 \\
 &\quad 00:24 \quad \therefore 24 \text{ minutes}
 \end{aligned}$$

4.2 07:33 bus [arrives at 07:57 at the Convention Centre]

4.3 17:36 [as she catches the 17:13 bus home]

4.4 $\triangleright$ Rocklands to Convention Centre:
06:49 - 07:13 (24 minutes)

$\triangleright$ Convention Centre to Waterfront:
18:37 - 18:50 (13 minutes)

$\triangleright$ Waterfront to Rocklands:
21:09 - 21:16 (7 minutes)

$$\begin{aligned}
 \therefore \text{Total bus travelling time} \\
 &= 24 + 13 + 7 \\
 &= 44 \text{ min}
 \end{aligned}$$



- 4.5 ▶ Sally travels to and from work during the peak fare period.
- ▶ The distance Sally travels per trip (3,7 km) falls within the 0 - 5 km distance group.
- ▶ Number of working days per week (Monday → Friday) = 5 days
- ▶ Total number of working days per month = 5 days × 4 weeks = 20 days
- ▶ Number of bus trips per day (to and from work) = 2 trips
- ▶ Monthly MyCiti Movers fare = R6,30 × 2 trips × 20 days = R252
- ▶ Monthly MyCiti Standard fare = R8,90 × 2 trips × 20 days = R356
- ∴ Sally could save R356 - R252 = R104



- 5.1 Sunrise (05:43) → 5.43 am
Sunset (19:16) → 7.16 pm

- 5.2 Lowest tide: 21:02
- Spring tide: -14:43
6:19 ∴ 6 hours 19 minutes

- 5.3 3 - 8 November as the 'Fish-O-Meter' indicates these are the 'Best fishing days'.

- 5.4 First low tide on 5 November: 07:55
- First low tide on 4 November: -07:11
00:44
∴ 44 min later

- 5.5 Fishing Time out:
- ▶ 2.15 pm - 5.00 pm
16 60
∴ 17:00
- 14:15
2:45 ∴ 2 h 45 min fishing time out



- ▶ Best fishing time: 14:24 - 15:26
∴ 15:26
- 14:24
1:02 ∴ 1 h 2 min best fishing time

$$\begin{aligned} \% &= \frac{\text{best fishing time}}{\text{fishing time out}} \times 100 \\ &= \frac{1 \text{ h } 2 \text{ min}}{2 \text{ h } 45 \text{ min}} \times 100 \\ &= \frac{62 \text{ min}}{165 \text{ min}} \times 100 \\ &= 37,58\% \end{aligned}$$

$$\begin{aligned} 1 \text{ h } 2 \text{ min} &= 60 + 2 = 62 \text{ min} \\ 2 \text{ h } 45 \text{ min} &= 120 + 45 = 165 \text{ min} \end{aligned}$$



UNIT 7

CALCULATING PERIMETER, AREA, TOTAL SURFACE AREA AND VOLUME

EXERCISE 1

1. Hypotenuse² = side₁² + side₂² (Pythagoras)
Length of cable² = (50)² + (20)²
Length of cable² = 2 900
Length of cable = $\sqrt{2 900}$
Length of cable = 53,85 m

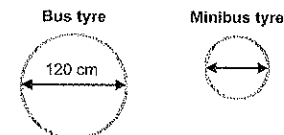
- 2.1 120 ÷ 1,5 = 80 times
∴ 80 × 100 m = 8 000 m

- 2.2 Time = $\frac{\text{distance}}{\text{speed}}$
= $\frac{8 000 \text{ m}}{1,8 \text{ m/s}}$
= 4 444,44 s (+ 60)
= 74,074 minutes
≈ 74 minutes and 5 seconds

$$\begin{aligned} 1 \text{ minute} &= 60 \text{ seconds} \\ \therefore 0,074 \times 60 &= 4,44 \text{ seconds} \\ \therefore \text{more than } 4 \text{ seconds} \\ \therefore 5 \text{ seconds} \end{aligned}$$



3.



First calculate the diameter of the minibus (taxi) tyre:
We know the ratio of the bus tyre to minibus tyre is

$$\times 10 \quad 12:7 \quad \div 10$$

∴ Diameter of minibus tyre = 70 cm

It can also be calculated like this:

$$120 \times \frac{7}{12} = 70 \text{ cm}$$

Since the formula to calculate circumference needs the **radius** of the tyre:

$$\text{radius} = \frac{1}{2} \cdot \text{diameter} = \frac{1}{2} \cdot 70 = 35 \text{ cm}$$

- ∴ We can now work out the circumference of the tyre:
circumference = $2 \times \pi \times r$
= $2 \times 3,142 \times 35$
= 219,94 cm

Using this information we can now calculate the distance travelled:

$$\begin{aligned} \text{Number of rotation} &= \frac{\text{distance travelled}}{\text{circumference of tyre}} \\ \therefore 1 862 &= \frac{\text{distance travelled}}{219,94} \end{aligned}$$

$$\begin{aligned} \therefore \text{Distance travelled} &= 1 862 \times 219,94 \\ &= 409 528,28 \text{ cm} \\ &= 4,095 \text{ km} \approx 4 \text{ km} \end{aligned}$$

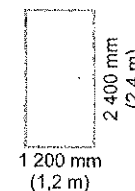
We divide by 100 000 to convert cm to km.

EXERCISE 2

- 1.1 Dimensions
= (800 + 400) × (500 + 500 + 500 + 900)
= 1 200 mm × 2 400 mm
= 1,2 m × 2,4 m

- 1.2 Side panels:

$$\begin{aligned} A &= \ell \times b \\ &= (1,2 \times 0,5) \times 2 \text{ panels} \\ &= 0,6 \times 2 \\ &= 1,2 \text{ m}^2 \end{aligned}$$

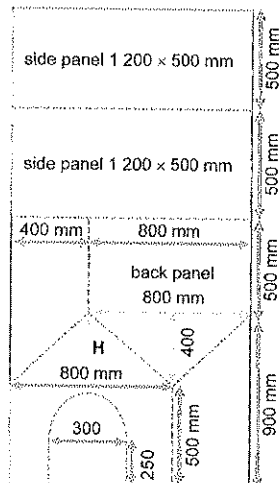


1.3 $H = 900 - 500 = 400 \text{ mm}$

1.4 Back panel
= rectangle + triangle
 $= (\ell \times b) + \left(\frac{1}{2} \cdot b \cdot h\right)$
 $= (800 \times 500) + \left(\frac{1}{2} \cdot 800 \cdot 400\right)$

$800 \text{ mm} = 0,8 \text{ m}$
 $400 \text{ mm} = 0,4 \text{ m}$
 $500 \text{ mm} = 0,5 \text{ m}$

$= (0,8 \times 0,5) + \left(\frac{1}{2} \cdot 0,8 \cdot 0,4\right)$
 $= 0,4 \text{ m} + 0,16 \text{ m}$
 $= 0,56 \text{ m}^2$



1.5 Door = rectangle + $\frac{1}{2}$ circle

$= (\ell \times b) + \left(\frac{1}{2} \cdot \pi r^2\right)$
 $= (0,30 \times 0,25) + \left(\frac{1}{2} \times 3,142 \times 0,15^2\right)$
 $= 0,075 + 0,0353$
 $= 0,11 \text{ m}^2$

$250 \text{ mm} = 0,25 \text{ m}$
 $300 \text{ mm} = 0,3 \text{ m}$
 $\text{diameter} = \frac{0,3}{2}$
 $= 0,15 \text{ m}$

1.6 Area front panel = area back panel - area door
 $= 0,56 - 0,11 = 0,45 \text{ m}^2$

1.7 TSA = front + back + side panels
 $= 0,45 + 0,56 + 1,2 = 2,21 \text{ m}^2$

1.8 Unshaded area = (total area of plywood - used area)
 $= (1,2 \times 2,4) - 2,21$
 $= 2,88 - 2,21 = 0,67 \text{ m}^2$

2. Convert cm to m because the height of the door is in m.

2.1 Area window opening: $(160 \text{ cm} \times 130 \text{ cm})$
 $= 1,6 \text{ m} \times 1,3 \text{ m}$
 $= 2,08 \text{ m}^2$

Area door opening: $2,08 \text{ m}^2 + (9\% \text{ of } 2,08 \text{ m}^2)$
 $= 2,08 \text{ m}^2 + \left(\frac{9}{100} \times 2,08 \text{ m}^2\right)$
 $= 2,08 \text{ m}^2 + 0,1872$
 $= 2,2672 \text{ m}^2$

Area door opening = length $\times$ breadth/width
 $2,2672 \text{ m}^2 = 2,14 \text{ m} \times \text{width}$
 $\frac{2,2672}{2,14} = \frac{2,14 \times \text{width}}{2,14}$
 $1,06 \text{ m} = \text{width of the door}$

2.2.1 Height of walls: $2\,400 \text{ mm} = 2,4 \text{ m}$
Area walls = $2 (3,304 \text{ m} \times 2,4 \text{ m})$
 $+ 2(2,984 \text{ m} \times 2,4 \text{ m})$
 $= 15,8592 \text{ m}^2 + 14,3232 \text{ m}^2$
 $= 30,1824 \text{ m}^2$

Area walls excluding door and window
 $= 30,1824 \text{ m}^2 - 2,2672 \text{ m}^2 - 2,08 \text{ m}^2$
 $= 25,84 \text{ m}^2$

Only round off at the end.

2.2.2 Area of both bedrooms:
 $25,84 \text{ m}^2 + 28,44 \text{ m}^2 = 54,28 \text{ m}^2$

No. of litres of paint = $\frac{54,28 \text{ m}^2}{4 \text{ m}^2} = 13,57 \ell$

Paint is only sold in 5 litre tins.
Mrs Wong must buy three 5-litre tins.

Cost: $3 \times \text{R}169,99 = \text{R}509,97$

Her estimation was incorrect. It will cost just over R500 to paint the two rooms. (R9,97 more)

EXERCISE 3

1. $\triangleright$ TSA of Cambrieni cheese box
 $= (2 \times \text{length} \times \text{breadth}) + (2 \times \text{length} \times \text{height})$
 $+ (2 \times \text{breadth} \times \text{height})$
 $= (2 \times 5 \times 5) + (2 \times 5 \times 1,5) + (2 \times 5 \times 1,5)$
 $= 50 + 15 + 15$
 $= 80 \text{ cm}^2$

$\triangleright$ TSA of Laughing Cow cheese box
 $= [2 \times \pi \times (\text{radius})^2] + [2 \times \pi \times \text{radius} \times \text{height}]$
 $= [2 \times 3,142 \times (3,5)^2] + [2 \times 3,142 \times 3,5 \times 2]$
 $= 76,979 + 43,988$
 $= 120,97 \text{ cm}^2$

$\text{Radius} = \frac{7}{2} = 3,5$

$\therefore$ the Cambrieni cheese will require the least amount of cardboard for the cheese box.

A28

REMEMBER! Only round off your final answer to 2 decimal places!



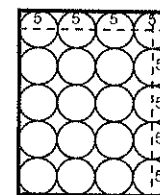
2.1 TSA = $[2 \times 3,142 \times (2,5)^2] + [2 \times 3,142 \times 2,5 \times 12]$
 $= 39,275 + 188,52$
 $= 227,80 \text{ cm}^2$

Radius $5 \div 2 = 2,5$



2.2 TSA = $2 \times \pi \times \text{radius} \times \text{height}$
 $= 2 \times 3,142 \times 2,5 \times 8$
 $= 125,68 \text{ cm}^2$

2.3

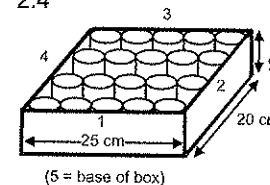


Dimensions of one tin:
diameter = 5 cm

$\therefore$ dimensions of the box:

breadth = $5 \text{ cm} \times 4 = 20 \text{ cm}$
length = $5 \text{ cm} \times 5 = 25 \text{ cm}$

2.4



(5 = base of box)

5 areas (surfaces) need to be calculated and then added together.

height = 12 cm
breadth = 20 cm
length = 25 cm



Surfaces 1 and 3 will be the same:

Area = $2 \times (\ell \times b)$
 $= 2 \times (25 \times 12) = 600 \text{ cm}^2$

Surfaces 2 and 4 will be the same:

Area = $2 \times (\ell \times b)$
 $= 2 \times (20 \times 12) = 480 \text{ cm}^2$

Surface 5 is the base area of the box:

Area = $(\ell \times b)$
 $= 25 \times 20$
 $= 500 \text{ cm}^2$

TSA = $600 + 480 + 500 = 1\,580 \text{ cm}^2$

3.1 $\triangleright$ Front and back of house = $2 \times (\ell \times b)$
 $= 2 \times 100 \times 60$
 $= 12\,000 \text{ cm}^2$

$\triangleright$ Two sides of house = $2 \times (\ell \times b)$
 $= 2 \times 80 \times 60$
 $= 9\,600 \text{ cm}^2$



$$\begin{aligned} \text{Front and back of roof} &= 2 \times \left(\frac{1}{2} \times b \times \perp h \right) \\ &= 2 \times \frac{1}{2} \times 100 \times 50 \\ &= 5\,000 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Two sides of roof} &= 2 \times (\ell \times b) \\ &= 2 \times 80 \times 71 \\ &= 11\,360 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Therefore, TSA of house and roof} &= 12\,000 + 9\,600 + 5\,000 + 11\,360 \\ &= 37\,960 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 3.2 \text{ No. of ml of paint needed} &= 37\,960 \text{ cm}^2 \div 10 \text{ cm}^2 \\ &= 3\,796 \text{ ml} \end{aligned}$$

$$\begin{aligned} \text{No. of tins of paint} &= 3\,796 \text{ ml} \div 500 \text{ ml} \\ &= 7,592 \\ &\approx 8 \text{ tins} \end{aligned}$$

$$\begin{aligned} \text{Total cost of paint} &= 8 \text{ tins} \times \text{R}27,99 \\ &= \text{R}223,92 \end{aligned}$$

EXERCISE 4

$$\begin{aligned} 1.1 \text{ } V &= 9 \times 2,75 \times 0,07 \\ &= 1,73 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} 1.2 \text{ } 95\% \text{ of Volume} &= \frac{95}{100} \times 1,73 \text{ m}^3 \\ &= 1,6435 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{No. of bags of sand} &= \frac{1,6435 \text{ m}^3}{0,75 \text{ m}^3} \\ &= 2,19 \text{ bags} \\ &\approx 3 \text{ bags} \end{aligned}$$

$$\begin{aligned} \text{Total cost of sand} &= \text{R}129,95 \times 3 \text{ bags} \\ &= \text{R}389,85 \end{aligned}$$

$$\begin{aligned} 2.1 \text{ Volume of wood} &= \ell \times b \times h \\ &= 80 \times 30 \times 45 \\ &= 108\,000 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of door stop} &= \pi \times r^2 \times h \\ &= 3,142 \times (17,5)^2 \times 77 \\ &= 74\,092,29 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of left-over wood} &= 108\,000 - 74\,092,29 \\ &= 33\,907,71 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} 0,3 \text{ m} &= 0,3 \times 100 = 30 \text{ cm} \\ 0,35 \text{ m} &= 0,35 \times 100 = 35 \text{ cm} \\ \text{Radius} &= 35 \div 2 = 17,5 \text{ cm} \end{aligned}$$



$$\begin{aligned} 2.2 \text{ Volume of 1 wooden block} &= 12 \times 50 \times 6 \\ &= 3\,600 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{No. of blocks from left-over wood} &= \frac{33\,907,71}{3\,600} \\ &= 9,42 \text{ blocks} \\ &\approx 9 \text{ blocks} \end{aligned}$$

Therefore, Lazarus would not have enough wood for 10 blocks.

$$3.1 \text{ 26 litres} = 26 \times 1\,000 \text{ cm}^3 = 26\,000 \text{ cm}^3$$

$$3.2 \text{ } V = \ell \times b \times h \quad \therefore b = \frac{V}{\ell \times h} = \frac{26\,000}{40 \times 25} = 26 \text{ cm}$$

$$\begin{aligned} 3.3 \text{ length} &= 25 \div 5 \\ &= 5 \text{ tins} \end{aligned}$$

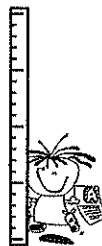
$$\begin{aligned} 3.4 \text{ breadth} &= 26 \div 5 \\ &= 5,2 \\ &\approx 5 \text{ tins} \end{aligned}$$

$$\begin{aligned} 3.5 \text{ height} &= 40 \div 16 \\ &= 2,5 \\ &\approx 2 \text{ tins} \end{aligned}$$

$$\begin{aligned} 3.6 \text{ Number of tins} &= \ell \times b \times h \\ &= 5 \times 5 \times 2 \\ &= 50 \text{ tins} \end{aligned}$$

3.7 No. There was space along the breadth and the height that the tins could not occupy. Also, the tins are cylindrical $\therefore$ there will be spaces between the tins when they are packed together.

You cannot round up to 3 tins - they will not fit!



3.8 Volume of the cooler bag:

$$V = 26\,000 \text{ cm}^3$$

Volume of one tin:

$$\begin{aligned} V &= \pi r^2 \times h \\ &= \pi \times 2,5^2 \times 16 \\ &= 3,142 \times 2,5^2 \times 16 \\ &= 314,2 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of the cooler bag} \div \text{volume of a tin} &= 26\,000 \text{ cm}^3 \div 314,2 \text{ cm}^3 \\ &= 82,75 \end{aligned}$$

$\therefore$ 82 tins, which is far more than 50!

**Test Your Understanding**

1.1 First work out the perimeter of the pool area:

$$\begin{aligned} \text{Perimeter of square} &= 4 \times \text{side length} \\ &= 4 \times (3 + 10 + 4) \\ &= 4 \times 17 = 68 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{length of fencing needed} &= \text{total perimeter} - \text{length of gate} \\ &= 68 - 1,3 \\ &= 66,7 \text{ m} \end{aligned}$$

Now work out the number of 5 m rolls of fencing needed:

$$\begin{aligned} \text{Number of rolls} &= \frac{\text{length needed}}{5 \text{ m}} \\ &= \frac{66,7}{5} \\ &= 13,34 \text{ rolls} \\ &\approx 14 \text{ rolls} \end{aligned}$$

$$\begin{aligned} \therefore \text{Cost of fencing} &= \text{cost per roll} \times \text{number of rolls} \\ &= \text{R}389,95 \times 14 \\ &= \text{R}5\,459,30 \end{aligned}$$

NOTE! The context of the question when rounding off your answer. If you only bought 13 rolls of fencing, you would be short by 0,34 m. $\therefore$ You round up your answer in this case.



1.2 First work out the total area of square:

$$\begin{aligned}\text{Area} &= \text{side} \times \text{side} \\ &= 17 \times 17 \\ &= 289 \text{ m}^2\end{aligned}$$

Now work out the area of the semi-circular pool:

$$\begin{aligned}\text{Area of semi-circle} &= \frac{1}{2} \times \pi \times r^2 \\ &= \frac{1}{2} \times 3,142 \times (1,5)^2 \\ &= 3,53475 \text{ m}^2\end{aligned}$$

Kiddies pool is half a circle.

radius = $3 \text{ m} \div 2 = 1,5 \text{ m}$

Now work out the area of the wall:

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{breadth} \\ &= 3 \times 0,5 \\ &= 1,5 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{Area to be paved (excl. rectangular pool)} &= \text{area of square} - \text{area of semicircle} - \text{area of wall} \\ &= 289 - 3,53475 - 1,5 \\ &= 283,96525 \\ &\approx 284 \text{ m}^2\end{aligned}$$

$\therefore$ The area of the rectangular pool can now be calculated:

$$\begin{aligned}\text{Area of rectangular pool} &= \text{area paved (excl. rectangular pool)} - \text{net area to be paved} \\ &= 284 - 244 = 40 \text{ m}^2\end{aligned}$$

Now, to work out the breadth of the pool:

$$\begin{aligned}\therefore \text{Area of rectangular pool} &= \text{length} \times \text{breadth} \\ \therefore \text{breadth} &= \frac{\text{area}}{\text{length}} = \frac{40}{10} = 4 \text{ m}\end{aligned}$$

1.3.1 First work out the volume of the kiddies pool:

$$\begin{aligned}V &= \frac{1}{2} \times \pi \times r^2 \times h \\ &= \frac{1}{2} \times 3,142 \times (1,5)^2 \times 0,45 \\ &= 1,59 \text{ m}^3\end{aligned}$$

45 cm = $45 \div 100 = 0,45 \text{ m}$

$$\begin{aligned}\therefore \text{Total volume} &= 61 \text{ m}^3 + 1,59 \text{ m}^3 \\ &= 62,59 \text{ m}^3 \\ &\approx 63 \text{ m}^3\end{aligned}$$

$$1.3.2 \quad 63 \text{ m}^3 \times 95\% = 63 \times \frac{95}{100} = 59,85 \text{ m}^3 \approx 60 \text{ k\ell}$$

2. Store room

$$\begin{aligned}\text{Area of side walls} &= 2 \times (1,8 \text{ m} \times 2,7 \text{ m}) \\ &= 2 \times 4,86 \text{ m}^2 \\ &= 9,72 \text{ m}^2\end{aligned}$$

$$\text{Area of back wall} = 2,5 \text{ m} \times 2,7 \text{ m} = 6,75 \text{ m}^2$$

$$\begin{aligned}\text{Area of front wall (with the door)} &= \text{area of whole wall} - \text{area of door} \\ &= (2,5 \text{ m} \times 2,7 \text{ m}) - (2,2 \text{ m} \times 0,8 \text{ m}) \\ &= 6,75 \text{ m}^2 - 1,76 \text{ m}^2 \\ &= 4,99 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Surface area of the storeroom to be painted} &= 9,72 \text{ m}^2 + 6,75 \text{ m}^2 + 4,99 \text{ m}^2 \\ &= 21,46 \text{ m}^2\end{aligned}$$

Garage

$$\begin{aligned}\text{Area of front wall (with window only)} &= \text{area of whole wall} - \text{area of window} \\ &= (6 \text{ m} \times 2,7 \text{ m}) - (1,2 \text{ m} \times 0,85 \text{ m}) \\ &= 16,2 \text{ m}^2 - 1,02 \text{ m}^2 \\ &= 15,18 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of back wall (with door only)} &= \text{area of whole wall} - \text{area of door} \\ &= (6 \text{ m} \times 2,7 \text{ m}) - (2,2 \text{ m} \times 0,8 \text{ m}) \\ &= 16,2 \text{ m}^2 - 1,76 \text{ m}^2 \\ &= 14,44 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of side wall (with window and door)} &= \text{area of whole wall} - \text{area of door} - \text{area of window} \\ &= (3,6 \text{ m} \times 2,7 \text{ m}) - (2,2 \text{ m} \times 0,8 \text{ m}) - (0,7 \text{ m} \times 0,5 \text{ m}) \\ &= 9,72 \text{ m}^2 - 1,76 \text{ m}^2 - 0,35 \text{ m}^2 \\ &= 7,61 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Surface area of the garage to be painted} &= 15,18 \text{ m}^2 + 14,44 \text{ m}^2 + 7,61 \text{ m}^2 \\ &= 37,23 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area of the walls to be painted} &= 21,46 \text{ m}^2 + 37,23 \text{ m}^2 \\ &= 58,69 \text{ m}^2 \\ &\approx 58,7 \text{ m}^2\end{aligned}$$

rounded off to one decimal place

Undercoat

$$\begin{aligned}\text{Paint coverage} &= 9 \text{ m}^2 : 1 \ell \\ &\rightarrow 1 \text{ m}^2 : (1 \ell \div 9)\end{aligned}$$

$$58,7 \text{ m}^2 : (1 \ell \div 9) \times 58,7 \approx 6,5 \ell$$

rounded off to one decimal place

Possible options for Bob's Builders:

$$\text{Buy } 4 \times 2 \ell \text{ tins} = 8 \ell$$

$$\text{Cost} = 4 \times \text{R}65,00 = \text{R}260,00$$

$$\text{Buy } 2 \times 5 \ell \text{ tins} = 10 \ell$$

$$\text{Cost} = 2 \times \text{R}112,50 = \text{R}225,00$$

$$\text{Buy } 1 \times 5 \ell + 1 \times 2 \ell = 7 \ell$$

$$\text{Cost} = 1 \times \text{R}112,50 + 1 \times \text{R}65,00 = \text{R}177,50$$

So, the best option would be for Bob's Builders to buy $1 \times 5 \ell$ tin and $1 \times 2 \ell$ tin of undercoat.

Acrylic

Bob's Builders will be painting two coats of Acrylic paint. So, quantity of paint needed is for:

$$58,7 \text{ m}^2 \times 2 = 117,4 \text{ m}^2$$

$$\text{Paint coverage} = 7,5 \text{ m}^2 : 1 \ell$$

$$\rightarrow 1 \text{ m}^2 : (1 \ell \div 7,5)$$

$$\begin{aligned}117,4 \text{ m}^2 : (1 \ell \div 7,5) &\times 117,4 \\ &\approx 15,7 \ell\end{aligned}$$

rounded off to one decimal place

Possible options for Bob's Builders:

$$\text{Buy } 3 \times 5 \ell \text{ tins} + 1 \times 1 \ell = 16 \ell$$

$$\begin{aligned}\text{Cost} &= 3 \times \text{R}82,00 + \text{R}48,00 = \text{R}246,00 + \text{R}48,00 \\ &= \text{R}294,00\end{aligned}$$

$$\text{Buy } 1 \times 10 \ell \text{ tins} + 1 \times 5 \ell + 1 \times 1 \ell = 16 \ell$$

$$\text{Cost} = \text{R}147,00 + \text{R}82,00 + \text{R}48,00 = \text{R}277$$

So, the best option would be for Bob's Builders to buy $1 \times 10 \ell + 1 \times 5 \ell + 1 \times 1 \ell$ tins of Acrylic paint.

$$3.1.1 \quad A = \pi \times r^2$$

$$\begin{aligned}&= 3,142 \times (0,25)^2 \\ &= 0,2 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Radius} &= 0,5 \text{ m} \div 2 \\ &= 0,25 \text{ m}\end{aligned}$$

3.1.2 Grass needed

$$\begin{aligned}&= \text{area garden} - \text{area stone tiles} - \text{area pond} \\ &= (10 \text{ m} \times 4 \text{ m}) - (5 \times 0,20 \text{ m}^2) - (2 \text{ m} \times 2 \text{ m}) \\ &= 40 \text{ m}^2 - 1 \text{ m}^2 - 4 \text{ m}^2 \\ &= 35 \text{ m}^2\end{aligned}$$

$$3.1.3 \quad \text{Cost of grass: } \text{R}25 \text{ per } \frac{1}{2} \text{ m}^2 = \text{R}50 \text{ per m}^2$$

$$35 \text{ m}^2 \times \text{R}50 = \text{R}1\,750$$

OR

$$70 \text{ m}^2 \times \text{R}25 = \text{R}1\,750$$



3.2.1 Volume = $2\text{ m} \times 2\text{ m} \times 1,5\text{ m}$
 $= 6\text{ m}^3$

$150\text{ cm} = 150 \div 100$
 $= 1,5\text{ m}$

3.2.2 $6\text{ m}^3 = 6\,000\text{ l}$

$\therefore$ Water needed = $\frac{90}{100} \times 6\,000\text{ ml}$
 $= 5\,400\text{ l}$

3.2.3 No. of litres/fish = $200\text{ l} \div 2 = 100\text{ l}$

$\therefore$ No. of fish in pond = $\frac{5\,400\text{ l}}{100\text{ l}}$
 $= 54\text{ fish}$

3.2.4 $\triangleright$ TSA = bottom of pond + $4 \times$ side walls
 $= (2 \times 2) + (4 \times 2 \times 1,5)$
 $= 4 + 12$
 $= 16\text{ m}^2$

$\triangleright$ Paint needed = $\frac{16\text{ m}^2}{10\text{ m}^2}$
 $= 1,6\text{ l}$



3.3 $1\text{ h} = 60\text{ min}$

$\therefore$ Amount of water = $20\text{ l} \times 60\text{ min}$
 $= 1\,200\text{ l}$

4.1 Perimeter = sum of all sides
 $= 250 + 200 + 150 + 200 + 224$
 $= 1\,024\text{ m}$

4.2 Height = 2 m
 Radius = 10 m

Volume = $\pi \times \text{radius}^2 \times \text{height}$
 $= 3,142 \times 10^2 \times 2$
 $= 628,4\text{ m}^3$



NOTE! Slant height is irrelevant to the calculation, as we are interested in the $\perp$ height.

4.3 $\perp$ Height = 200 m
 Base = 200 m

Area of triangle = $\frac{1}{2} \times \text{base} \times \perp \text{ height}$
 $= \frac{1}{2} \times 200 \times 200$
 $= 20\,000\text{ m}^2$

4.4 Area of a trapezium
 $= \frac{1}{2} \times (\text{sum of parallel sides}) \times \perp \text{ height}$
 $= \frac{1}{2} \times (250 + 200 + 150) \times 200$
 $= \frac{1}{2} \times 600 \times 200$
 $= 60\,000\text{ m}^2$

The $\perp$ height of the plot is the side that is perpendicular to the parallel sides (200 m).

5.1 Height of box = 143 mm
 % height of box compared to bottle = 102%
 Height of bottle = ?

If 143 is 102% of the bottle, we need to find out the value of 100% :

$\therefore 143 \times \frac{100}{102} = 140,196 \approx 140\text{ mm}$

5.2 First work out the area of the base of the bottle:
 Area of circle = $\pi \times \text{radius}^2$

$= \pi \times \left(\frac{\text{diameter}}{2}\right)^2$
 $= 3,142 \times \left(\frac{58}{2}\right)^2$
 $= 3,142 \times 29^2$
 $= 3,142 \times 841$
 $= 2\,642,42\text{ mm}^2$
 $= 26,42\text{ cm}^2$



REMEMBER!
 radius = $\frac{\text{diameter}}{2}$

To work out the area of the base of the box we first need to find the length of the side of the box:

Diameter of bottle base = 58 mm
 % length of box compared to diameter of bottle = 105%
 Length of box = $58 \times 105\%$
 $= 58 \times \frac{105}{100}$
 $= 60,9\text{ mm}$

$\therefore$ Area of base of box = $(\text{side length})^2$
 $= 60,9^2$
 $= 3\,708,81\text{ mm}^2$
 $= 37,09\text{ cm}^2$

$\therefore$ Difference between area of bottle base and box base
 $= 37,09 - 26,42 = 10,67\text{ cm}^2$

$10,67\text{ cm}^2 < 11\text{ cm}^2$

$\therefore$ the dimensions do satisfy this guideline

5.3.1 Area of section B is the same as the base of the box calculated above = $3\,708,81\text{ mm}^2$

The area of section C can be worked out using the dimensions of B:

Length of base of box (side length) = $60,9\text{ mm}$

$\therefore$ diameter of semicircle C = $60,9\text{ mm}$

$\therefore$ radius of C = $\frac{60,9}{2} = 30,45\text{ mm}$

$\therefore$ Area of 2C = $\pi \times \text{radius}^2$
 $= 3,142 \times 30,45^2$
 $= 3,142 \times 927,2025$
 $= 2\,913,42\text{ mm}^2$

REMEMBER!
 C is a semi-circle, so you must divide the area of the circle by 2.

$\therefore$ C = $\frac{2\,913,42}{2}$
 $= 1\,456,6351\text{ mm}^2$

Area of A = height $\times$ side length
 $= 143 \times 60,9$
 $= 8\,708,7\text{ mm}^2$

$\therefore$ Area of opened cardboard box
 $= 4(A + D) + 2(B + C) + E$
 $= 4(8\,708,7 + 1\,832) + 2(3\,708,81 + 1\,456,6351) + 2\,855$
 $= 4(10\,540,7) + 2(5\,165,4451) + 2\,855$
 $= 42\,162,8 + 10\,330,89 + 2\,855$
 $= 55\,348,69\text{ mm}^2$
 $= 553,49\text{ cm}^2$
 $= 0,055349\text{ m}^2$



$\therefore$ Mass of cardboard needed
 $= 240 \times 0,055349$
 $= 13,284\text{ g} \approx 14\text{ g}$

5.3.2 First convert from g to kg:

$1\text{ g} = 0,001\text{ kg}$

$\therefore 14\text{ g} = 0,014\text{ kg}$

$\therefore$ Mass of box in kg = $0,014\text{ kg}$

Total cost

$= R16 + (\text{mass of cardboard box in kg}) \times R20\text{ per kg}$
 $= 16 + (0,014) \times R20,00$
 $= 16 + 0,28$
 $= R16,28$

MODULE 5

MAPS, PLANS AND REPRESENTATIONS

UNIT 1

1

SCALE

EXERCISE 1

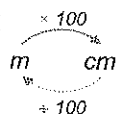
1. $A = 1,5 \text{ m} \div 50 = 0,03 \text{ m} = 3 \text{ cm}$

$B = 4 \text{ cm} \times 50 = 200 \text{ cm} = 2 \text{ m}$

$C = 0,8 \text{ m} \div 50 = 0,016 \text{ m} = 1,6 \text{ cm}$

$D = 2,4 \text{ cm} \times 50 = 120 \text{ cm} = 1,2 \text{ m}$

Actual length
= map length
 $\times$ scale factor
OR
Map length =
actual length
 $\div$ scale factor



REMEMBER! Only round off your final answer to 2 decimal places!

2. Actual height = 1,2 m

Height on coin = 15 mm
= 0,015 m

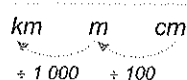
$\therefore \text{Scale} = \frac{0,015 \text{ m}}{1,2 \text{ m}} = \frac{1}{80}$

$\text{Scale} = \frac{\text{map length}}{\text{actual length}}$

$\therefore \text{Scale is } 1 : 80$

3.1.1 Map distance from Somerset West to Arniston = $\pm 8,4 \text{ cm}$

$\therefore$ Actual distance from Somerset West to Arniston
= $8,4 \text{ cm} \times 4\,000\,000$
= $33\,600\,000 \text{ cm}$
= 336 km



3.1.2 Map distance from Somerset West to Hermanus = $\pm 3,2 \text{ cm}$

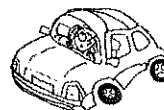
$\therefore$ Actual distance from Somerset West to Hermanus
= $3,2 \text{ cm} \times 4\,000\,000$
= $12\,800\,000 \text{ cm}$
= 128 km

3.2.1 Actual distance to Arniston = 336 km (from Q.3.1.1)
Petrol consumption = $7,4 \text{ l}/100 \text{ km}$

$\therefore$ No. of litres of petrol = $\frac{672 \text{ km}}{100 \text{ km}} \times 7,4 \text{ l}$
= $49,73 \text{ l}$

3.2.2 Total cost of petrol = $49,73 \text{ l} \times R9,79$
= $R486,86$

Cost per boy = $R486,86 \div 3$
= $R162,29$

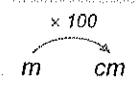


EXERCISE 2

1.1 1,2 cm on the plan represents 0,9 m in reality

$\therefore 1,2 \text{ cm} : 0,9 \text{ m}$
 $1,2 \text{ cm} : 90 \text{ cm}$

$\frac{1,2}{1,2} : \frac{90}{1,2}$
 $1 : 75$



$\therefore 1 \text{ cm on the plan} = 75 \text{ cm in actual length.}$

1.2

Dimension	Measurement on Plan (cm)	Actual Measurement (metres)
Wall length	4,9 cm	$4,9 \text{ cm} \times 75 = 367,5 \text{ cm} = 3,68 \text{ m}$
Wall height	2,4 cm	$2,4 \text{ cm} \times 75 = 180 \text{ cm} = 1,8 \text{ m}$
Roof height	1,2 cm	$1,2 \text{ cm} \times 75 = 90 \text{ cm} = 0,9 \text{ m}$
Window length	1,6 cm	$1,6 \text{ cm} \times 75 = 120 \text{ cm} = 1,2 \text{ m}$
Window height	0,8 cm	$0,8 \text{ cm} \times 75 = 60 \text{ cm} = 0,6 \text{ m}$

Actual length = plan length $\times$ scale factor

2.1.1 $2,3 \text{ cm} : 250 \text{ m}$
 $2,3 \text{ cm} : 25\,000 \text{ cm}$
 $\frac{2,3}{2,3} : \frac{25\,000}{2,3}$
 $1 : 10\,869,57$
 $\approx 1 : 10\,870$

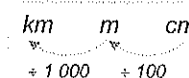
First determine the numerical scale using the bar scale and the length of the bar scale on the map.

Round-up to the nearest whole number.



Distance from the North End Railway Station to the Police Station on the map = 4,3 cm

$\therefore$ Actual distance
= $4,3 \text{ cm} \times 10\,870$
= $46\,741 \text{ cm}$
= $0,47 \text{ km}$



$\text{Speed} = \frac{\text{distance}}{\text{time}}$



2.1.2 Time = $\frac{\text{distance}}{\text{speed}}$
= $\frac{0,47 \text{ km}}{10 \text{ km/h}}$
= $0,047 \text{ h}$
= $0,046 \text{ h} \times 60 \text{ min}$
= $2,82 \text{ minutes}$

Refer to Module 4: Measurement to convert fractions of time into different units.



2.2.1 Distance from the Police Station to the Nelson Mandela Bay Stadium entrance on the map = 5,7 cm

$\therefore$ Actual distance
= $5,7 \text{ cm} \times 10\,870$
= $61\,959 \text{ cm}$
= $0,62 \text{ km}$

Scale factor of 10 870 calculated in Question 2.1.1.

2.2.2 $17 : 38$
 $0 : 23$
 $18 : 01 \text{ min}$

$\therefore$ they will be late for the start of the soccer match by 1 minute!

Test Your Understanding



$$\begin{aligned}
 1. \text{ Scale} &= \frac{\text{map length}}{\text{actual length}} \\
 &= \frac{13,5 \text{ cm}}{325 \text{ m}} \\
 &= \frac{0,135 \text{ m}}{325 \text{ m}} \\
 &= \frac{27}{65\,000}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Scale is } \frac{27}{27} &: \frac{65\,000}{27} \\
 &1 : 2\,407,41 \\
 &\approx 1 : 2\,407
 \end{aligned}$$

REMEMBER!
A scale is
always written in
the form 1: ...



$$\begin{aligned}
 2.1 \text{ Actual distance} &= 18 \text{ cm} \times 500\,000 \\
 &= 9\,000\,000 \text{ cm} \\
 &= 90 \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 2.2 \text{ Map distance} &= 295 \text{ km} \div 500\,000 \\
 &= 0,00059 \text{ km} \\
 &= 59 \text{ cm}
 \end{aligned}$$



$$\begin{aligned}
 3.1 \text{ A length of } 1,5 \text{ cm on diagram represents } 1,0 \text{ m in reality} \\
 \therefore 1,5 \text{ cm} : 1,0 \text{ m} \\
 1,5 \text{ cm} : 100 \text{ cm} \\
 \frac{1,5}{1,5} : \frac{100}{1,5} \\
 1 : 66,67 \\
 \approx 1 : 67
 \end{aligned}$$

$$\begin{aligned}
 \text{Length of library on diagram} &= 8 \text{ cm} \\
 \therefore \text{Actual length} &= 8 \text{ cm} \times 67 = 536 \text{ cm} = 5,36 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Breadth of library on diagram} &= 4 \text{ cm} \\
 \therefore \text{Actual breadth} &= 4 \text{ cm} \times 67 = 268 \text{ cm} = 2,68 \text{ m} \\
 \therefore \text{Actual dimensions of library} &= 5,36 \text{ m} \times 2,68 \text{ m}
 \end{aligned}$$



$$\begin{aligned}
 3.2 \text{ Length of carpet on diagram} &= 7 \text{ cm} \\
 \therefore \text{Actual length of carpet} &= 7 \text{ cm} \times 67 = 469 \text{ cm} \\
 &= 4,69 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Breadth of carpet on diagram} &= 3 \text{ cm} \\
 \therefore \text{Actual breadth of carpet} &= 3 \text{ cm} \times 67 = 201 \text{ cm} \\
 &= 2,01 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Actual area of carpet} &= \ell \times b \\
 &= 4,69 \text{ m} \times 2,01 \text{ m} \\
 &= 9,4 \text{ m}^2 \approx 10 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Cost of carpeting} &= 10 \text{ m}^2 \times R98,20 \\
 &= R982
 \end{aligned}$$

Round up as
carpet is sold
per whole m^2 .

$$4.1.1 \text{ Distance between Pietermaritzburg and Johannesburg on the map} = 2,1 \text{ cm}$$

$$\begin{aligned}
 4.1.2 \text{ } 1 \text{ cm} : 300 \text{ km} \\
 1 \text{ cm} : 30\,000\,000 \text{ cm} \\
 1 : 30\,000\,000
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Actual distance} &= 2,1 \text{ cm} \times 30\,000\,000 \\
 &= 63\,000\,000 \text{ cm} \\
 &= 630 \text{ km}
 \end{aligned}$$

$$4.2 \text{ Distance} = \text{average speed} \times \text{time}$$

$$\begin{aligned}
 \therefore \text{Time} &= \frac{\text{distance}}{\text{speed}} \\
 &= \frac{630 \text{ km}}{110 \text{ km/h}} \\
 &= 5,727 \text{ h} \\
 &= 5 \text{ h} + 0,727 \text{ h} \\
 &= 5 \text{ h} + (0,727 \text{ h} \times 60 \text{ min}) \\
 &= 5 \text{ h} + 43,64 \text{ min} \\
 &\approx 5 \text{ h } 44 \text{ min}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Time elapsed: } 14:30 \\
 - 08:15 \\
 \hline
 06:15
 \end{aligned}$$

$$\begin{aligned}
 08:15 \\
 + 05:44 \\
 \hline
 13:59
 \end{aligned}$$

$\therefore$ They planned to take
6 h 45 min;
but instead they took
approximately 5 h 44 min

$\therefore$ They arrived
31 minutes earlier.

$\therefore$ They arrived at a different time than predicted.

$$\begin{aligned}
 4.3.1 \text{ Cost of a half-full tank} &= 30 \ell \times R10,12 \\
 &= R303,60
 \end{aligned}$$

$\therefore$ Since the cost of refuelling was more than
R303,60 (i.e. R455,40); it means that the
fuel gauge is not working properly.

OR

$$\begin{aligned}
 \text{No. of litres to fill the tank} &= R455,40 \div R10,12 \\
 &= 45 \ell
 \end{aligned}$$

Since $\frac{1}{2}$ tank = 30 ℓ ; the fuel gauge was faulty.

$$\begin{aligned}
 4.3.2 \text{ } 9 \ell \text{ per } 100 \text{ km} \\
 \therefore 1 \ell \text{ per } (100 \div 9) \text{ km} \\
 \therefore 1 \ell \text{ per } 11,11 \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore, since they travelled using } 45 \ell \text{ of petrol} \\
 \text{Distance covered} &= 45 \ell \times 11,11 = 499,95 \text{ km} \\
 \therefore \text{Distance from Johannesburg} \\
 &= 630 \text{ km} - 499,95 \text{ km} \\
 &= 130,05 \text{ km}
 \end{aligned}$$

UNIT 2 MAPS

EXERCISE 1

- Travel west on the N1 and take exit 20. Turn left and travel south on Mike Pienaar Boulevard; crossing over 3 sets of robots (indicated by $\cdot \cdot \cdot$); then the railway tracks (indicated by $+++$); and at the next set of robots, the entrance to the faculty will be on your left.

- The Tygerberg Hospital is south-west from the Fisan Building.

OR

The Fisan Building is north-east from the Tygerberg Hospital.



3. The Francie van Zijl ladies residence is situated on the westerly side of the campus and is the first building on your right as you enter the Faculty of Health Services. The residence is in front of the Tygerberg Student Centre.

EXERCISE 2

1.1 Line 3

1.2 Hatfield

1.3 3 stops (i.e. Malboro, Midrand and then Centurion)

1.4 East-west commuter line / line 2

1.5 Midrand

1.6 Total cost = $(2 \times R49) + (2 \times R135)$
= R368

*Two trips
as the cost
is only for
1-way.*

2.1 927 km

2.2 Total distance = $58 \text{ km} + 342 \text{ km} = 400 \text{ km}$

2.3 Distance from Johannesburg via Kimberley to Cape Town
= $467 + 960$
= 1 427 km

Distance from Johannesburg via Bloemfontein to Cape Town
= $417 + 998$
= 1 415 km

$\therefore$ Travelling via Bloemfontein is the shortest route, by 12 km.

2.4 Average speed: $\frac{\text{distance}}{\text{time}}$
= $\frac{1\,029 \text{ km}}{12,5 \text{ h}}$
= 82,32 km/h

3.1 X6

3.2.1 1,4 cm



3.2.2 A distance of 1 cm on the map represents 2 km in reality

$\therefore 1 \text{ cm} : 2 \text{ km}$

$1 \text{ cm} : 200\,000 \text{ cm}$

$1 : 200\,000$

$\therefore$ Actual distance = $1,4 \text{ cm} \times 200\,000$
= 280 000 cm
= 2 800 m
= 2,8 km

3.3 Distance from party house (B) to hired house (A) is 9,6 cm on map

$\therefore$ Actual distance = $9,6 \text{ cm} \times 200\,000$
= 1 920 000 cm
= 19,2 km

Total cost = $R50 + (19,2 \text{ km} \times R3/\text{km})$
= $R50 + R57,60$
= R107,60

$\therefore$ Cost per friend = $R107,60 \div 4 = R26,90$

4.1 300 km

4.2 Speed = $\frac{\text{distance}}{\text{time}}$

$\therefore$ Time = $\frac{\text{distance}}{\text{speed}}$
= $\frac{300 \text{ km}}{80 \text{ km/h}}$
= 3,75 h
= 3 h + $(0,75 \text{ h} \times 60 \text{ min})$
= 3 h + 45 min

*Look at the
distance table
on p. 160.*



4.3 $\triangleright$ Total distance travelled = $600 \text{ km} + 500 \text{ km}$
= 1 100 km

$\triangleright$ Fixed annual cost = R52 033

$\therefore$ Weekly fixed cost = $R52\,033 \div 52 \text{ weeks}$
= R1 000,63

$\triangleright$ Fuel cost = $1\,100 \text{ km} \times 90,8 \text{ c/km}$
= 99 880c
= R998,80

$\triangleright$ Maintenance cost = $1\,100 \text{ km} \times 32,8 \text{ c/km}$
= 36 080c
= R360,80

$\triangleright$ Total car transport cost
= $R1\,000,63 + R998,80 + R360,80$
= R2 360,23

4.4 A distance of 1 cm on the map represents 2 km in reality.

$\therefore 1 \text{ cm} : 2 \text{ km}$
 $1 \text{ cm} : 200\,000 \text{ cm}$
 $1 : 200\,000$

Distance from Humewood to Summerstrand is 2 cm on the map.

$\therefore$ Actual distance = $2 \text{ cm} \times 200\,000$
= 400 000 cm
= 4 km

$\therefore$ Total distance there and back = $4 \text{ km} \times 2 = 8 \text{ km}$

Therefore, it would not be feasible to walk there and back after supper, as it is too far and may be unsafe.

4.5 Port Elizabeth $\rightarrow$ Coega: 15 km

Coega $\rightarrow$ Addo: 40 km

Addo $\rightarrow$ Coerney: 16 km

Coerney $\rightarrow$ Paterson: 24 km

Paterson $\rightarrow$ N2: 16 km

N2 $\rightarrow$ Port Elizabeth: $15 \text{ km} + 44 \text{ km} = 59 \text{ km}$

$\therefore$ Total distance of round-trip
= $15 + 40 + 16 + 24 + 16 + 59 = 170 \text{ km}$

4.6 South-easterly direction



Test Your Understanding

1.1 Row 5 Column 2

1.2 3 students

1.3 south-east

1.4 $0,75 \text{ m}^2 \times (8 \text{ rows} \times 4 \text{ columns})$
= $0,75 \text{ m}^2 \times 32$
= 24 m^2

- 2.1 ① West stand ② North stand
 ③ East stand ④ South stand

2.2 East stand (i.e. stand ③)

- 2.3 A = North C7 B = North B2
 C = West D16 D = West B6
 E = South A7 F = South D4
 G = East B9 H = East C5

3.1 9 holes

3.2 Any explanation with a good reason, e.g.:

Hole 8 - it appears to be the longest hole and there are two sand bunkers. **OR**

Hole 5 - you have to hit over the lake and then get through two sand bunkers. **OR**

Hole 6 or 9 - there are three sand bunkers.

3.3 Hole 5 - measured on the map it is approximately 125 mm from the edge of the tee off point to the hole.

3.4 Approximately south

3.5.1 The ball will probably fly towards the northeast or the east and end up off the fairway.

3.5.2 The ball will be flying almost directly into the wind which means it will lose speed very quickly and will not travel as far as when there is no wind.

3.6 Distance of 2,5 cm on map represents 250 m in reality

$$\therefore 2,5 \text{ cm} : 250 \text{ m}$$

$$2,5 \text{ cm} : 25\,000 \text{ cm}$$

$$\frac{2,5}{2,5} : \frac{25\,000}{2,5}$$

$$1 : 10\,000$$

Distance on map for Hole 7 = 3,1 cm

$$\therefore \text{Actual distance} = 3,1 \text{ cm} \times 10\,000 \\ = 31\,000 \text{ cm} \\ = 310 \text{ m}$$



REMEMBER!
 Measure from
 'the edge' of the
 tee off point.



4.1 The destination is the restaurant.

4.2 Distance on map is 0,5 cm; which represents 2 km in reality

$$\therefore 0,5 \text{ cm} : 2 \text{ km}$$

$$0,5 \text{ cm} : 200\,000 \text{ cm}$$

$$\frac{0,5}{0,5} : \frac{200\,000}{0,5}$$

$$1 : 400\,000$$

4.3 Distance on map between Entrance 2 and Shop is 1,7 cm

$$\therefore \text{Actual distance} = 1,7 \text{ cm} \times 400\,000 \\ = 680\,000 \text{ cm} \\ = 6,8 \text{ km}$$

5.1.1 79 km

$$5.1.2 \text{ Senekal to Bloemfontein} = 628 - 450 \\ = 178 \text{ km}$$

$$\text{Bloemfontein to Springfontein} = 143 \text{ km}$$

$$\therefore \text{Senekal to Springfontein} = 178 + 143 \\ = 321 \text{ km}$$

$$5.1.3 \text{ Durban to Cape Town} = 628 + 1\,004 \\ = 1\,632 \text{ km}$$

$$5.2.1 \text{ Distance} = 518 - 304 = 214 \text{ km}$$

$$5.2.2 9:15 - 6:45 = 2 \text{ hour } 30 \text{ min}$$

5.2.3 It is recommended that you stop every 2 hours for a rest break. He might have needed to go to the toilet, have something to eat or fill up with petrol.

5.3.1 Litres of petrol : Distance travelled

$$120 : 2\,000$$

$$6 : 100$$

$$\therefore 6 \ell / 100 \text{ km}$$

Express as
a ratio.



$$5.3.2 \frac{120}{45} = 2,6$$

He will use 2,6 tanks of petrol, therefore he needs to stop twice to fill the tank.

Divide the total
petrol consumed by
the amount of petrol
one tank holds.

$$5.4.1 45 \times \frac{1}{4} = 11,25 \ell$$

$$5.4.2 \text{ Distance from Colesberg to Three Sisters} \\ = 467 - 226 \\ = 241 \text{ km}$$

Use the strip chart on p. 164
to determine the distance.

He uses 6 ℓ /100 km, so for the 241 km trip he will need:

$$\frac{241}{100} \times 6 = 14,46 \ell \text{ of petrol}$$

Refer to Question
5.3.2 for the
amount of petrol
in a tank.

Therefore he will have to fill up in Colesberg.

UNIT 3

FLOOR, ELEVATION AND DESIGN PLANS

Test Your Understanding

1.1 Two bedrooms with cupboards in each bedroom.
 2 bathrooms - one bathroom has a toilet, basin and bath, the other only has a toilet and basin. There is an open-plan lounge, dining room and kitchen.

1.2 11

1.3 The door between the kitchen and the one bedroom, the entrance to the long passage.

1.4 Bar scale: 1 cm = 1,5 m



Garage

On plan: length = 4,1 cm width = 2,5 cm

Actual size: length = 1,5 m $\times$ 4,1 = 6,15 m
 width = 1,5 m $\times$ 2,5 = 3,75 m

Store Room

On plan: length = 1,7 cm width = 1,2 cm

Actual size: length = 1,5 m $\times$ 1,7 = 2,55 m = 2,6 m
 width = 1,5 m $\times$ 1,2 = 1,8 m

1.5 Bedroom

On plan: length = 2,9 cm width = 2,6 cm

Actual size: length = $1,5 \text{ m} \times 2,9 = 4,35 \text{ m}$
width = $1,5 \text{ m} \times 2,6 = 3,9 \text{ m}$

→ Area of floor = $4,35 \text{ m} \times 3,9 \text{ m}$
= $16,965 \text{ m}^2$

Cupboard

On plan: length = 1,9 cm width = 0,5 cm

Actual size: length = $1,5 \text{ m} \times 1,9 = 2,85 \text{ m}$
width = $1,5 \text{ m} \times 0,5 = 0,75 \text{ m}$

→ Area of cupboard = $2,85 \text{ m} \times 0,75 \text{ m} = 2,138 \text{ m}^2$

∴ Area of floor to be carpeted

= total floor - cupboard

= $16,965 \text{ m}^2 - 2,138 \text{ m}^2$

= $14,827 \text{ m}^2$

≈ $14,8 \text{ m}^2$

round up
to 15 m^2

Cost of carpeting: $1,5 \text{ m}^2$ costs R129,00

→ $1 \text{ m}^2 = (\text{R}129,00 \div 1,5)$
= R86,00

∴ $15 \text{ m}^2 = \text{R}86,00 \times 15 \text{ m}^2$
= R1 290,00

1.6 Learner's own design of house.

2.1 The floor plan provides dimensions for each of the walls and rooms inside the change room which are not visible from outside or on any of the elevation plans. The floor plan also provides an indication of the location of fittings (toilets, basins, benches, etc.).

2.2 $0,8 \text{ cm} : 1 \text{ m}$

$0,8 \text{ cm} : 100 \text{ cm}$

$\frac{0,8}{0,8} \cdot \frac{100}{0,8}$

1 : 125

∴ Length of bathroom on plan = 1,7 cm

∴ Actual bathroom length = $1,7 \text{ cm} \times 125$
= 212,5 cm
≈ 2,1 m



Width of bathroom = $1,3 \text{ cm} \times 125$
= 162,5 cm
≈ 1,6 m

2.3 Yes, there is plenty of space for changing. There are plenty of windows to let air and light into the building. There are options for shower or bath. Showers are separate cubicles for privacy. Basins are provided next to toilets to encourage hand washing. Circular bench provides more space for sport bags/clothing. Toilets are not visible from door.

UNIT 4

INSTRUCTIONS AND ASSEMBLY DIAGRAMS

Test Your Understanding



1. Step 1: Take the large tongue of the tie (A) and cross it over the top of the small tongue of the tie (B). Ensure that tongue A is larger in length than tongue B.
- Step 2: Cross tongue A back around tongue B.
- Step 3: Lift tongue A and bring it back in front and across tongue B.
- Step 4: Take tongue A back around tongue B, but this time, push tongue A up between the left and right-hand collar of the shirt.
- Step 5: Flap tongue A down and through the knot created. Tongue A should lie over tongue B and also Tongue A should be longer in length than tongue B.

2.1 18 bolts

- 2.2 ▶ Diagram B: Since you put 4 bolts into the front panel, one would assume that you would also need 4 bolts to attach the back panel. The diagram only shows 3 bolts for the back panel.

▶ Diagram A: Since the right roof panel requires 4 bolts, one would again assume that you would also need 4 bolts to attach the left roof panel. The diagram only shows 3 bolts for the left roof panel.

2.3 Step 1: Diagram B - Instruction 3

Step 2: Diagram D - Instruction 2

Step 3: Diagram A - Instruction 4

Step 4: Diagram C - Instruction 1

3.1 1 Backrest height adjustment knob

3.2 Mechanism Screw

3.3 Step 1:

- ▶ Fit five casters (1) to the base (2) of the chair.
- ▶ Attach the seat post (3) to the top of the base (2) of the chair.
- ▶ Attach the seat post cover (4) to the top of the seat post (3).

Step 2:

- ▶ Take the seat (8) and turn it upside down.
- ▶ Place the mechanism (6) on top of the upside seat (8) and attach using six mechanism screws (5).

Step 3:

- ▶ Take the backrest (7) and turn it upside down; resting it on the ridge of the top of the backrest.
- ▶ Attach the plastic stopper (11) to the top of the backrest (7).
- ▶ Attach the mechanism plastic bellow (10) to the top of the plastic stopper (11).
- ▶ Take the seat (8) and mechanism (6) assembled in Step 2 and attach to the left of the mechanism plastic bellow (10).

Step 4:

- ▶ Take the base (2) and seat post and cover (3 and 4) in Step 1 and attach to the top of the mechanism (6).
- ▶ Turn the chair around, so that the casters (5) are on the floor.
- ▶ The chair is now assembled and ready to use!

UNIT 5 MODELS

Test Your Understanding



$$\begin{aligned} 1.1 \text{ Volume} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 27,5 \text{ cm} \times 15 \text{ cm} \times 11,9 \text{ cm} \\ &= 4\,908,75 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} 1.2 \text{ Number of shoe boxes: } &118 \text{ cm} \div 11,9 \text{ cm} = 9,915 \\ \therefore \text{ You will only fit } &9 \text{ shoe boxes on the shelf.} \end{aligned}$$

2. Practical method:

- ▶ Number of boxes across the length
 $= 4,6 \div 0,23 \text{ m}$
 $= 20 \text{ boxes}$
- ▶ Number of boxes across the breadth
 $= 2 \text{ m} \div 0,2 \text{ m}$
 $= 10 \text{ boxes}$
- ▶ Number of boxes to fit the height
 $= 1,8 \div 0,12 \text{ m}$
 $= 15 \text{ boxes}$

$$\begin{aligned} \therefore \text{ Total number of boxes} &= 20 \times 10 \times 15 \\ &= 3\,000 \text{ boxes} \end{aligned}$$

Mathematical method:

$$\begin{aligned} \text{▶ Volume of display} &= 4,6 \text{ m} \times 2 \text{ m} \times 1,8 \text{ m} \\ &= 16,56 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{▶ Volume of box} &= 0,23 \times 0,2 \text{ m} \times 0,12 \text{ m} \\ &= 0,00552 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total number of boxes} &= \frac{16,56 \text{ m}^3}{0,00552 \text{ m}^3} \\ &= 3\,000 \text{ boxes} \end{aligned}$$

The answers are the same for both methods, because there are no empty spaces to be filled i.e. the boxes fit exactly into the display.

$$3.1 \text{ Length} = 967 \text{ mm}$$

$$\begin{aligned} \therefore \text{ Extra 20\% length} &= \frac{20}{100} \times 967 \text{ mm} \\ &= 193,4 \text{ mm} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total length} &= 967 + 193,4 \text{ mm} \\ &= 1\,160,4 \text{ mm} \\ &= 116,04 \text{ cm} \end{aligned}$$

$$\text{Height (with stand)} = 626,1 \text{ mm}$$

You must work with the maximum height in order for the TV to fit into the box.

$$\begin{aligned} \therefore \text{ Extra 20\% height} &= \frac{20}{100} \times 626,1 \text{ mm} \\ &= 125,22 \text{ mm} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total height} &= 626,1 + 125,22 \\ &= 751,32 \text{ mm} \\ &= 75,13 \text{ cm} \end{aligned}$$

$$\text{Width} = 199,9 \text{ mm}$$



You must work with the maximum width in order for the TV to fit into the box.

$$\begin{aligned} \therefore \text{ Extra 20\% width} &= \frac{20}{100} \times 199,9 \text{ mm} \\ &= 39,98 \text{ mm} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total width} &= 199,9 + 39,98 \text{ mm} \\ &= 239,88 \text{ mm} \\ &= 23,99 \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Dimensions of the box:} \\ 116,04 \text{ cm} \times 23,99 \text{ cm} \times 75,13 \text{ cm} \end{aligned}$$

$$\begin{aligned} 3.2 \text{ TSA} &= (2 \times l \times b) + (2 \times l \times h) + (2 \times b \times h) \\ &= (2 \times 116,04 \times 23,99) + (2 \times 116,04 \times 75,13) \\ &\quad + (2 \times 23,99 \times 75,13) \\ &= 5\,567,5992 + 17\,436,1704 + 3\,604,7374 \\ &= 26\,608,51 \text{ cm}^2 \end{aligned}$$

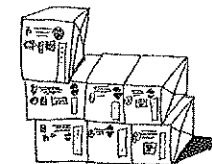
$$\begin{aligned} \therefore \text{ Cost of cardboard} &= 26\,608,51 \times R0,02 \\ &= R532,17 \end{aligned}$$



$$3.3 \text{ ▶ Length of TV box} = 116,04 \text{ cm}$$

$$\text{Length of store-room} = 3 \text{ m} = 3 \times 100 = 300 \text{ cm}$$

$$\begin{aligned} \therefore \text{ Number of boxes across length of store-room} \\ &= \frac{300 \text{ cm}}{116,04 \text{ cm}} \\ &= 2,59 \text{ cm} \\ &\approx 2 \text{ boxes} \end{aligned}$$



$$\text{▶ Width of TV box} = 23,99 \text{ cm}$$

$$\text{Width of store-room} = 2,5 \text{ m} = 2,5 \times 100 = 250 \text{ cm}$$

$$\begin{aligned} \therefore \text{ Number of boxes across width of store-room} \\ &= \frac{250 \text{ cm}}{23,99 \text{ cm}} \\ &= 10,42 \text{ cm} \\ &\approx 10 \text{ boxes} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total number of boxes that will fit in store-room is} \\ 2 \times 10 = 20 \text{ boxes} \end{aligned}$$

$$4. \text{ Area of office} = 18,5 \text{ m} \times 12 \text{ m} = 222 \text{ m}^2$$

$$\begin{aligned} \text{Open space in office} &= \frac{15}{100} \times 222 \text{ m}^2 \\ &= 33,3 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{ Area of office available for hubs} \\ &= 222 - 33,3 \text{ m}^2 = 188,7 \text{ m}^2 \end{aligned}$$

Area of circular table

$$\begin{aligned} &= \pi r^2 \\ &= 3,142 \times (0,8)^2 \\ &= 2,01 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Radius} &= \text{diameter} \div 2 \\ &= 1,6 \text{ m} \div 2 \\ &= 0,8 \text{ m} \end{aligned}$$

$$\text{Area of 3 chairs} = 3 \times 1 \text{ m}^2 = 3 \text{ m}^2$$

$$\text{Total area of a hub} = 2,01 \text{ m}^2 + 3 \text{ m}^2 = 5,01 \text{ m}^2$$

$$\begin{aligned} \therefore \text{ Number of hubs that can fit in available office space} \\ &= \frac{188,07 \text{ m}^2}{5,01 \text{ m}^2} \\ &= 37,66 \\ &\approx 37 \text{ hubs} \end{aligned}$$



REMEMBER!
 Round down, otherwise
 38 hubs won't fit!

MODULE 6

DATA HANDLING

UNIT 1

DEVELOPING QUESTIONS

Test Your Understanding

Possible sub-questions which would enable meaningful data collection:

1. What type of vehicle was the victim driving?
 2. Was the vehicle roadworthy?
 3. Did the victim have a valid driver's licence?
 4. What was the age of the victim?
 5. What time of day did the accident occur?
 6. Were there any obvious obstructions in the road that led to the accident?
 7. Did the victim die on impact or later in hospital?
 8. Were there any other passengers in the vehicle?
 9. Was the victim intoxicated?
 10. Was the driver of the other vehicle intoxicated?
1. Which provinces in South Africa have the lowest matric pass rate?
 2. How many qualified teachers are there per province?
 3. How many academic/teaching school days are there?
 4. Do all schools have the required textbooks and teaching resources?
 5. How long is the school day?
 6. Are schools supported by members of the community?
 7. Are matrics educated in terms of different study methods?
 8. Are matrics given the opportunity to study at home?
 9. What is the government education budget and how is it being spent?
 10. How many teachers are being trained at universities per year?

1. How much did you spend on your dress/suit?
2. How much did you spend on your shoes?
3. How much did you spend on your accessories e.g. jewellery?
4. How much did you spend on your hair, nails and skin?
5. How much did you spend on your transport to and from the venue?
6. Were there any other additional costs?
7. What percentage of your family's total income was spent on the total costs of the Matric Dance?
8. On a scale of 1 - 10, where 1 is bad and 10 is excellent; rate the Matric Dance experience.
9. In hindsight, would you have spent less money on the Matric Dance?
10. If you were given the money that you spent on your Matric Dance, how would you have used it in another way?

UNIT 2

COLLECTING DATA

Test Your Understanding

- 1.1 Observation. This information can easily be observed and recorded by sitting at the different shopping centre entrances, without interacting with the shoppers.
- 1.2 Survey using a questionnaire. The information needed can be gathered simultaneously by handing out a questionnaire to all the delegates at the end of the workshop. Therefore, the information can be collected as a whole, in a fast and easy manner.
- 1.3 Interview. The information needed is complex and would need an in-depth discussion with the researcher.

2. A possible questionnaire that Penny could use to do market research for her coffee shop.

Coffee Shop Questionnaire for Local Shopping Centre

Please tick the appropriate answer:

1. What is your gender?	☐ Male	☐ Female
2. In which age group do you belong?	☐ 0 - 20 yrs ☐ 21 - 30 yrs ☐ 31 - 40 yrs	☐ 41 - 50 yrs ☐ 51 - 60 yrs ☐ 60+ yrs
3. How often do you shop here at your local shopping centre?	☐ very seldom ☐ once a week ☐ twice a week	☐ 3 times a week ☐ 4 or more times a week
4. How often do you usually go for coffee or a bite to eat at any coffee shop?	☐ very seldom ☐ once a week ☐ twice a week	☐ 3+ times a week
5. What do you usually drink at a coffee shop? (You may tick more than one)	☐ Coffee/Cappuccino/Latte ☐ Ceylon Tea ☐ Rooibos Tea ☐ Fruit Juice ☐ Coke/Sprite/Fanta/Cream Soda ☐ Milkshake ☐ Hot Chocolate ☐ Other: Please specify	
6. What food would appeal to you at a coffee shop? (You may tick more than one)	☐ Toasted sandwiches ☐ Burger ☐ Pita ☐ Salad ☐ Other: Please specify	☐ Pancakes ☐ Home-made food (i.e. pie, bobotie, lasagne)
7. What sweet delights would appeal to you at a coffee shop? (You may tick more than one)	☐ Cakes ☐ Muffins ☐ Tarts ☐ Scones ☐ Brownies	☐ Biscuits ☐ Warm desserts (e.g. malva pudding, apple pie)

- 3.1 All the Two Oceans Marathon runners.
- 3.2 The group of 100 athletes.
4. The poll excluded all people who do not have a radio, who do not have e-mail and who do not listen to that particular radio station. This excludes a large majority of the South African population. As such, the information collected would not lead to accurate deductions being made about the outcome of the election.
5. No - the data is not representative. The owner has interviewed more white people than black people even though there are significantly more black people in the company. The employer has also only interviewed 2 Indians and no coloured people and so the data is biased towards white and black people. Finally, the owner has interviewed more males than females even though there are more females in the company.

UNIT 3 CLASSIFYING AND ORGANISING DATA

Test Your Understanding

- 1.1 Numerical, discrete data
- 1.2 Numerical, continuous data
- 1.3 Categorical data
- 1.4 Numerical, continuous data
- 1.5 Categorical data
- 1.6 Numerical, discrete data



2.1

Frequency Table of Perfume Classification by Shoppers

Small	Tally	Frequency
Sweet		3
Fresh		2
Musky		7
Citrus		3
Total		15

$$2.2 \quad \% \text{ Musky} = \frac{7}{15} \times 100 = 46,67\%$$

3.1

FREQUENCY TABLE 1 - HEIGHT DISTRIBUTION BY GENDER

Height Category	Females		Males	
	Tally	No. of People (Frequency)	Tally	No. of People (Frequency)
1,5 - 1,59 m		2		1
1,6 - 1,69 m		7		3
1,7 - 1,79 m		4		10
1,8 - 1,89 m		2		7
1,9 - 1,99 m		0		3
≥ 2 m		0		1
Total		15		25

3.2

FREQUENCY TABLE 2 - WEIGHT DISTRIBUTION BY GENDER

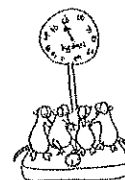
Weight Category	Females		Males	
	Tally	No. of People (Frequency)	Tally	No. of People (Frequency)
50 - 59 kg		5		0
60 - 69 kg		5		1
70 - 79 kg		2		11
80 - 89 kg		2		4
90 - 99 kg		1		5
100 - 109 kg		0		0
110 - 119 kg		0		2
≥ 120 kg		0		2
Total		15		25

3.3.1 1,6 - 1,69 m

3.3.2 3 men

$$3.3.3 \quad 5 + 5 = 10 \text{ women}$$

$$3.3.4 \quad \% \text{ men} = \frac{2+2}{25} \times 100 = \frac{4}{25} \times 100 = 16\%$$



$$3.3.5 \quad \text{BMI} = \frac{75 \text{ kg}}{(1,62 \text{ m})^2} = 28,58 \text{ kg/m}^2$$

UNIT 4 SUMMARISING DATA

EXERCISE 1

$$1.1 \quad \text{Mean} = \frac{747}{12} = 62,25$$

Ordered set:
24 ; 28 ; 32 ; 48 ; 53 ; 53 ; 55 ; 67 ;
77 ; 100 ; 101 ; 109

$$\therefore \text{Median} = \frac{53+55}{2} = 54$$

Mode = 53

$$\text{Mean} = \frac{\text{sum of all data values}}{\text{total no. of data values}}$$

Remember to arrange your data into ascending order (smallest to biggest) in order to find the median.



$$1.2 \quad \text{Mean} = \frac{710\,688}{5} = 142\,137,6$$

Ordered set:
100 421 ; 127 034 ; 153 003 ;
160 421 ; 169 809
 $\therefore$ Median = 153 003

Mode
= 100 421 ; 127 034 ; 153 003 ; 160 421 ; 169 809



All data points occur once in the data set
 $\therefore$ the mode is every value in the data set.

$$1.3 \quad \text{Mean} = \frac{497,54}{8} = 62,19$$

Ordered set: 23,04 ; 58,7 ; 61,98 ; 66,01 ; 66,01 ;
66,01 ; 72,39 ; 83,4
 $\therefore$ Median = $\frac{66,01+66,01}{2} = 66,01$

Mode = 66,01

$$2.1 \quad \text{Mean} = \frac{(2\text{h}:00 + 2\text{h}:05 + 2\text{h}:00 + 2\text{h}:00 + 1\text{h}:58 + 2\text{h}:01 + 2\text{h}:45) \div 7}{7} = \frac{120+125+120+120+118+121+165}{7} = \frac{889 \text{ min}}{7} = 127 \text{ min} = 2\text{h}:07 \text{ min}$$

2.2 Ordered set:
1h:58 2h:00 2h:00 2h:00 2h:01 2h:05 2h:45
 $\therefore$ Median = 2h:00

2.3 Mode = 2h:00

2.4 The median or mode is the best indication of her running time, as they are not skewed by the outlier of 2h:45 min.

2.5 Perhaps Lizelle was sick; or tired; or had an injury.

- 3.1 Mean, because it is useful in science to compare many data values.
- 3.2 Median, because it is not skewed by the outlier of the one millionaire in town.

EXERCISE 2

- 1.1 > Range = Highest value - Lowest value
= 25 - 5
= 20

> Ordered set: 5; 7; 8; 11; 14; 15; 16; 19; 21; 25
Quartiles: Q_1 Q_2 Q_3

$$\therefore Q_2 = \frac{14+15}{2} = 14,5$$

- > Interquartile Range = $Q_3 - Q_1$
= 19 - 8
= 11

- 1.2 > Range = 35 - 4
= 31

> Ordered set: 4; 8; 11; 12; 15; 16; 17; 21; 25; 35
Quartiles: Q_1 Q_2 Q_3

$$\therefore Q_2 = \frac{15+16}{2} = 15,5$$

- > Interquartile Range = 21 - 11
= 10

Remember, you need to arrange the data in ascending order, before finding quartiles.

- 1.3 > Range = 50 - 11
= 39

> Ordered set: 11; 14; 17; 22; 23; 29; 29; 31; 32;
35; 41; 44; 45; 47; 50
 Q_1 Q_2 Q_3

- > Interquartile Range = 44 - 22
= 22

2.1 88 2.2 % = $\frac{126}{130} \times 100$
= 96,92%

2.3 112

- 2.4 Min. value = 50
 Q_1 = 100
 Q_2 = 106
 Q_3 = 116
Max. value = 126



- 2.5 > Paper 1: Range = 120 - 88 = 32
Interquartile Range = 118 - 104 = 14
> Paper 2: Range = 126 - 50 = 76
Interquartile Range = 116 - 100 = 16
> 50% of the Paper 1 results fell between 104 and 118 out of 130.
> 50% of the Paper 2 results fell between 100 and 116 out of 130.
> The spread of marks was wider for Paper 2 than for Paper 1, as the range was larger for Paper 2.

- 2.6 The interquartile range is a better measure of spread; as 50% of the data values are represented; and the IQR is not affected by outliers.

- 3.1 5 7 8 11 14 15 16 19 21 25
 $\therefore$ 30th percentile = 8 70th percentile = 16

- 3.2 1 1 3 4 5 5 5 7 8 8 10 11 13 14 16 16 18 19 20 25
 $\therefore$ 30th percentile = 5 70th percentile = 14

Each number represents 5%; as $\frac{100}{20 \text{ numbers}} = 5\%$

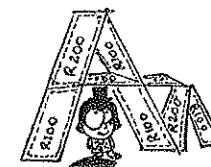
4. BMI = $\frac{50 \text{ kg}}{(1,5 \text{ m})^2} = 22,22 \text{ kg/m}^2$

- 5.1 75th percentile 5.2 nearly 22 (50th percentile)
5.3 18,2 to 25,6 5.4 25,6 to 28,9
5.5 less than 18,2
5.6 A - Age 4 years 5.7 B - Age 14 years and 3 mths
BMI of 18 kg/m² BMI of 21,4 kg/m²
5.8 The girls will have a similar curve but towards the teenage years girls grow more quickly and this will influence their BMI's.

Test Your Understanding



1.1 % Increase = $\frac{48,5 \text{ million} - 44,8 \text{ million}}{44,8 \text{ million}} \times 100$
= $\frac{3,7 \text{ million}}{44,8 \text{ million}} \times 100$
= $\frac{3\,700\,000}{44\,800\,000} \times 100$
= 8,26%



- 1.2 Eastern Cape

1.3 Mean Community Survey 2007
= $\frac{70,4+97,5+97,9+79,4+83,6+91,3+94,8+89,9+98,9}{9} \%$
= $\frac{803,7}{9} \%$
= 89,3%

- 1.4 63,2; 72,5; 78,1; 85,7; 86,6; 93,9; 95,7; 97,1; 98,3
 $\therefore$ Median = 86,6

- 1.5 > Range = 98,3 - 63,2
= 35,1

> Ordered set: $Q_1 = \frac{72,5+78,1}{2} = 75,3$

63,2; 72,5; 78,1; 85,7; 86,6; 93,9;

95,7; 97,1; 98,3
 Q_2

$$Q_3 = \frac{95,7+97,1}{2} = 96,4$$

- $\therefore$ IQR = 96,4 - 75,3 = 21,1

2.1 > Mean = $\frac{1624}{16} = 101,5$

> Ordered set: Paper 2
64 74 87 90 90 98 100 103 105 109
110 112 117 119 120 126

$\therefore$ Median = $\frac{103+105}{2} = 104$

- > Mode = 90

2.2 Box-and-whisker plot **A** is Paper 2; **B** is Paper 1

2.3 Min. value: 64
 Q_1 : 90
 Q_2 : 104
 Q_3 : 114
 Max. value: 126



2.4 > Range = 130 - 82
 = 48

Range
 = highest value - lowest value

> IQR = 120 - 103
 = 17

IQR = $Q_3 - Q_1$

2.5 Paper 2 (plot **A**) has a wider spread than Paper 1 (plot **B**) - the range is bigger. The middle 50% of the data (the box) is more condensed for Paper 1 than Paper 2. That means the values in the middle 50% are closer together for Paper 1 than Paper 2.

3.1.1 Vuka Secondary School

Ordered set:

49; 50; 54; 57; 67; 67; 67; 78; 78; 89; 90; 90; 95; 98

> $P = \text{Median} = \frac{67+78}{2} = 72,5\%$

> $Q = \text{Mode} = 67$

> $R = \text{Mean} = \frac{1029}{14} = 73,5\%$

Bathini High School

> $S = \text{Range} = 99 - 59 = 40$

3.1.2 Bathini High performed better in the competition. Their mean (average) was higher than that of Vuka Secondary School. Bathini also had a lower range in marks meaning that marks were less spread around the mean.

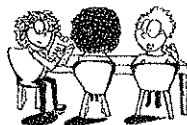
3.2.1 $75\% \times 14 = 10,5$

scores 11 - 14

3.2.2 $26\% \times 11 = 2,75$

i.e. learners 1 (59) and 2 (67) of Bathini

i.e. 4 learners of Vuka scored less than 59 (49, 50, 54, 57)



UNIT 5

REPRESENTING DATA

Test Your Understanding



1.1 R71,3 billion

1.2 Science, technology and environment

1.3 % Protection services
 = 100 - 10 - 8 - 15 - 4 - 17 - 11 - 7 - 15
 = 13%

1.4 Size of 'welfare' sector = $\frac{15}{100} \times 360^\circ = 54^\circ$

Size of sector = fraction of whole (%) $\times 360^\circ$
 (angle of revolution)



1.5 Total budget (in billions) = 118 + 48 + 93 + 50 + 254
 + 57 + 65 + 146 + 143 + 116 + 19 + 144
 = R1 253 billion

$\therefore$ Size of 'Social Protection' sector

= $\frac{\text{R144 billion}}{\text{R1 253 billion}} \times 360^\circ$
 = 41,37%

Size of sector (in degrees)
 = fraction of whole (%) $\times 360^\circ$
 = $\frac{\text{Social Protection budget}}{\text{Total budget}} \times 360^\circ$



1.6 % Increase = $\frac{\text{R254 billion} - \text{R121,1 billion}}{\text{R121,1 billion}} \times 100$
 = $\frac{\text{R132,9 billion}}{\text{R121,1 billion}} \times 100$
 = 109,74%

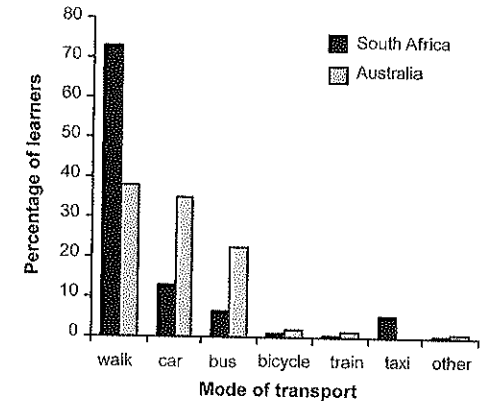
2.1 Walking

2.2 73,1% of learners walk
 $\therefore$ 73,1% of 1,2 million:

$1\,200\,000 \times \frac{73,1}{100} = 877\,200$ learners



2.3 MODES OF TRANSPORT USED BY LEARNERS IN SOUTH AFRICA AND AUSTRALIA



3.1 Stacked bar graph

3.2 2001

3.3 7 200

To find the middle of 2 400 and 4 800:

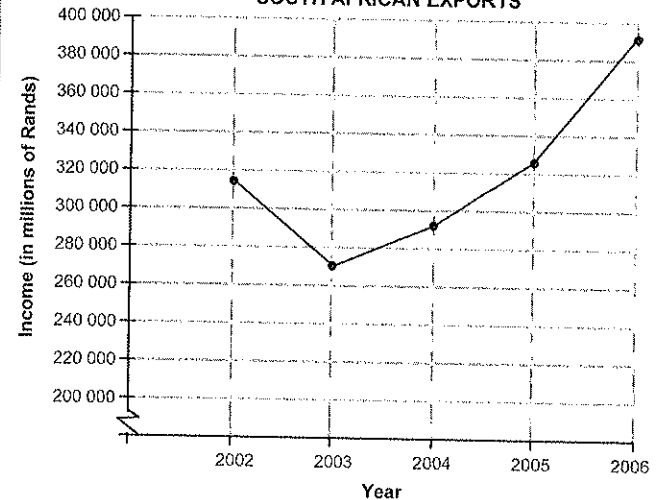
3.4 3 600 i.e. Mean = $\frac{2\,400 + 4\,800}{2} = 3\,600$

3.5 2001

4.1 Total income generated by agricultural exports
 = 25 460 + 22 670 + 22 074 + 25 458 + 26 978
 = R122 640 million

4.2 % = $\frac{22\,074}{292\,079} \times 100 = 7,56\%$

4.3 INCOME GENERATED BY TOTAL SOUTH AFRICAN EXPORTS



5.1 8 class intervals

Class intervals = categories
e.g. $0 < d \leq 20$; $20 < d \leq 30$; etc.



5.2 Total = $1 + 7 + 40 + 67 + 75 + 48 + 10 + 2$
= 250

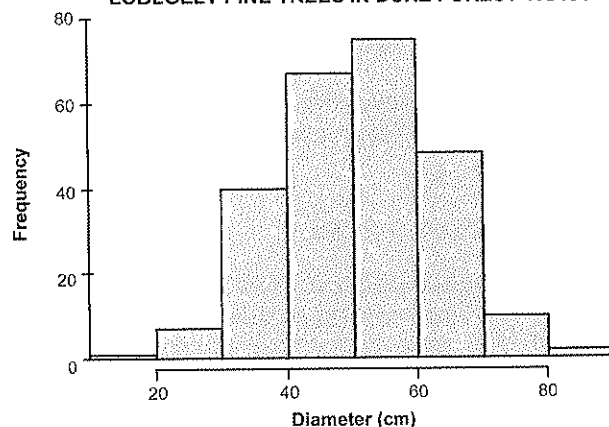
Modal class is the class interval that occurs most frequently.

5.3 Modal class: $50 < d \leq 60$

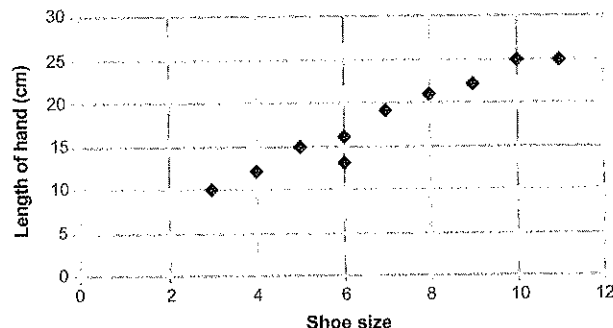


5.4 Continuous, numerical data

5.5 HISTOGRAM OF DIAMETERS OF LOBLOLLY PINE TREES IN DUKE FOREST TRACT



6.1 SCATTER PLOT OF SHOE SIZES VS LENGTH OF HANDS



6.2 Yes, there is a strong positive linear correlation.

UNIT 6

INTERPRETING AND ANALYSING DATA

Test Your Understanding



1.1 The % population growth decreased from approximately 3,4% in 1961 to approximately 2,2% in 1968.

1.2 Increase = $\frac{1,73}{100} \times 44\,000\,000$
= 761 200

$\therefore$ Total population for 2009 = $44\,000\,000 + 761\,200$
= 44 761 200

OR Total population for 2009
= 101,73% of the population in 2008
= $\frac{101,73}{100} \times 44\,000\,000$
= 44 761 200



1.3 Possible reasons could include:

- ▶ There could possibly have been famine, disease, drought, civil unrest or war in developing countries that could have decreased population growth.
- ▶ The generation of women who were of child-bearing age may have decided not to have children; or decided to have fewer children than the previous generation; and/or decided to focus on their careers and rather have children later in life.

2. ▶ He has split the causes of smoking-related deaths into categories and there is no value to determine the actual number of deaths from smoking.
- ▶ He cleverly arranged the bars from biggest to smallest (tallest to shortest) causing the reader to think that there is a decrease in deaths.
- ▶ There are no figures relating to actual smoking-related deaths. The percentages indicate the part each cause forms of all smoking-related deaths.

3.1 Males who started = $8\,325 + 449$
= 8 774

A42

3.2 Females who started = total starters - male starters
= $10\,700 - 8\,774$
= 1 926

3.3 No. of females who finished = 94,2% of 1 926
= 1 814,29
= 1 814 females

3.4 % of males who dropped out
= $(449 \div 8\,774) \times 100$
= 5,1%

$\therefore$ A marginally higher percentage of females dropped out of the race - i.e. 5,8% compared to 5,1%.

MODULE 7

PROBABILITY

UNIT 1

EXPRESSIONS OF PROBABILITY

Test Your Understanding



1. $\frac{9}{11}$; 90% ; 0,87 ; $\frac{7}{8}$; 0,25 ; 80%
= 81,82% ; 90% ; 87% ; 87,5% ; 25% ; 80%
 $\therefore$ Most probable (likely) to most improbable (unlikely)
= 90% ; 87,5% ; 87% ; 81,82% ; 80% ; 25%
= 90% ; $\frac{7}{8}$; 0,87 ; $\frac{9}{11}$; 80% ; 0,25

Always remember to answer the question using the original values given (e.g. $\frac{9}{11}$) and **not** the equivalent values that you worked out (e.g. 81,82%).

2.1 $P(\text{banana is chosen}) = \frac{2}{10} = \frac{1}{5} = 20\%$

$P(\text{event}) = \frac{\text{no. of favourable outcomes}}{\text{total no. of possible outcomes}}$

2.2 $P(\text{pear is chosen}) = \frac{5}{10} = \frac{1}{2} = 50\%$

2.3 $P(\text{banana or apple is chosen}) = \frac{2+3}{10} = \frac{5}{10} = \frac{1}{2} = 50\%$



2.4 $P(\text{no apples are chosen}) = P(\text{banana or pear is chosen}) = \frac{2+5}{10} = \frac{7}{10} = 70\%$

3.1 $P(\text{choosing number 5}) = \frac{1}{9}$

3.2 Even numbers = 2, 4, 6, 8, 10
 $\rightarrow P(\text{choosing even number}) = \frac{5}{9}$

4.1.1 'Have another turn'

4.1.2 None
'Equally likely' means there is a 50-50% chance of an event occurring. $\therefore$ we would need $\frac{4}{8}$ sectors allocated to 1 event.



4.1.3 'Collect R500'

4.2.1 $\frac{2}{8} = \frac{1}{4} (= 25\%)$

Remember to simplify the fraction or far as possible!

4.2.2 $\frac{3}{8} (= 37,5\%)$



4.2.3 $\frac{0}{8} = 0 (= 0\%)$

Remember a standard deck of cards has 52 cards!

5.1 $\frac{1}{52}$

5.2 $\frac{13}{52} = \frac{1}{4}$

*4 suits in a deck: hearts, diamonds, spades, clubs
 $\therefore \frac{52}{4} = 13$ cards per suit.*

5.3 $\frac{4}{52} = \frac{1}{13}$

5.4 $\frac{51}{52}$

'Not a King of Clubs' implies every other card in the deck $\therefore 52 - 1$ card = 51 cards.

UNIT

2

PREDICTION

Test Your Understanding



1.1 $P(\text{helped skin}) = \frac{37}{50} = 74\%$

1.2 $P(\text{made no difference}) = \frac{13}{50} = 26\%$

Since 26% of the people did not find the test useful, the cream is probably not very effective, i.e. 26% means that 26 people out of every 100 people who use the cream might be unhappy. Based on this, the manufacturers could receive a great many complaints if hundreds of people buy the cream.

1.3 Number of people who might be unhappy
 $= P(\text{unhappy}) \times \text{no. of people}$
 $= \frac{13}{50} \times 500$
 $= 130$

OR

Number of people who might be unhappy
 $= P(\text{unhappy}) \times \text{no. of people}$
 $= 26\% \times 500$
 $= 130$



2.1.1 $P(\text{Mint Fresh}) = \frac{1}{5} = 20\%$

2.1.2 27

2.1.3 $\frac{10}{150} = \frac{1}{15} = 6,67\%$

2.2.1 $P(\text{Caramel Cup}) = \frac{3}{15} = \frac{1}{5} = 20\%$

2.2.2 $P(\text{Nutty Delight}) = \frac{5}{14} = 35,71\%$



Remember that the first potential customer already picked and ate a chocolate, so now the tester plate has $15 - 1 = 14$ chocolates left.

2.2.3 $P(\text{Caramel Cup}) = \frac{2}{13} = 15,38\%$

Remember that the first customer already picked a Caramel Cup $\therefore$ no. of Caramel Cups left = $3 - 1 = 2$ Caramel Cups. Also, now that the first and second customers have eaten their 2 tester chocolates, there are now $15 - 2 = 13$ chocolate testers on the plate.



3.1 No. of roadworthy cars

$= \text{Total no. of cars on road} - \text{unroadworthy cars}$
 $= 9\,168\,000 - 375\,000$
 $= 8\,793\,000$

$\therefore P(\text{roadworthy car}) = \frac{8\,793\,000}{9\,168\,000} = \frac{2\,931}{3\,056} = 95,91\%$

3.2 $P(\text{unroadworthy cars}) = \frac{375\,000}{9\,168\,000} = \frac{125}{3\,056} = 4,09\%$

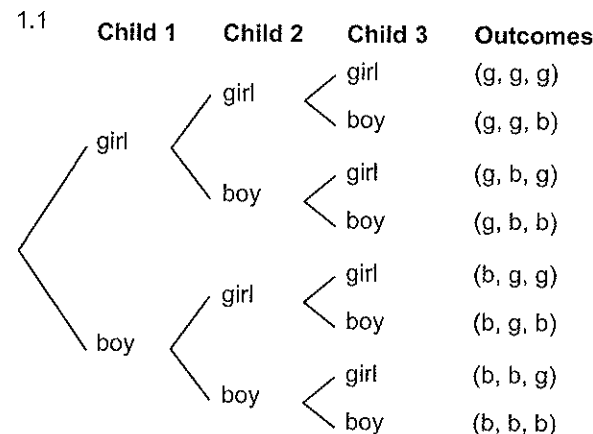
$\therefore$ No. of cars expected to be unroadworthy
 $= 4,09\% \text{ of } 30 \text{ cars pulled over}$
 $= \frac{4,09}{100} \times 30$
 $= 1,227 \text{ cars} \approx 2 \text{ cars}$

UNIT

3

REPRESENTATIONS FOR DETERMINING POSSIBLE OUTCOMES

Test Your Understanding



1.2 GGG; GGB; GBG; GBB; BGG; BGB; BBG; BBB

1.3 $P(\text{GGG}) = \frac{1}{8} (= 12,5\%)$

1.4 $P(\text{GGB}) = \frac{1}{8} (= 12,5\%)$



1.5 $P(\text{2 boys and 1 girl; any order}) = \frac{3}{8} (= 37,5\%)$

2.1

	Can swim	Cannot swim	Total
Boys	350	778	1 128
Girls	228	432	660
Total	578	1 210	1 788

2.2 1 128

2.3 % Girls who couldn't swim

$$= \frac{\text{no. of girls who couldn't swim}}{\text{total no. of girls}} \times 100$$

$$= \frac{432}{660} \times 100$$

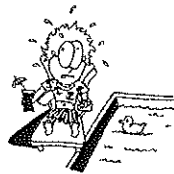
$$= 65,45\%$$

2.4 $P(\text{any child could swim})$

$$= \frac{\text{no. of children who could swim}}{\text{total no. of children surveyed}}$$

$$= \frac{578}{1 788}$$

$$= \frac{289}{894} (= 32,33\%)$$

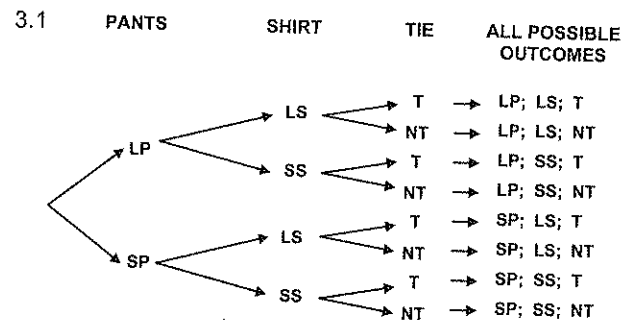


2.5 $P(\text{boy couldn't swim at school})$

$$= \frac{\text{no. of boys who couldn't swim}}{\text{total no. of boys surveyed}}$$

$$= \frac{778}{1 128}$$

$$= \frac{389}{564} (= 68,97\%)$$



3.2 $P(\text{LP; SS; NT}) = \frac{1}{8} (= 12,5\%)$

3.3 $P(\text{T}) = \frac{4}{8} = \frac{1}{2} (= 50\%)$

3.4 $P(\text{boy wearing LP; LS; T or SP; SS; NT})$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

$$= 25\%$$



4.1 $P(\text{pedestrian fatality}) = \frac{5 272}{14 630}$

$$= 36,04\%$$

4.2 $P(\text{pedestrian in Gauteng fatality}) = \frac{1389}{14 630}$

$$= 9,49\%$$

4.3 Number of expected driver fatalities in following year

$$= P(\text{drivers in WC}) \times \text{number of expected road users}$$

$$= \frac{460}{14 630} \times 17 800$$

$$= 559,67$$

$$\approx 560$$

4.4 The populations of these two provinces are much bigger than any of the other provinces. This means that there are more people using the roads in these two provinces than in any of the other provinces.

UNIT

4

EVALUATING EXPRESSIONS INVOLVING PROBABILITY

Test Your Understanding



1. $P(6) = \frac{1}{6} (= 16,67\%)$

Just because the relative frequency was $\frac{7}{20}$, it does not affect the theoretical probability of $\frac{1}{6}$.

2. No, the family has an equal chance of having a boy or girl.

i.e. $P(\text{boy}) = \frac{1}{2}$



Also, the events of the family having 3 girls previously, is independent of the next event of having another child.

3. Possible factors that would increase a sports team's chance of winning a match, could include:

- Having the best players.
- Home-ground advantage i.e. there is a psychological benefit to players as they know their sports facilities and often have more supporters.
- Certain teams favour certain weather conditions.

4. False. Just because the theoretical probability is 50% (i.e. $P(\text{T}) = \frac{1}{2}$) does not mean that the relative frequency or experimental probability will be the same. [This only occurs when the number of experimental trials increases to infinity.]

5. No. A probability statement of '98% effective' means that you are most likely not going to get wrinkles, but it is not a certain event.

50 Imam Haron Road
Claremont
7708 Cape Town
www.theanswer.co.za
Consult our website!



the **ANSWER**
SERIES...your key to exam success

Tel: (021) 671 0837
Fax: (021) 671 2546

Faxtoemail: 088 021 671 2546
email: info@theanswer.co.za
or orders@theanswer.co.za

240517 BK

PLEASE USE BLACK PEN

PRICES VALID UNTIL
30 JUNE 2018

Date: Prov:

School/Institution:

Mr/Miss/Mrs:
TITLE NAME SURNAME

Address:

Tel: (.....) Code:

Fax: (.....)

Cell:

Email:

NB
Choose
ENG or AFR

MATHS (SENIOR PHASE)

	PRICE	QUANTITY	TOTAL
		ENG AFR	(Rand)
Gr 8 Maths 2 in 1	R119		
Gr 9 Maths 2 in 1	R119		
Gr 8 Maths Companion WB 1 (Terms 1 & 2)	R119		
Gr 8 Maths Companion WB 2 (Terms 3 & 4)	R119		
Gr 8 Maths Companion AB (for both WBs)	R219		
Gr 9 Maths Companion WB 1 (Terms 1 & 2)	R119		
Gr 9 Maths Companion WB 2 (Terms 3 & 4)	R119	O	
Gr 9 Maths Companion AB (for both WBs)	R219	O	

NB: Consult us re combinations of Companion books.

MATHEMATICS (FET)

Gr 10 Maths 3 in 1	R149		
Gr 10 Maths – A Class Text LB	R199		
Gr 10 Maths – A Class Text TG	R199		
Gr 10 & 11 Patterns & Modelling	R99		
Gr 11 Maths 3 in 1	R169		
Gr 11 Maths p & a	R99		
Gr 12 Maths 2 in 1	R169		
Gr 12 Maths p & a	R99		
Varsity Maths Prep	R119		
Gr 12 EULER Maths Ruler	R20		

MATHS LITERACY

Gr 10 Maths Literacy 3 in 1	R149		
Gr 11 Maths Literacy 3 in 1	R169		
Gr 12 Maths Literacy 3 in 1	R169		

PHYSICAL SCIENCES

Gr 10 Physical Sciences 3 in 1	R149		
Gr 11 Physical Sciences 3 in 1	R169		
Gr 12 Physical Sciences 3 in 1	R169		
Gr 12 Physical Sciences 2 in 1	R169		
The GENIE Science Ruler	R20		

LIFE SCIENCES

	PRICE	ENG	AFR	TOTAL
		CAPS IEB	CAPS IEB	(Rand)
NB: Choose CAPS or IEB				
Gr 10 Life Sciences 3 in 1	R149			
Gr 11 Life Sciences 3 in 1	R169			
Gr 12 Life Sc. P.1 3 in 1	R149			
Gr 12 Life Sc. P.2 3 in 1	R149			

Notes, Questions and Answers per topic

Qs & As per topic and Exam memos

Exam papers & Answers

3 in 1 =

2 in 1 =

p & a =

KEY: ☐ In production ☐ Not available for ordering



ECONOMIC & MANAGEMENT SCIENCES

Gr 9 EMS 3 in 1 Bk & Template bk (SET)	R149		
---	------	--	--

ACCOUNTING

Gr 10 Accounting 3 in 1	R149		
Gr 11 Accounting 3 in 1	R169		
Gr 12 Accounting 3 in 1	R169		
Gr 12 Accounting 2 in 1	R169		

BUSINESS STUDIES/ECONOMICS

Gr 10 Business Studies 3 in 1	R149		
Gr 11 Business Studies 3 in 1	R169		
Gr 12 Business Studies 3 in 1	R169		
Gr 10 Economics 3 in 1	R149		
Gr 12 Economics 3 in 1	R169		

CONSUMER STUDIES

Gr 10 Consumer Studies 3 in 1	R149		
Gr 12 Consumer Studies 3 in 1	R169		

GEOGRAPHY & HISTORY

Gr 12 Geography 3 in 1	R169		
Gr 10 History 3 in 1	R149		
NEW: Gr 12 History 3 in 1 (2017)	R169		

LANGUAGES

	PRICE	QUANTITY	TOTAL
Gr 12 Afrikaans EAT 3 in 1	R169	O	
Gr 10 English HL 3 in 1	R149		
Gr 12 English HL 3 in 1	R169		
Gr 10 English FAL 3 in 1	R149		
Gr 11 English FAL 3 in 1	R169		
Gr 12 English FAL 3 in 1	R169		

Please include **PROOF OF PAYMENT** with all orders!

Date of Payment

Please tick (✓) Method of Payment

EFT ☐ Bank deposit ☐

NO. OF BOOKS less % =
COST OF BOOKS **% DISCOUNT** **DISCOUNTED**
(consult reverse) **COST OF BOOKS**

+ postage: or
REGULAR 5-7 working days **FASTMAIL** 2-4 working days **SPEED SERV.** 1-2 working days

+ rulers: =
RULERS (R18 each) **FINAL AMOUNT**

Note:
No discount or postage for rulers.

Regular: R18/book for 1st 10, R9/book thereafter, to a max. of R300.
Fastmail: R35/book to a maximum of R300.

Speed Services: R90/book (confirm telephonically)

Door-to-door services: arrange telephonically

For postage outside South Africa, contact us for a quotation.

DISCOUNT STRUCTURE

≥ 10 books: less 5% ≥ 100 books: less 15%
≥ 50 books: less 10% ≥ 300 books: less 20%

BANK DETAILS

THE ANSWER, at Standard Bank, Rondebosch,
Branch ID: 05 1001, Account number: 07 142 4903

